Computer algebra independent integration tests

`1_Algebraic_functions/1.2_Trinomial_products/1.2.1Quadratic/1.2.1.1(a+bx+cx^2)^p`

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Contents

1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

1.1 Listing of CAS systems tested

The following systems were tested at this time.

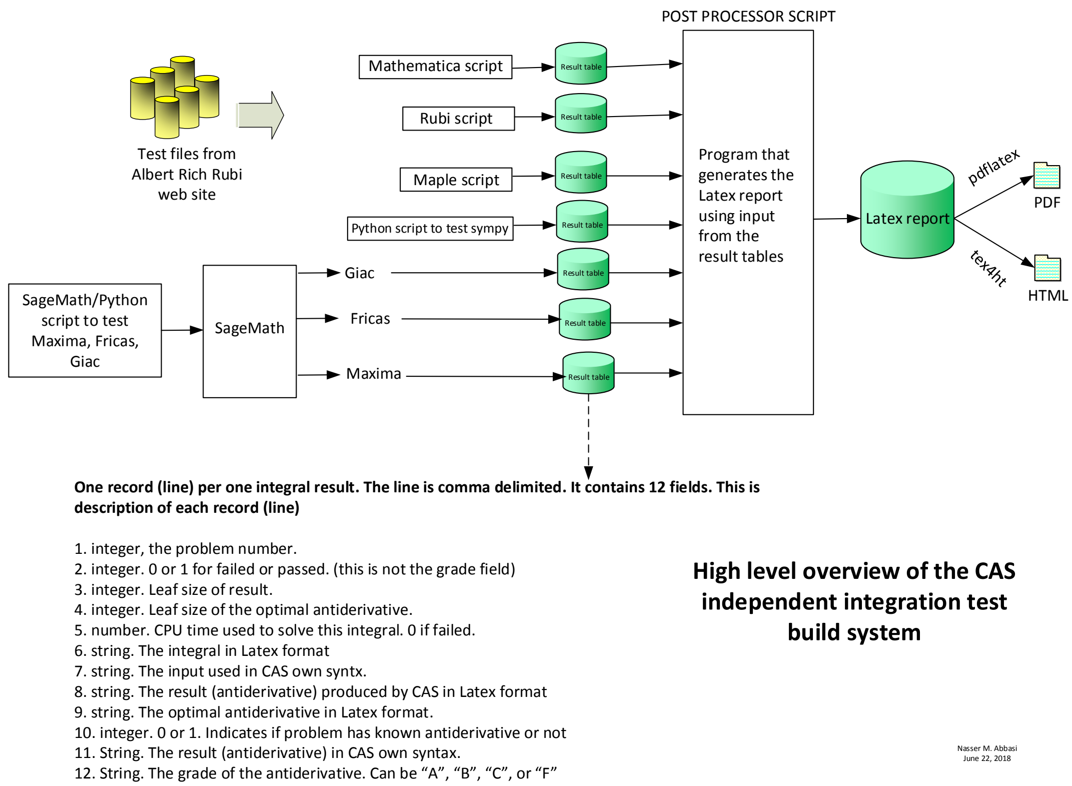
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



1.3 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python’s `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (143)	% 0. (0)
Rubi in Sympy	% 97.2 (139)	% 2.8 (4)
Mathematica	% 100. (143)	% 0. (0)
Maple	% 79.02 (113)	% 20.98 (30)
Maxima	% 65.03 (93)	% 34.97 (50)
Fricas	% 78.32 (112)	% 21.68 (31)
Sympy	% 32.87 (47)	% 67.13 (96)
Giac	% 73.43 (105)	% 26.57 (38)

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

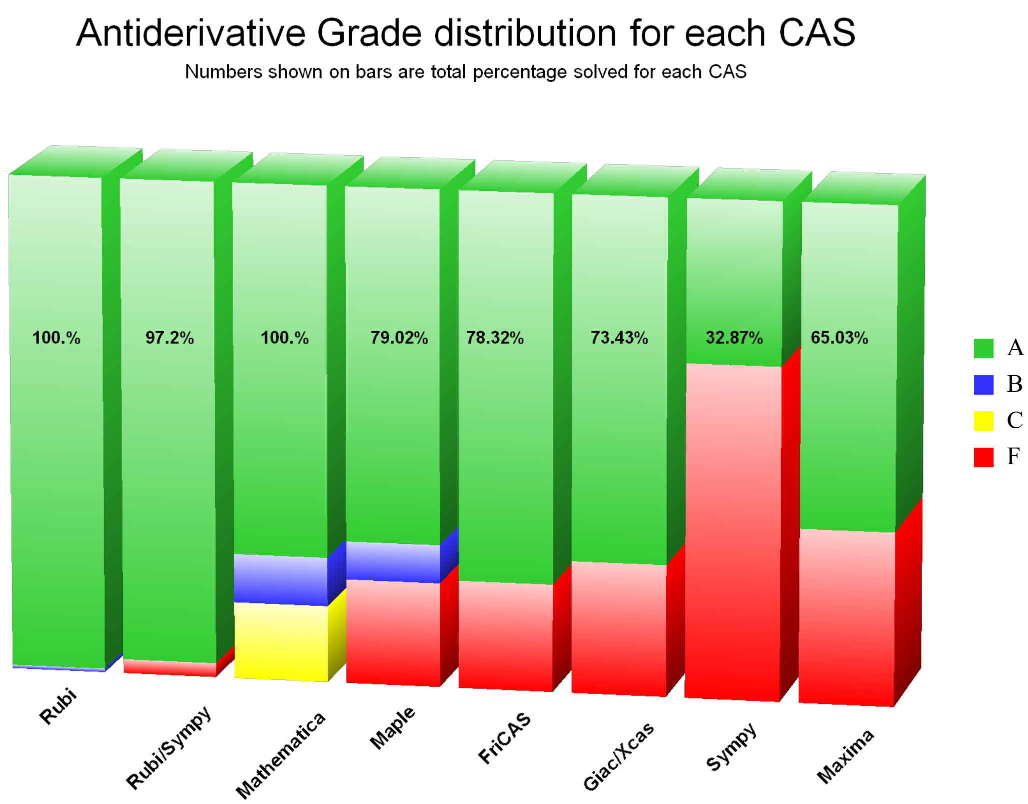
Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented.

For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

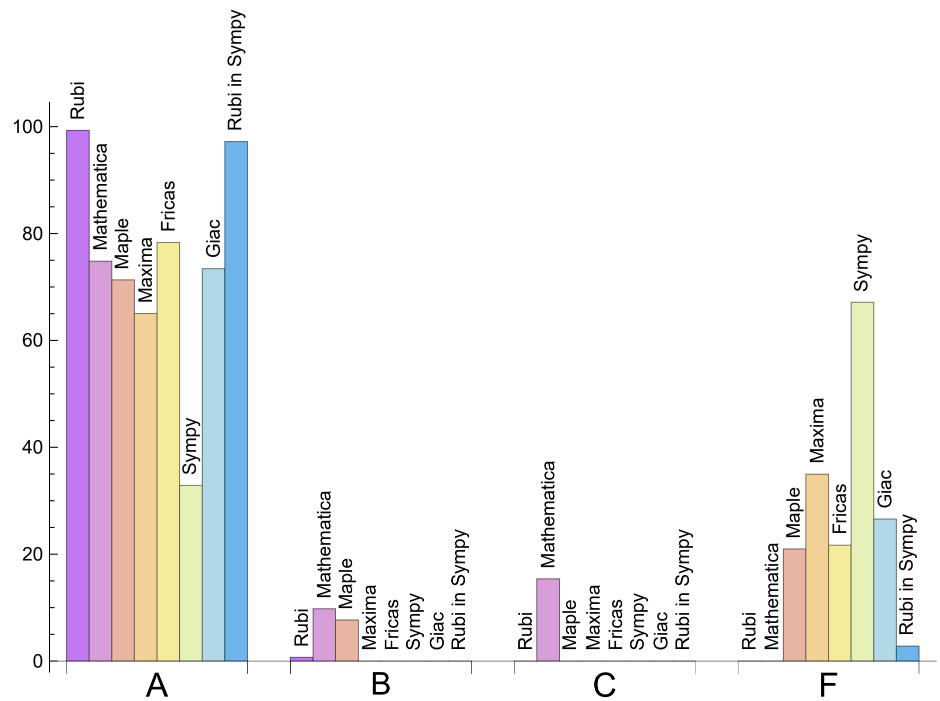
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.3	0.7	0.	0.
Rubi in Sympy	97.2	0.	0.	2.8
Mathematica	74.83	9.79	15.38	0.
Maple	71.33	7.69	0.	20.98
Maxima	65.03	0.	0.	34.97
Fricas	78.32	0.	0.	21.68
Sympy	32.87	0.	0.	67.13
Giac	73.43	0.	0.	26.57

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.14	83.41	1.02	38.	1.
Rubi in Sympy	8.89	75.39	0.97	34.	0.87
Mathematica	0.04	52.81	1.19	45.	1.
Maple	0.01	56.69	1.15	32.	0.86
Maxima	0.77	63.89	1.48	46.	1.41
Fricas	0.22	71.08	1.65	46.5	1.54
Sympy	1.74	118.6	2.39	70.	1.81
Giac	0.22	64.3	1.52	46.	1.21

1.8 list of integrals that has no closed form antiderivative

{}

1.9 list of integrals not solved by each system

Not solved by Rubi {}

Not solved by Rubi in Sympy {49, 50, 51, 52}

Not solved by Mathematica {}

Not solved by Maple {30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143}

Not solved by Maxima {1, 25, 26, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 88, 89, 90, 95, 96, 97, 125, 126, 127, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143}

Not solved by Fricas {30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 79, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143}

Not solved by Sympy {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 64, 65, 66, 67, 70, 71, 72, 73, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143}

Not solved by Giac {2, 3, 4, 5, 17, 18, 19, 20, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143}

1.10 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Rubi in Sympy {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {35, 36, 37, 38, 39}

Mathematica {102, 133, 134, 135, 136, 140, 141, 143}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Rubi in Sympy Verification phase not implemented yet.

2 detailed summary tables of results

2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	158	173	0	1	0	178	139
normalized size	1	1.	1.07	1.18	0.	0.01	0.	1.21	0.95
time (sec)	N/A	0.129	0.217	0.007	0.	0.243	0.	0.226	16.086

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	117	91	176	494	0	0	104
normalized size	1	1.	0.97	0.75	1.45	4.08	0.	0.	0.86
time (sec)	N/A	0.072	0.116	0.029	0.79	0.226	0.	0.	4.326

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	86	71	139	386	0	0	80
normalized size	1	1.	0.91	0.75	1.46	4.06	0.	0.	0.84
time (sec)	N/A	0.053	0.096	0.01	0.783	0.235	0.	0.	3.273

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	83	51	103	278	0	0	56
normalized size	1	1.	1.2	0.74	1.49	4.03	0.	0.	0.81
time (sec)	N/A	0.037	0.073	0.01	0.793	0.219	0.	0.	2.431

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	62	31	66	170	0	0	32
normalized size	1	1.	1.44	0.72	1.53	3.95	0.	0.	0.74
time (sec)	N/A	0.025	0.054	0.01	0.807	0.218	0.	0.	1.829

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	82	158	92	0	77	90
normalized size	1	1.	1.01	0.81	1.56	0.91	0.	0.76	0.89
time (sec)	N/A	0.064	0.11	0.005	0.79	0.22	0.	0.215	3.579

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	92	64	122	78	0	63	70
normalized size	1	1.	1.16	0.81	1.54	0.99	0.	0.8	0.89
time (sec)	N/A	0.047	0.061	0.006	0.798	0.216	0.	0.214	2.818

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	74	46	85	65	0	50	49
normalized size	1	1.	1.3	0.81	1.49	1.14	0.	0.88	0.86
time (sec)	N/A	0.033	0.071	0.004	0.794	0.21	0.	0.216	2.184

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	72	28	49	51	0	36	29
normalized size	1	1.	2.06	0.8	1.4	1.46	0.	1.03	0.83
time (sec)	N/A	0.023	0.033	0.004	0.804	0.212	0.	0.211	1.726

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	45	28	49	47	0	34	26
normalized size	1	1.	1.29	0.8	1.4	1.34	0.	0.97	0.74
time (sec)	N/A	0.024	0.043	0.005	0.799	0.213	0.	0.209	1.689

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	72	28	49	51	0	36	29
normalized size	1	1.	2.06	0.8	1.4	1.46	0.	1.03	0.83
time (sec)	N/A	0.024	0.039	0.005	0.852	0.211	0.	0.211	1.729

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	63	42	74	58	0	47	41
normalized size	1	1.	1.24	0.82	1.45	1.14	0.	0.92	0.8
time (sec)	N/A	0.027	0.069	0.005	0.795	0.231	0.	0.213	1.894

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	40	33	55	144	0	45	31
normalized size	1	1.	1.14	0.94	1.57	4.11	0.	1.29	0.89
time (sec)	N/A	0.02	0.043	0.005	0.75	0.232	0.	0.211	1.478

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	44	33	58	144	0	45	32
normalized size	1	1.	1.19	0.89	1.57	3.89	0.	1.22	0.86
time (sec)	N/A	0.02	0.044	0.005	0.727	0.215	0.	0.211	1.488

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	46	33	58	163	0	50	29
normalized size	1	1.	1.18	0.85	1.49	4.18	0.	1.28	0.74
time (sec)	N/A	0.02	0.045	0.005	0.716	0.216	0.	0.211	1.482

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	70	75	150	127	0	100	82
normalized size	1	1.	0.84	0.9	1.81	1.53	0.	1.2	0.99
time (sec)	N/A	0.052	0.066	0.006	0.739	0.217	0.	0.222	4.947

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	50	10	28	26	0	0	8
normalized size	1	1.	3.12	0.62	1.75	1.62	0.	0.	0.5
time (sec)	N/A	0.016	0.018	0.01	0.792	0.213	0.	0.	1.361

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	21	38	42	0	0	20
normalized size	1	1.	0.92	0.81	1.46	1.62	0.	0.	0.77
time (sec)	N/A	0.01	0.015	0.01	0.711	0.212	0.	0.	1.279

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	36	42	74	127	0	0	42
normalized size	1	1.	0.68	0.79	1.4	2.4	0.	0.	0.79
time (sec)	N/A	0.022	0.024	0.01	0.694	0.223	0.	0.	1.841

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	48	62	111	197	0	0	65
normalized size	1	1.	0.61	0.78	1.41	2.49	0.	0.	0.82
time (sec)	N/A	0.035	0.032	0.012	0.695	0.225	0.	0.	2.58

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	45	9	11	26	0	11	8
normalized size	1	1.	3.75	0.75	0.92	2.17	0.	0.92	0.67
time (sec)	N/A	0.014	0.016	0.005	0.778	0.216	0.	0.214	1.353

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	21	25	38	24	0	39	19
normalized size	1	1.	0.95	1.14	1.73	1.09	0.	1.77	0.86
time (sec)	N/A	0.01	0.014	0.005	0.698	0.221	0.	0.217	1.182

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	31	35	74	53	0	53	37
normalized size	1	1.	0.69	0.78	1.64	1.18	0.	1.18	0.82
time (sec)	N/A	0.02	0.023	0.004	0.69	0.216	0.	0.219	1.616

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	51	45	111	76	0	66	56
normalized size	1	1.	0.76	0.67	1.66	1.13	0.	0.99	0.84
time (sec)	N/A	0.033	0.032	0.005	0.704	0.226	0.	0.221	2.153

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	58	35	0	36	0	20	8
normalized size	1	1.	4.83	2.92	0.	3.	0.	1.67	0.67
time (sec)	N/A	0.019	0.032	0.009	0.	0.216	0.	0.221	3.317

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	45	37	0	36	0	49	20
normalized size	1	1.	1.88	1.54	0.	1.5	0.	2.04	0.83
time (sec)	N/A	0.02	0.039	0.004	0.	0.212	0.	0.222	2.541

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	38	7	11	24	0	8	5
normalized size	1	1.	3.8	0.7	1.1	2.4	0.	0.8	0.5
time (sec)	N/A	0.015	0.014	0.005	0.782	0.22	0.	0.213	1.342

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	33	14	23	23	0	24	14
normalized size	1	1.	2.06	0.88	1.44	1.44	0.	1.5	0.88
time (sec)	N/A	0.011	0.013	0.004	0.728	0.212	0.	0.212	1.191

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	37	14	23	23	0	24	14
normalized size	1	1.	2.31	0.88	1.44	1.44	0.	1.5	0.88
time (sec)	N/A	0.011	0.014	0.005	0.704	0.212	0.	0.213	1.186

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	94	0	0	0	0	0	398
normalized size	1	1.	0.21	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	1.338	0.084	0.057	0.	0.	0.	0.	41.363

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	70	0	0	0	0	0	332
normalized size	1	1.	0.18	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.932	0.057	0.044	0.	0.	0.	0.	32.401

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	44	0	0	0	0	0	264
normalized size	1	1.	0.14	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.788	0.023	0.055	0.	0.	0.	0.	23.123

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	57	0	0	0	0	0	328
normalized size	1	1.	0.15	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.922	0.049	0.084	0.	0.	0.	0.	32.462

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	90	0	0	0	0	0	400
normalized size	1	1.	0.2	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	1.063	0.097	0.142	0.	0.	0.	0.	41.483

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	842	842	94	0	0	0	0	0	731
normalized size	1	1.	0.11	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	2.02	0.07	0.058	0.	0.	0.	0.	82.292

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	781	781	70	0	0	0	0	0	663
normalized size	1	1.	0.09	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	1.829	0.053	0.054	0.	0.	0.	0.	68.578

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	715	715	46	0	0	0	0	0	595
normalized size	1	1.	0.06	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	1.665	0.026	0.047	0.	0.	0.	0.	55.449

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	773	773	57	0	0	0	0	0	660
normalized size	1	1.	0.07	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	1.835	0.046	0.087	0.	0.	0.	0.	68.937

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	838	838	90	0	0	0	0	0	728
normalized size	1	1.	0.11	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	1.994	0.099	0.144	0.	0.	0.	0.	82.681

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	94	0	0	0	0	0	109
normalized size	1	1.	0.79	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.111	0.07	0.063	0.	0.	0.	0.	17.234

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	70	0	0	0	0	0	80
normalized size	1	1.	0.78	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.078	0.064	0.056	0.	0.	0.	0.	14.028

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	70	0	0	0	0	0	78
normalized size	1	1.	0.78	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.075	0.052	0.049	0.	0.	0.	0.	13.896

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	46	0	0	0	0	0	51
normalized size	1	1.	0.79	0.	0.	0.	0.	0.	0.88
time (sec)	N/A	0.051	0.029	0.047	0.	0.	0.	0.	11.858

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	44	0	0	0	0	0	53
normalized size	1	1.	0.75	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.05	0.022	0.058	0.	0.	0.	0.	11.931

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	59	0	0	0	0	0	75
normalized size	1	1.	0.71	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.073	0.053	0.09	0.	0.	0.	0.	13.592

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	90	0	0	0	0	0	109
normalized size	1	1.	0.78	0.	0.	0.	0.	0.	0.95
time (sec)	N/A	0.101	0.105	0.153	0.	0.	0.	0.	16.242

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	114	0	0	0	0	0	139
normalized size	1	1.	0.78	0.	0.	0.	0.	0.	0.95
time (sec)	N/A	0.13	0.144	0.1	0.	0.	0.	0.	19.614

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	45	0	0	0	0	0	42
normalized size	1	1.	0.82	0.	0.	0.	0.	0.	0.76
time (sec)	N/A	0.032	0.041	0.044	0.	0.	0.	0.	2.753

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	44	58	1	49	58	0
normalized size	1	1.	1.	0.86	1.14	0.02	0.96	1.14	0.
time (sec)	N/A	0.044	0.003	0.001	0.701	0.185	0.107	0.209	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	42	1	32	42	0
normalized size	1	1.	1.	0.91	1.2	0.03	0.91	1.2	0.
time (sec)	N/A	0.029	0.002	0.002	0.694	0.187	0.098	0.208	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	22	28	1	22	28	0
normalized size	1	1.	1.	0.88	1.12	0.04	0.88	1.12	0.
time (sec)	N/A	0.019	0.002	0.002	0.704	0.191	0.088	0.21	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	14	1	8	14	0
normalized size	1	1.	1.	0.92	1.17	0.08	0.67	1.17	0.
time (sec)	N/A	0.008	0.	0.001	0.698	0.19	0.067	0.209	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	16	0	1	53	20	22
normalized size	1	1.	1.	0.67	0.	0.04	2.21	0.83	0.92
time (sec)	N/A	0.019	0.008	0.005	0.	0.221	0.29	0.209	2.447

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	36	0	1	78	47	36
normalized size	1	1.	1.	0.8	0.	0.02	1.73	1.04	0.8
time (sec)	N/A	0.029	0.045	0.002	0.	0.229	1.41	0.21	3.773

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	51	0	1	105	61	54
normalized size	1	1.	0.89	0.82	0.	0.02	1.69	0.98	0.87
time (sec)	N/A	0.041	0.088	0.002	0.	0.23	1.834	0.209	5.708

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	71	66	0	1	97	85	78
normalized size	1	1.	0.85	0.79	0.	0.01	1.15	1.01	0.93
time (sec)	N/A	0.054	0.081	0.006	0.	0.249	14.343	0.215	5.836

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	62	51	0	1	70	66	60
normalized size	1	1.	0.95	0.78	0.	0.02	1.08	1.02	0.92
time (sec)	N/A	0.037	0.058	0.004	0.	0.24	9.943	0.215	4.418

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	49	36	0	1	41	50	39
normalized size	1	1.	1.07	0.78	0.	0.02	0.89	1.09	0.85
time (sec)	N/A	0.024	0.024	0.003	0.	0.236	6.226	0.211	3.174

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	0	1	17	31	22
normalized size	1	1.	1.	0.84	0.	0.04	0.68	1.24	0.88
time (sec)	N/A	0.015	0.01	0.003	0.	0.227	3.573	0.215	2.432

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	19	31	17	19	12
normalized size	1	1.	1.	0.94	1.19	1.94	1.06	1.19	0.75
time (sec)	N/A	0.008	0.012	0.004	0.706	0.215	1.803	0.215	1.274

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	29	26	42	63	95	36	32
normalized size	1	1.	0.74	0.67	1.08	1.62	2.44	0.92	0.82
time (sec)	N/A	0.021	0.021	0.004	0.702	0.225	2.666	0.215	2.014

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	40	37	62	93	413	55	51
normalized size	1	1.	0.69	0.64	1.07	1.6	7.12	0.95	0.88
time (sec)	N/A	0.03	0.027	0.004	0.708	0.235	5.259	0.217	3.327

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	51	48	82	123	1265	74	70
normalized size	1	1.	0.66	0.62	1.06	1.6	16.43	0.96	0.91
time (sec)	N/A	0.043	0.032	0.006	0.699	0.252	9.963	0.217	5.014

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	35	41	26	0	69	19
normalized size	1	1.	0.87	1.52	1.78	1.13	0.	3.	0.83
time (sec)	N/A	0.01	0.016	0.004	0.804	0.209	0.	0.209	1.22

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	25	41	12	0	35	19
normalized size	1	1.	1.09	1.09	1.78	0.52	0.	1.52	0.83
time (sec)	N/A	0.01	0.009	0.004	0.769	0.219	0.	0.21	1.246

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	23	8	11	0	34	26
normalized size	1	1.	0.9	0.79	0.28	0.38	0.	1.17	0.9
time (sec)	N/A	0.014	0.01	0.007	0.791	0.212	0.	0.212	1.546

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	20	17	12	19	0	4	20
normalized size	1	1.	0.8	0.68	0.48	0.76	0.	0.16	0.8
time (sec)	N/A	0.01	0.01	0.005	0.787	0.221	0.	0.562	1.224

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	25	41	12	8	35	20
normalized size	1	1.	1.09	1.09	1.78	0.52	0.35	1.52	0.87
time (sec)	N/A	0.01	0.02	0.005	0.799	0.216	0.105	0.211	1.401

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	26	23	8	11	7	20	27
normalized size	1	1.	0.9	0.79	0.28	0.38	0.24	0.69	0.93
time (sec)	N/A	0.014	0.018	0.007	0.805	0.233	0.116	0.21	1.754

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	27	27	41	12	0	39	20
normalized size	1	1.	1.17	1.17	1.78	0.52	0.	1.7	0.87
time (sec)	N/A	0.009	0.013	0.003	0.776	0.209	0.	0.21	1.458

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	25	8	8	0	35	27
normalized size	1	1.	0.97	0.86	0.28	0.28	0.	1.21	0.93
time (sec)	N/A	0.014	0.012	0.005	0.806	0.221	0.	0.211	1.798

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	27	27	41	12	0	39	20
normalized size	1	1.	1.17	1.17	1.78	0.52	0.	1.7	0.87
time (sec)	N/A	0.01	0.011	0.003	0.792	0.215	0.	0.209	1.395

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	28	25	8	8	0	35	27
normalized size	1	1.	0.97	0.86	0.28	0.28	0.	1.21	0.93
time (sec)	N/A	0.017	0.007	0.003	0.796	0.216	0.	0.212	1.89

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	207	636	316	315	253	489	95
normalized size	1	1.	1.9	5.83	2.9	2.89	2.32	4.49	0.87
time (sec)	N/A	0.302	0.055	0.007	0.721	0.22	0.433	0.209	48.389

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	207	636	316	317	250	489	95
normalized size	1	1.	1.9	5.83	2.9	2.91	2.29	4.49	0.87
time (sec)	N/A	0.295	0.076	0.006	0.718	0.213	0.419	0.209	47.351

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	199	636	316	306	253	489	99
normalized size	1	1.	1.83	5.83	2.9	2.81	2.32	4.49	0.91
time (sec)	N/A	0.289	0.052	0.006	0.732	0.212	0.431	0.209	47.709

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	207	636	316	317	248	489	97
normalized size	1	1.	1.9	5.83	2.9	2.91	2.28	4.49	0.89
time (sec)	N/A	0.292	0.078	0.006	0.726	0.217	0.411	0.209	44.205

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	22	22	22	22	19
normalized size	1	1.	1.	0.94	1.22	1.22	1.22	1.22	1.06
time (sec)	N/A	0.027	0.008	0.005	0.795	0.224	0.225	0.206	1.416

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	9	11	0	7	11	7
normalized size	1	1.	1.	0.75	0.92	0.	0.58	0.92	0.58
time (sec)	N/A	0.017	0.023	0.018	0.792	0.	0.393	0.21	1.358

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	34	17	36	51	39	42	22
normalized size	1	1.	1.79	0.89	1.89	2.68	2.05	2.21	1.16
time (sec)	N/A	0.036	0.035	0.002	0.82	0.22	0.22	0.211	1.437

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	18	18	10	20	10
normalized size	1	1.	1.	1.08	1.38	1.38	0.77	1.54	0.77
time (sec)	N/A	0.012	0.005	0.007	0.739	0.214	0.19	0.208	1.513

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	16	20	20	14	23	14
normalized size	1	1.	1.	0.76	0.95	0.95	0.67	1.1	0.67
time (sec)	N/A	0.014	0.005	0.008	0.728	0.213	0.212	0.21	1.655

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	B	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	17	17	14	18	18	12	20	12
normalized size	1	2.83	2.83	2.33	3.	3.	2.	3.33	2.
time (sec)	N/A	0.012	0.005	0.008	0.727	0.226	0.179	0.209	1.421

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	24	51	82	76	54	26
normalized size	1	1.	1.	0.89	1.89	3.04	2.81	2.	0.96
time (sec)	N/A	0.04	0.014	0.005	0.742	0.244	0.526	0.21	3.156

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	26	53	81	76	61	26
normalized size	1	1.	1.07	0.96	1.96	3.	2.81	2.26	0.96
time (sec)	N/A	0.041	0.014	0.004	0.734	0.219	0.566	0.211	3.085

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	28	36	46	87	36	24
normalized size	1	1.	1.	0.9	1.16	1.48	2.81	1.16	0.77
time (sec)	N/A	0.046	0.014	0.004	0.728	0.23	0.442	0.208	2.863

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	29	26	53	81	76	61	26
normalized size	1	1.	1.07	0.96	1.96	3.	2.81	2.26	0.96
time (sec)	N/A	0.039	0.014	0.006	0.716	0.224	0.584	0.209	2.923

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	35	0	1	124	46	34
normalized size	1	1.	1.	0.92	0.	0.03	3.26	1.21	0.89
time (sec)	N/A	0.065	0.019	0.008	0.	0.217	0.553	0.209	4.595

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	34	35	0	1	100	41	27
normalized size	1	1.	0.97	1.	0.	0.03	2.86	1.17	0.77
time (sec)	N/A	0.065	0.016	0.008	0.	0.218	0.578	0.208	7.517

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	41	31	0	115	102	74	27
normalized size	1	1.	1.28	0.97	0.	3.59	3.19	2.31	0.84
time (sec)	N/A	0.051	0.018	0.005	0.	0.216	0.654	0.213	7.222

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	37	49	68	39	49	36
normalized size	1	1.	1.	0.86	1.14	1.58	0.91	1.14	0.84
time (sec)	N/A	0.037	0.04	0.005	0.791	0.212	0.335	0.207	1.864

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	62	37	63	99	58	69	39
normalized size	1	1.	1.44	0.86	1.47	2.3	1.35	1.6	0.91
time (sec)	N/A	0.04	0.054	0.003	0.785	0.215	0.325	0.208	1.865

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	33	32	46	72	29	49	29
normalized size	1	1.	0.97	0.94	1.35	2.12	0.85	1.44	0.85
time (sec)	N/A	0.022	0.019	0.014	0.722	0.217	0.312	0.21	1.951

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	32	46	72	32	49	32
normalized size	1	1.	1.	0.76	1.1	1.71	0.76	1.17	0.76
time (sec)	N/A	0.026	0.023	0.013	0.716	0.22	0.324	0.21	2.156

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	70	68	0	1	265	90	60
normalized size	1	1.	0.99	0.96	0.	0.01	3.73	1.27	0.85
time (sec)	N/A	0.082	0.105	0.005	0.	0.232	2.781	0.208	7.53

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	86	0	1	228	101	58
normalized size	1	1.	1.	1.19	0.	0.01	3.17	1.4	0.81
time (sec)	N/A	0.096	0.072	0.004	0.	0.225	2.589	0.209	11.565

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	78	84	0	216	218	122	60
normalized size	1	1.	1.13	1.22	0.	3.13	3.16	1.77	0.87
time (sec)	N/A	0.07	0.11	0.004	0.	0.237	2.674	0.214	9.552

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	65	111	215	120	212	135	94
normalized size	1	1.	1.05	1.79	3.47	1.94	3.42	2.18	1.52
time (sec)	N/A	0.268	0.154	0.044	0.828	0.244	2.99	0.222	80.535

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	58	34	31	88	85	56	76	49
normalized size	1	1.76	1.03	0.94	2.67	2.58	1.7	2.3	1.48
time (sec)	N/A	0.074	0.056	0.013	0.703	0.235	0.806	0.218	4.474

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	58	32	31	82	80	56	73	49
normalized size	1	1.87	1.03	1.	2.65	2.58	1.81	2.35	1.58
time (sec)	N/A	0.064	0.051	0.009	0.756	0.23	0.795	0.22	4.648

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	19	33	36	20	165	36	17
normalized size	1	1.	1.12	1.94	2.12	1.18	9.71	2.12	1.
time (sec)	N/A	0.039	0.034	0.02	0.795	0.226	0.45	0.207	3.907

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	A	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	56	39	45	28	70	45	17
normalized size	1	1.	2.43	1.7	1.96	1.22	3.04	1.96	0.74
time (sec)	N/A	0.054	0.061	0.044	0.788	0.231	2.255	0.208	4.857

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	39	29	51	177	0	54	44
normalized size	1	1.	1.03	0.76	1.34	4.66	0.	1.42	1.16
time (sec)	N/A	0.027	0.028	0.005	0.791	0.215	0.	0.21	2.024

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	51	53	0	32	44
normalized size	1	1.	1.	0.83	1.7	1.77	0.	1.07	1.47
time (sec)	N/A	0.024	0.025	0.004	0.829	0.221	0.	0.209	1.936

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	50	70	177	0	55	44
normalized size	1	1.	1.	1.02	1.43	3.61	0.	1.12	0.9
time (sec)	N/A	0.028	0.027	0.006	0.804	0.216	0.	0.209	1.983

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	35	62	85	0	72	53
normalized size	1	1.	1.02	0.78	1.38	1.89	0.	1.6	1.18
time (sec)	N/A	0.036	0.03	0.003	0.816	0.23	0.	0.212	2.054

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	35	62	70	0	49	56
normalized size	1	1.	1.02	0.78	1.38	1.56	0.	1.09	1.24
time (sec)	N/A	0.043	0.035	0.005	0.832	0.221	0.	0.21	2.027

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	50	78	85	0	73	53
normalized size	1	1.	0.89	0.81	1.26	1.37	0.	1.18	0.85
time (sec)	N/A	0.034	0.046	0.004	0.847	0.223	0.	0.213	2.013

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	44	32	55	72	0	42	56
normalized size	1	1.	1.02	0.74	1.28	1.67	0.	0.98	1.3
time (sec)	N/A	0.029	0.034	0.004	0.821	0.224	0.	0.21	2.079

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	53	50	78	86	0	73	54
normalized size	1	1.	0.9	0.85	1.32	1.46	0.	1.24	0.92
time (sec)	N/A	0.032	0.038	0.005	0.826	0.227	0.	0.212	2.004

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	46	62	115	0	51	53
normalized size	1	1.	0.92	0.78	1.05	1.95	0.	0.86	0.9
time (sec)	N/A	0.033	0.049	0.007	0.917	0.22	0.	0.211	2.008

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	50	78	88	0	73	54
normalized size	1	1.	0.89	0.81	1.26	1.42	0.	1.18	0.87
time (sec)	N/A	0.033	0.047	0.005	0.833	0.224	0.	0.212	2.102

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	32	55	69	0	42	54
normalized size	1	1.	1.03	0.82	1.41	1.77	0.	1.08	1.38
time (sec)	N/A	0.022	0.033	0.005	0.865	0.223	0.	0.21	2.061

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	9	11	27	0	27	22
normalized size	1	1.	1.	0.64	0.79	1.93	0.	1.93	1.57
time (sec)	N/A	0.017	0.009	0.004	0.871	0.218	0.	0.212	1.508

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	14	7	11	28	0	8	26
normalized size	1	1.	1.4	0.7	1.1	2.8	0.	0.8	2.6
time (sec)	N/A	0.015	0.01	0.003	0.805	0.229	0.	0.212	1.528

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	24	30	30	27	0	28	22
normalized size	1	1.	0.96	1.2	1.2	1.08	0.	1.12	0.88
time (sec)	N/A	0.016	0.01	0.005	0.872	0.233	0.	0.212	1.486

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	15	22	55	0	45	32
normalized size	1	1.	1.	0.83	1.22	3.06	0.	2.5	1.78
time (sec)	N/A	0.022	0.011	0.004	0.829	0.232	0.	0.214	1.481

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	15	22	38	0	22	34
normalized size	1	1.	1.	0.79	1.16	2.	0.	1.16	1.79
time (sec)	N/A	0.026	0.016	0.004	0.884	0.233	0.	0.215	1.532

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	30	38	54	0	46	32
normalized size	1	1.	0.8	0.86	1.09	1.54	0.	1.31	0.91
time (sec)	N/A	0.02	0.017	0.004	0.848	0.231	0.	0.216	1.49

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	12	15	38	0	15	34
normalized size	1	1.	1.	0.71	0.88	2.24	0.	0.88	2.
time (sec)	N/A	0.017	0.01	0.005	0.848	0.22	0.	0.216	1.498

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	30	38	54	0	46	32
normalized size	1	1.	0.81	0.94	1.19	1.69	0.	1.44	1.
time (sec)	N/A	0.02	0.01	0.004	0.834	0.223	0.	0.214	1.481

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	28	26	22	86	0	24	34
normalized size	1	1.	0.85	0.79	0.67	2.61	0.	0.73	1.03
time (sec)	N/A	0.02	0.022	0.003	0.797	0.223	0.	0.215	1.486

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	30	38	54	0	46	32
normalized size	1	1.	0.8	0.86	1.09	1.54	0.	1.31	0.91
time (sec)	N/A	0.02	0.016	0.003	0.813	0.223	0.	0.216	1.495

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	15	38	0	15	34
normalized size	1	1.	1.	0.92	1.15	2.92	0.	1.15	2.62
time (sec)	N/A	0.011	0.01	0.004	0.789	0.223	0.	0.215	1.525

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	51	0	1	0	66	42
normalized size	1	1.	1.	2.32	0.	0.05	0.	3.	1.91
time (sec)	N/A	0.029	0.045	0.015	0.	0.237	0.	0.233	6.632

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	44	0	1	0	72	44
normalized size	1	1.	1.	1.91	0.	0.04	0.	3.13	1.91
time (sec)	N/A	0.028	0.045	0.018	0.	0.236	0.	0.229	6.409

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	44	0	1	0	72	44
normalized size	1	1.	1.	2.2	0.	0.05	0.	3.6	2.2
time (sec)	N/A	0.025	0.042	0.016	0.	0.247	0.	0.23	6.575

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	24	35	42	0	23	17
normalized size	1	1.	1.	1.26	1.84	2.21	0.	1.21	0.89
time (sec)	N/A	0.011	0.012	0.005	0.746	0.221	0.	0.213	1.191

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	28	41	42	0	23	19
normalized size	1	1.	1.	1.22	1.78	1.83	0.	1.	0.83
time (sec)	N/A	0.01	0.015	0.004	0.763	0.21	0.	0.212	1.286

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	26	41	39	0	39	17
normalized size	1	1.	1.	1.13	1.78	1.7	0.	1.7	0.74
time (sec)	N/A	0.019	0.022	0.005	0.78	0.22	0.	0.214	3.757

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	36	80	66	0	49	36
normalized size	1	1.	0.72	0.84	1.86	1.53	0.	1.14	0.84
time (sec)	N/A	0.02	0.027	0.004	0.75	0.218	0.	0.216	1.683

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	126	0	0	0	0	0	105
normalized size	1	1.	1.03	0.	0.	0.	0.	0.	0.86
time (sec)	N/A	0.087	0.155	0.157	0.	0.	0.	0.	4.819

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	93	0	0	0	0	0	27
normalized size	1	1.	2.51	0.	0.	0.	0.	0.	0.73
time (sec)	N/A	0.037	0.11	0.26	0.	0.	0.	0.	2.114

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	94	0	0	0	0	0	26
normalized size	1	1.	2.94	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	0.033	0.146	0.231	0.	0.	0.	0.	1.917

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	93	0	0	0	0	0	27
normalized size	1	1.	2.51	0.	0.	0.	0.	0.	0.73
time (sec)	N/A	0.036	0.109	0.184	0.	0.	0.	0.	1.984

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	92	0	0	0	0	0	17
normalized size	1	1.	4.38	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	0.023	0.101	0.228	0.	0.	0.	0.	1.73

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	48	0	0	0	0	0	44
normalized size	1	1.	0.89	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	0.028	0.029	0.068	0.	0.	0.	0.	1.935

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	17	17	0	26	20	22	12
normalized size	1	1.	0.94	0.94	0.	1.44	1.11	1.22	0.67
time (sec)	N/A	0.008	0.008	0.003	0.	0.236	0.074	0.206	1.484

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	83	0	0	0	0	0	26
normalized size	1	1.	2.68	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.028	0.08	0.151	0.	0.	0.	0.	1.956

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	86	0	0	0	0	0	26
normalized size	1	1.	2.77	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.026	0.173	0.154	0.	0.	0.	0.	2.04

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	81	0	0	0	0	0	27
normalized size	1	1.	2.13	0.	0.	0.	0.	0.	0.71
time (sec)	N/A	0.032	0.112	0.157	0.	0.	0.	0.	2.068

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	58	0	0	0	0	0	26
normalized size	1	1.	1.66	0.	0.	0.	0.	0.	0.74
time (sec)	N/A	0.033	0.036	0.096	0.	0.	0.	0.	1.955

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	81	0	0	0	0	0	27
normalized size	1	1.	2.13	0.	0.	0.	0.	0.	0.71
time (sec)	N/A	0.03	0.114	0.158	0.	0.	0.	0.	2.102

2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [35] had the largest ratio of [0.5385]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	3	1.	13	0.231
2	A	6	3	1.	15	0.2
3	A	5	3	1.	15	0.2
4	A	4	3	1.	15	0.2
5	A	3	3	1.	15	0.2
6	A	6	3	1.	13	0.231
7	A	5	3	1.	13	0.231
8	A	4	3	1.	13	0.231
9	A	3	3	1.	13	0.231
10	A	3	3	1.	13	0.231
11	A	3	3	1.	13	0.231
12	A	4	3	1.	11	0.273
13	A	3	3	1.	11	0.273
14	A	3	3	1.	11	0.273
15	A	3	3	1.	11	0.273
16	A	3	2	1.	13	0.154
17	A	2	2	1.	15	0.133
18	A	1	1	1.	15	0.067
19	A	2	2	1.	15	0.133
20	A	3	2	1.	15	0.133
21	A	2	2	1.	13	0.154
22	A	1	1	1.	13	0.077
23	A	2	2	1.	13	0.154
24	A	3	2	1.	13	0.154
25	A	2	2	1.	16	0.125
26	A	2	2	1.	15	0.133
27	A	2	2	1.	13	0.154
28	A	2	2	1.	11	0.182
29	A	2	2	1.	11	0.182
30	A	6	5	1.	13	0.385
31	A	5	5	1.	13	0.385
32	A	4	4	1.	13	0.308
33	A	5	5	1.	13	0.385

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
34	A	6	5	1.	13	0.385
35	A	8	7	1.	13	0.538
36	A	7	7	1.	13	0.538
37	A	6	6	1.	13	0.462
38	A	7	7	1.	13	0.538
39	A	8	7	1.	13	0.538
40	A	5	4	1.	13	0.308
41	A	4	4	1.	13	0.308
42	A	4	4	1.	13	0.308
43	A	3	3	1.	13	0.231
44	A	3	3	1.	13	0.231
45	A	4	4	1.	13	0.308
46	A	5	4	1.	13	0.308
47	A	6	4	1.	13	0.308
48	A	1	1	1.	11	0.091
49	A	2	1	1.	9	0.111
50	A	2	1	1.	9	0.111
51	A	2	1	1.	9	0.111
52	A	1	0	1.	7	0.
53	A	1	1	1.	9	0.111
54	A	2	2	1.	9	0.222
55	A	3	2	1.	9	0.222
56	A	5	3	1.	11	0.273
57	A	4	3	1.	11	0.273
58	A	3	3	1.	11	0.273
59	A	2	2	1.	11	0.182
60	A	1	1	1.	11	0.091
61	A	2	2	1.	11	0.182
62	A	3	2	1.	11	0.182
63	A	4	2	1.	11	0.182
64	A	1	1	1.	14	0.071
65	A	1	1	1.	14	0.071
66	A	2	2	1.	14	0.143
67	A	1	1	1.	14	0.071

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
68	A	1	1	1.	14	0.071
69	A	2	2	1.	14	0.143
70	A	1	1	1.	14	0.071
71	A	2	2	1.	14	0.143
72	A	1	1	1.	14	0.071
73	A	2	2	1.	14	0.143
74	A	3	2	1.	23	0.087
75	A	3	2	1.	23	0.087
76	A	3	2	1.	23	0.087
77	A	3	2	1.	23	0.087
78	A	2	2	1.	12	0.167
79	A	2	2	1.	15	0.133
80	A	2	2	1.	12	0.167
81	A	3	2	1.	12	0.167
82	A	3	2	1.	12	0.167
83	B	3	2	2.83	10	0.2
84	A	2	2	1.	12	0.167
85	A	2	2	1.	12	0.167
86	A	2	2	1.	12	0.167
87	A	2	2	1.	12	0.167
88	A	2	2	1.	12	0.167
89	A	2	2	1.	13	0.154
90	A	2	2	1.	14	0.143
91	A	3	3	1.	12	0.25
92	A	3	3	1.	12	0.25
93	A	4	3	1.	12	0.25
94	A	4	3	1.	12	0.25
95	A	3	3	1.	12	0.25
96	A	3	3	1.	13	0.231
97	A	3	3	1.	14	0.214
98	A	2	2	1.	40	0.05
99	A	3	2	1.76	30	0.067
100	A	3	2	1.87	31	0.065
101	A	2	2	1.	14	0.143

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
102	A	2	2	1.	16	0.125
103	A	3	3	1.	14	0.214
104	A	3	3	1.	14	0.214
105	A	3	3	1.	14	0.214
106	A	3	3	1.	14	0.214
107	A	3	3	1.	14	0.214
108	A	3	3	1.	14	0.214
109	A	3	3	1.	14	0.214
110	A	3	3	1.	14	0.214
111	A	3	3	1.	14	0.214
112	A	3	3	1.	14	0.214
113	A	3	3	1.	14	0.214
114	A	2	2	1.	14	0.143
115	A	2	2	1.	14	0.143
116	A	2	2	1.	14	0.143
117	A	2	2	1.	14	0.143
118	A	2	2	1.	14	0.143
119	A	2	2	1.	14	0.143
120	A	2	2	1.	14	0.143
121	A	2	2	1.	14	0.143
122	A	2	2	1.	14	0.143
123	A	2	2	1.	14	0.143
124	A	2	2	1.	14	0.143
125	A	2	2	1.	27	0.074
126	A	2	2	1.	30	0.067
127	A	2	2	1.	28	0.071
128	A	1	1	1.	12	0.083
129	A	1	1	1.	14	0.071
130	A	1	1	1.	16	0.062
131	A	2	2	1.	14	0.143
132	A	1	1	1.	12	0.083
133	A	2	2	1.	12	0.167
134	A	2	2	1.	12	0.167
135	A	2	2	1.	12	0.167

Continued on next page

Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	2	2	1.	12	0.167
137	A	1	1	1.	10	0.1
138	A	1	1	1.	7	0.143
139	A	2	2	1.	12	0.167
140	A	2	2	1.	12	0.167
141	A	2	2	1.	12	0.167
142	A	2	2	1.	12	0.167
143	A	2	2	1.	12	0.167

3 Listing of integrals

3.1 $\int \left(bx+cx^2\right)^{7/2}\,dx$

Optimal. Leaf size=147

$$\frac{35b^8\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{16384c^{9/2}}-\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4}+\frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3}-\frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2}+\frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c}$$

```
[Out] (-35*b^6*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(16384*c^4) + (35*b^4*(b
+ 2*c*x)*(b*x + c*x^2)^(3/2))/(6144*c^3) - (7*b^2*(b + 2*c*x)*(b*
x + c*x^2)^(5/2))/(384*c^2) + ((b + 2*c*x)*(b*x + c*x^2)^(7/2))/(
16*c) + (35*b^8*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(16384*c^
(9/2))
```

Rubi [A] time = 0.129437, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{35b^8\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{16384c^{9/2}}-\frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4}+\frac{35b^4(b+2cx)\left(bx+cx^2\right)^{3/2}}{6144c^3}-\frac{7b^2(b+2cx)\left(bx+cx^2\right)^{5/2}}{384c^2}+\frac{(b+2cx)\left(bx+cx^2\right)^{7/2}}{16c}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)^(7/2),x]
```

```
[Out] (-35*b^6*(b + 2*c*x)*Sqrt[b*x + c*x^2])/(16384*c^4) + (35*b^4*(b
+ 2*c*x)*(b*x + c*x^2)^(3/2))/(6144*c^3) - (7*b^2*(b + 2*c*x)*(b*
x + c*x^2)^(5/2))/(384*c^2) + ((b + 2*c*x)*(b*x + c*x^2)^(7/2))/(
16*c) + (35*b^8*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(16384*c^
(9/2))
```

Rubi in Sympy [A] time = 16.0857, size = 139, normalized size = 0.95

$$\frac{35b^8 \operatorname{atanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{16384c^{\frac{9}{2}}} - \frac{35b^6(b+2cx)\sqrt{bx+cx^2}}{16384c^4} + \frac{35b^4(b+2cx)\left(bx+cx^2\right)^{\frac{3}{2}}}{6144c^3} - \frac{7b^2(b+2cx)\left(bx+cx^2\right)^{\frac{5}{2}}}{384c^2} + \frac{(b+2cx)\left(bx+cx^2\right)^{\frac{7}{2}}}{16c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x)**(7/2),x)
```

```
[Out] 35*b**8*atanh(sqrt(c)*x/sqrt(b*x + c*x**2))/(16384*c**(9/2)) - 35
*b**6*(b + 2*c*x)*sqrt(b*x + c*x**2)/(16384*c**4) + 35*b**4*(b +
2*c*x)*(b*x + c*x**2)**(3/2)/(6144*c**3) - 7*b**2*(b + 2*c*x)*(b*
x + c*x**2)**(5/2)/(384*c**2) + (b + 2*c*x)*(b*x + c*x**2)**(7/2)
/(16*c)
```

Mathematica [A] time = 0.2166, size = 158, normalized size = 1.07

$$\frac{\sqrt{x}\sqrt{b+cx}\left(105b^8\log\left(\sqrt{c}\sqrt{b+cx}+c\sqrt{x}\right)+\sqrt{c}\sqrt{x}\sqrt{b+cx}\left(-105b^7+70b^6cx-56b^5c^2x^2+48b^4c^3x^3+10880b^3c^4x^4+2585\right.\right.}{49152c^{9/2}\sqrt{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(7/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[b + c*x]*(Sqrt[c]*Sqrt[x]*Sqrt[b + c*x]*(-105*b^7 +
 70*b^6*c*x - 56*b^5*c^2*x^2 + 48*b^4*c^3*x^3 + 10880*b^3*c^4*x^4
 + 25856*b^2*c^5*x^5 + 21504*b*c^6*x^6 + 6144*c^7*x^7) + 105*b^8*
Log[c*Sqrt[x] + Sqrt[c]*Sqrt[b + c*x]]))/(49152*c^(9/2)*Sqrt[x*(b
 + c*x)])
```

Maple [A] time = 0.007, size = 173, normalized size = 1.2

$$\frac{2\,cx+b}{16\,c}\left(cx^2+bx\right)^{\frac{7}{2}} - \frac{7\,b^2x}{192\,c}\left(cx^2+bx\right)^{\frac{5}{2}} - \frac{7\,b^3}{384\,c^2}\left(cx^2+bx\right)^{\frac{5}{2}} + \frac{35\,b^4x}{3072\,c^2}\left(cx^2+bx\right)^{\frac{3}{2}} + \frac{35\,b^5}{6144\,c^3}\left(cx^2+bx\right)^{\frac{3}{2}} - \frac{35\,b^6x}{8192\,c^3}\sqrt{cx^2+bx} - \frac{35\,b^7}{16384\,c^4}\sqrt{cx^2+bx} + \frac{35\,b^8}{32768}\ln\left(1\left(\frac{b}{2}+cx\right)\frac{1}{\sqrt{c}}+\sqrt{cx^2+bx}\right)c^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x)^(7/2),x)
```

```
[Out] 1/16*(2*c*x+b)*(c*x^2+b*x)^(7/2)/c-7/192*b^2/c*(c*x^2+b*x)^(5/2)*
x-7/384*b^3/c^2*(c*x^2+b*x)^(5/2)+35/3072*b^4/c^2*(c*x^2+b*x)^(3/
2)*x+35/6144*b^5/c^3*(c*x^2+b*x)^(3/2)-35/8192*b^6/c^3*(c*x^2+b*x
)^(1/2)*x-35/16384*b^7/c^4*(c*x^2+b*x)^(1/2)+35/32768*b^8/c^(9/2)
*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.242545, size = 1, normalized size = 0.01

$$\left[\frac{105\,b^8\log\left((2\,cx+b)\sqrt{c}+2\,\sqrt{cx^2+bxc}\right)+2\left(6144\,c^7x^7+21504\,bc^6x^6+25856\,b^2c^5x^5+10880\,b^3c^4x^4+48\,b^4c^3x^3-56\,b^5c\right.}{98304\,c^{\frac{9}{2}}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(7/2),x, algorithm="fricas")
```

```
[Out] [1/98304*(105*b^8*log((2*c*x + b)*sqrt(c) + 2*sqrt(c*x^2 + b*x)*c
) + 2*(6144*c^7*x^7 + 21504*b*c^6*x^6 + 25856*b^2*c^5*x^5 + 10880
*b^3*c^4*x^4 + 48*b^4*c^3*x^3 - 56*b^5*c^2*x^2 + 70*b^6*c*x - 105
*b^7)*sqrt(c*x^2 + b*x)*sqrt(c))/c^(9/2), 1/49152*(105*b^8*arctan
(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x)) + (6144*c^7*x^7 + 21504*b*c^6*
x^6 + 25856*b^2*c^5*x^5 + 10880*b^3*c^4*x^4 + 48*b^4*c^3*x^3 - 56
*b^5*c^2*x^2 + 70*b^6*c*x - 105*b^7)*sqrt(c*x^2 + b*x)*sqrt(-c))/
(sqrt(-c)*c^4)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx + cx^2\right)^{\frac{7}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x)**(7/2),x)
```

```
[Out] Integral((b*x + c*x**2)**(7/2), x)
```

GIAC/XCAS [A] time = 0.226363, size = 178, normalized size = 1.21

$$-\frac{35\,b^8 \ln\left(\left|-2\left(\sqrt{c}x - \sqrt{cx^2 + bx}\right)\sqrt{c} - b\right|\right)}{32768\,c^{\frac{9}{2}}} - \frac{1}{49152}\left(\frac{105\,b^7}{c^4} - 2\left(\frac{35\,b^6}{c^3} - 4\left(\frac{7\,b^5}{c^2} - 2\left(\frac{3\,b^4}{c} + 8\left(85\,b^3 + 2\left(101\,b^2c + 12\left(2\,c^3x + 7\,bc^2\right)x\right)x\right)x\right)x\right)x\right)x\right)\sqrt{cx^2 + bx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(7/2),x, algorithm="giac")
```

```
[Out] -35/32768*b^8*ln(abs(-2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) -
 b))/c^(9/2) - 1/49152*(105*b^7/c^4 - 2*(35*b^6/c^3 - 4*(7*b^5/c^
2 - 2*(3*b^4/c + 8*(85*b^3 + 2*(101*b^2*c + 12*(2*c^3*x + 7*b*c^2
)*x)*x)*x)*x)*x)*x)*sqrt(c*x^2 + b*x)
```

3.2 $\int \left(3ix+4x^2\right)^{7/2} dx$

Optimal. Leaf size=121

$$\frac{1}{64}(8x+3i)\left(4x^2+3ix\right)^{7/2}+\frac{21(8x+3i)\left(4x^2+3ix\right)^{5/2}}{2048}+\frac{945(8x+3i)\left(4x^2+3ix\right)^{3/2}}{131072}+\frac{25515(8x+3i)\sqrt{4x^2+3ix}}{4194304}+\frac{229635i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{16777216}$$

```
[Out] (25515*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/4194304 + (945*(3*I + 8
*x)*((3*I)*x + 4*x^2)^(3/2))/131072 + (21*(3*I + 8*x)*((3*I)*x +
4*x^2)^(5/2))/2048 + ((3*I + 8*x)*((3*I)*x + 4*x^2)^(7/2))/64 + (
(229635*I)/16777216)*ArcSin[1 - ((8*I)/3)*x]
```

Rubi [A] time = 0.072225, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{1}{64}(8x+3i)\left(4x^2+3ix\right)^{7/2}+\frac{21(8x+3i)\left(4x^2+3ix\right)^{5/2}}{2048}+\frac{945(8x+3i)\left(4x^2+3ix\right)^{3/2}}{131072}+\frac{25515(8x+3i)\sqrt{4x^2+3ix}}{4194304}+\frac{229635i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{16777216}$$

Antiderivative was successfully verified.

```
[In]  Int[((3*I)*x + 4*x^2)^(7/2),x]
```

```
[Out] (25515*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/4194304 + (945*(3*I + 8
*x)*((3*I)*x + 4*x^2)^(3/2))/131072 + (21*(3*I + 8*x)*((3*I)*x +
4*x^2)^(5/2))/2048 + ((3*I + 8*x)*((3*I)*x + 4*x^2)^(7/2))/64 + (
(229635*I)/16777216)*ArcSin[1 - ((8*I)/3)*x]
```

Rubi in Sympy [A] time = 4.32622, size = 104, normalized size = 0.86

$$\frac{(8x+3i)\left(4x^2+3ix\right)^{\frac{7}{2}}}{64}+\frac{21\left(8x+3i\right)\left(4x^2+3ix\right)^{\frac{5}{2}}}{2048}+\frac{945\left(8x+3i\right)\left(4x^2+3ix\right)^{\frac{3}{2}}}{131072}+\frac{25515\left(8x+3i\right)\sqrt{4x^2+3ix}}{4194304}+\frac{229635\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{16777216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*I*x+4*x**2)**(7/2),x)
```

```
[Out] (8*x + 3*I)*(4*x**2 + 3*I*x)**(7/2)/64 + 21*(8*x + 3*I)*(4*x**2 +
 3*I*x)**(5/2)/2048 + 945*(8*x + 3*I)*(4*x**2 + 3*I*x)**(3/2)/131
072 + 25515*(8*x + 3*I)*sqrt(4*x**2 + 3*I*x)/4194304 + 229635*asi
nh(8*x/3 + I)/16777216
```

Mathematica [A] time = 0.116105, size = 117, normalized size = 0.97

$$\frac{\sqrt{x}\sqrt{4x+3i}\left(2\sqrt{x}\sqrt{4x+3i}\left(33554432x^7+88080384ix^6-79429632x^5-25067520ix^4+82944x^3-72576ix^2-68040x+76\right.\right.}{8388608\sqrt{x(4x+3i)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((3*I)*x + 4*x^2)^(7/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[3*I + 4*x]*(2*Sqrt[x]*Sqrt[3*I + 4*x]*(76545*I - 68
040*x - (72576*I)*x^2 + 82944*x^3 - (25067520*I)*x^4 - 79429632*x
^5 + (88080384*I)*x^6 + 33554432*x^7) + 229635*Log[2*Sqrt[x] + Sq
rt[3*I + 4*x]]))/(8388608*Sqrt[x*(3*I + 4*x)])
```

Maple [A] time = 0.029, size = 91, normalized size = 0.8

$$\frac{3i+8x}{64}\left(3ix+4x^2\right)^{\frac{7}{2}}+\frac{63i+168x}{2048}\left(3ix+4x^2\right)^{\frac{5}{2}}+\frac{2835i+7560x}{131072}\left(3ix+4x^2\right)^{\frac{3}{2}}+\frac{76545i+204120x}{4194304}\sqrt{3ix+4x^2}+\frac{229635}{16777216}\mathit{Arcsinh}\left(\frac{8x}{3}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*I*x+4*x^2)^(7/2),x)
```

```
[Out] 1/64*(3*I+8*x)*(3*I*x+4*x^2)^(7/2)+21/2048*(3*I+8*x)*(3*I*x+4*x^2
)^(5/2)+945/131072*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)+25515/4194304*(3
*I+8*x)*(3*I*x+4*x^2)^(1/2)+229635/16777216*arcsinh(8/3*x+I)
```

Maxima [A] time = 0.789948, size = 176, normalized size = 1.45

$$\frac{1}{8}\left(4x^2+3ix\right)^{\frac{7}{2}}x+\frac{3}{64}i\left(4x^2+3ix\right)^{\frac{7}{2}}+\frac{21}{256}\left(4x^2+3ix\right)^{\frac{5}{2}}x+\frac{63}{2048}i\left(4x^2+3ix\right)^{\frac{5}{2}}+\frac{945}{16384}\left(4x^2+3ix\right)^{\frac{3}{2}}x+\frac{2835}{131072}i\left(4x^2+3ix\right)^{\frac{3}{2}}+\frac{25515}{524288}\sqrt{4x^2+3ix}x+\frac{76545}{4194304}i\sqrt{4x^2+3ix}+\frac{229635}{16777216}\log\left(8x+4\sqrt{4x^2+3ix}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/8*(4*x^2 + 3*I*x)^(7/2)*x + 3/64*I*(4*x^2 + 3*I*x)^(7/2) + 21/2
56*(4*x^2 + 3*I*x)^(5/2)*x + 63/2048*I*(4*x^2 + 3*I*x)^(5/2) + 94
5/16384*(4*x^2 + 3*I*x)^(3/2)*x + 2835/131072*I*(4*x^2 + 3*I*x)^(
3/2) + 25515/524288*sqrt(4*x^2 + 3*I*x)*x + 76545/4194304*I*sqrt(
4*x^2 + 3*I*x) + 229635/16777216*log(8*x + 4*sqrt(4*x^2 + 3*I*x)
+ 3*I)
```

Fricas [A] time = 0.226256, size = 494, normalized size = 4.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -(9223372036854775808*x^16 + 55340232221128654848*I*x^15 - 146997
491837372989440*x^14 - 226981421219472998400*I*x^13 + 22483320419
7217271808*x^12 + 148652564700431646720*I*x^11 - 6613272170002907
1360*x^10 - 19461487753030533120*I*x^9 + 3598235800719851520*x^8
+ 341772402683805696*I*x^7 + 22469894033375232*x^6 + 193905371193
01632*I*x^5 - 4790430374952960*x^4 - 557082135429120*I*x^3 + 2150
7697870848*x^2 + (7890198520135680*x^8 + 23670595560407040*I*x^7
- 28848538339246080*x^6 - 18307726253752320*I*x^5 + 6436310011084
800*x^4 + 1228750093025280*I*x^3 - 115195321221120*x^2 - (3945099
260067840*x^7 + 10355885557678080*I*x^6 - 10818201877217280*x^5 -
 5721164454297600*I*x^4 + 1609077502771200*x^3 + 230390642442240*
I*x^2 - 14399415152640*x - 257132413440*I)*sqrt(4*x^2 + 3*I*x) -
4114118615040*I*x + 24106163760)*log(-2*x + sqrt(4*x^2 + 3*I*x) -
 3/4*I) - (4611686018427387904*x^15 + 25940733853654056960*I*x^14
 - 64095229896736899072*x^13 - 91157360057606209536*I*x^12 + 8211
2302418497634304*x^11 + 48544353159984709632*I*x^10 - 18872887292
844834816*x^9 - 4697147896158486528*I*x^8 + 692278034027249664*x^
7 + 36186599083474944*I*x^6 + 1290704607667814 4*x^5 + 54395023644
42624*I*x^4 - 996472905891840*x^3 - 75654070087680*I*x^2 + 888535
339776*x - 49436767584*I)*sqrt(4*x^2 + 3*I*x) - 276723454464*I*x
+ 7647967431)/(576460752303423488*x^8 + 1729382256910270464*I*x^7
 - 2107684625609392128*x^6 - 1337569089329037312*I*x^5 + 47023913
2967239680*x^4 + 89772925384654848*I*x^3 - 8416211754811392*x^2 -
 (288230376151711744*x^7 + 756604737398243328*I*x^6 - 79038173460
3522048*x^5 - 417990340415324160*I*x^4 + 117559783241809920*x^3 +
 16832423509622784*I*x^2 - 1052026469351424*x - 18786186952704*I)
*sqrt(4*x^2 + 3*I*x) - 300578991243264*I*x + 1761205026816)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4x^2 + 3ix\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*I*x+4*x**2)**(7/2),x)

[Out] Integral((4*x**2 + 3*I*x)**(7/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4\,x^2 + 3i\,x\right)^{\frac{7}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(7/2),x, algorithm="giac")

[Out] integrate((4*x^2 + 3*I*x)^(7/2), x)
```

3.3 $\int \left(3ix+4x^2\right)^{5/2} dx$

Optimal. Leaf size=95

$$\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}+\frac{15(8x+3i)\left(4x^2+3ix\right)^{3/2}}{1024}+\frac{405(8x+3i)\sqrt{4x^2+3ix}}{32768}+\frac{3645i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{131072}$$

```
[Out] (405*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/32768 + (15*(3*I + 8*x)*(
(3*I)*x + 4*x^2)^(3/2))/1024 + ((3*I + 8*x)*((3*I)*x + 4*x^2)^(5/
2))/48 + ((3645*I)/131072)*ArcSin[1 - ((8*I)/3)*x]
```

Rubi [A] time = 0.0527866, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}+\frac{15(8x+3i)\left(4x^2+3ix\right)^{3/2}}{1024}+\frac{405(8x+3i)\sqrt{4x^2+3ix}}{32768}+\frac{3645i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{131072}$$

Antiderivative was successfully verified.

```
[In]  Int[((3*I)*x + 4*x^2)^(5/2),x]
```

```
[Out] (405*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/32768 + (15*(3*I + 8*x)*(
(3*I)*x + 4*x^2)^(3/2))/1024 + ((3*I + 8*x)*((3*I)*x + 4*x^2)^(5/
2))/48 + ((3645*I)/131072)*ArcSin[1 - ((8*I)/3)*x]
```

Rubi in Sympy [A] time = 3.27274, size = 80, normalized size = 0.84

$$\frac{(8x+3i)\left(4x^2+3ix\right)^{\frac{5}{2}}}{48}+\frac{15\,(8x+3i)\left(4x^2+3ix\right)^{\frac{3}{2}}}{1024}+\frac{405\,(8x+3i)\sqrt{4x^2+3ix}}{32768}+\frac{3645\,\mathrm{asinh}\left(\frac{8x}{3}+i\right)}{131072}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*I*x+4*x**2)**(5/2),x)
```

```
[Out] (8*x + 3*I)*(4*x**2 + 3*I*x)**(5/2)/48 + 15*(8*x + 3*I)*(4*x**2 +
 3*I*x)**(3/2)/1024 + 405*(8*x + 3*I)*sqrt(4*x**2 + 3*I*x)/32768
+ 3645*asinh(8*x/3 + I)/131072
```

Mathematica [A] time = 0.0963258, size = 86, normalized size = 0.91

$$\frac{\sqrt{x(4x+3i)}\left(524288x^5+983040ix^4-497664x^3-6912ix^2-6480x+\frac{10935\log\left(2\sqrt{x}+\sqrt{4x+3i}\right)}{\sqrt{4x+3i}\sqrt{x}}+7290i\right)}{196608}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((3*I)*x + 4*x^2)^(5/2),x]
```

```
[Out] (Sqrt[x*(3*I + 4*x)]*(7290*I - 6480*x - (6912*I)*x^2 - 497664*x^3
 + (983040*I)*x^4 + 524288*x^5 + (10935*Log[2*Sqrt[x] + Sqrt[3*I
+ 4*x]])/(Sqrt[x]*Sqrt[3*I + 4*x])))/196608
```

Maple [A] time = 0.01, size = 71, normalized size = 0.8

$$\frac{3\,i+8\,x}{48}\left(3\,ix+4\,x^{2}\right)^{\frac{5}{2}}+\frac{45\,i+120\,x}{1024}\left(3\,ix+4\,x^{2}\right)^{\frac{3}{2}}+\frac{1215\,i+3240\,x}{32768}\sqrt{3\,ix+4\,x^{2}}+\frac{3645}{131072}Arcsinh\left(\frac{8\,x}{3}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*I*x+4*x^2)^(5/2),x)
```

```
[Out] 1/48*(3*I+8*x)*(3*I*x+4*x^2)^(5/2)+15/1024*(3*I+8*x)*(3*I*x+4*x^2
)^(3/2)+405/32768*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)+3645/131072*arcsi
nh(8/3*x+I)
```

Maxima [A] time = 0.782845, size = 139, normalized size = 1.46

$$\frac{1}{6}\left(4\,x^{2}+3i\,x\right)^{\frac{5}{2}}x+\frac{1}{16}i\left(4\,x^{2}+3i\,x\right)^{\frac{5}{2}}+\frac{15}{128}\left(4\,x^{2}+3i\,x\right)^{\frac{3}{2}}x+\frac{45}{1024}i\left(4\,x^{2}+3i\,x\right)^{\frac{3}{2}}+\frac{405}{4096}\sqrt{4\,x^{2}+3i\,x}x+\frac{1215}{32768}i\sqrt{4\,x^{2}+3i\,x}+\frac{3645}{131072}\log\left(8\,x+4\sqrt{4\,x^{2}+3i\,x}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(4*x^2 + 3*I*x)^(5/2)*x + 1/16*I*(4*x^2 + 3*I*x)^(5/2) + 15/1
28*(4*x^2 + 3*I*x)^(3/2)*x + 45/1024*I*(4*x^2 + 3*I*x)^(3/2) + 40
5/4096*sqrt(4*x^2 + 3*I*x)*x + 1215/32768*I*sqrt(4*x^2 + 3*I*x) +
```

```
3645/131072*log(8*x + 4*sqrt(4*x^2 + 3*I*x) + 3*I)
```

Fricas [A] time = 0.234746, size = 386, normalized size = 4.06

$$-\frac{140737488355328\,x^{12} + 633318697598976i\,x^{11} - 1202315964973056\,x^{10} - 1243135334154240i\,x^{9} + 753012993687552\,x^{8} + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(5/2),x, algorithm="fricas")
```

```
[Out] -(140737488355328*x^12 + 633318697598976*I*x^11 - 120231596497305
6*x^10 - 1243135334154240*I*x^9 + 753012993687552*x^8 + 267703164
076032*I*x^7 - 54548433469440*x^6 - 7815347306496*I*x^5 + 1795603
562496*x^4 + 402462867456*I*x^3 - 35032590720*x^2 + (733835427840
*x^6 + 1651129712640*I*x^5 - 1393140695040*x^4 - 541776936960*I*x
^3 + 95234227200*x^2 - (366917713920*x^5 + 687970713600*I*x^4 - 4
64380231680*x^3 - 135444234240*I*x^2 + 15872371200*x + 510183360*
I)*sqrt(4*x^2 + 3*I*x) + 6122200320*I*x - 63772920)*log(-2*x + sq
rt(4*x^2 + 3*I*x) - 3/4*I) - (70368744177664*x^11 + 2902710697328
64*I*x^10 - 497254133661696*x^9 - 453651625672704*I*x^8 + 2345664
17645568*x^7 + 67806393532416*I*x^6 - 10897456103424*x^5 - 169240
7955456*I*x^4 + 479859572736*x^3 + 78041677824*I*x^2 - 3193369920
*x + 72275976*I)*sqrt(4*x^2 + 3*I*x) - 153055008*I*x - 19663317)/
(26388279066624*x^6 + 59373627899904*I*x^5 - 50096498540544*x^4 -
 19481971654656*I*x^3 + 3424565329920*x^2 - (13194139533312*x^5 +
 24739011624960*I*x^4 - 16698832846848*x^3 - 4870492913664*I*x^2
+ 570760888320*x + 18345885696*I)*sqrt(4*x^2 + 3*I*x) + 220150628
352*I*x - 2293235712)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4x^2 + 3ix\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*I*x+4*x**2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(5/2), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4\,x^2 + 3i\,x\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((4*x^2 + 3*I*x)^(5/2), x)
```

3.4 $\int \left(3ix+4x^2\right)^{3/2}\,dx$

Optimal. Leaf size=69

$$\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}+\frac{27(8x+3i)\sqrt{4x^2+3ix}}{1024}+\frac{243i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{4096}$$

```
[Out] (27*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/1024 + ((3*I + 8*x)*((3*I)
*x + 4*x^2)^(3/2))/32 + ((243*I)/4096)*ArcSin[1 - ((8*I)/3)*x]
```

Rubi [A] time = 0.0368329, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}+\frac{27(8x+3i)\sqrt{4x^2+3ix}}{1024}+\frac{243i\sin^{-1}\left(1-\frac{8ix}{3}\right)}{4096}$$

Antiderivative was successfully verified.

```
[In]  Int[((3*I)*x + 4*x^2)^(3/2),x]
```

```
[Out] (27*(3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/1024 + ((3*I + 8*x)*((3*I)
*x + 4*x^2)^(3/2))/32 + ((243*I)/4096)*ArcSin[1 - ((8*I)/3)*x]
```

Rubi in Sympy [A] time = 2.43116, size = 56, normalized size = 0.81

$$\frac{(8x+3i)\left(4x^2+3ix\right)^{\frac{3}{2}}}{32}+\frac{27\,(8x+3i)\,\sqrt{4x^2+3ix}}{1024}+\frac{243\,\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{4096}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*I*x+4*x**2)**(3/2),x)
```

```
[Out] (8*x + 3*I)*(4*x**2 + 3*I*x)**(3/2)/32 + 27*(8*x + 3*I)*sqrt(4*x*
*2 + 3*I*x)/1024 + 243*asinh(8*x/3 + I)/4096
```

Mathematica [A] time = 0.0730044, size = 83, normalized size = 1.2

$$\frac{2x\left(4096x^4+7680ix^3-3744x^2+108ix-243\right)+243\sqrt{x}\sqrt{4x+3i}\log\left(2\sqrt{x}+\sqrt{4x+3i}\right)}{2048\sqrt{x(4x+3i)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((3*I)*x + 4*x^2)^(3/2),x]
```

```
[Out] (2*x*(-243 + (108*I)*x - 3744*x^2 + (7680*I)*x^3 + 4096*x^4) + 24
3*Sqrt[x]*Sqrt[3*I + 4*x]*Log[2*Sqrt[x] + Sqrt[3*I + 4*x]])/(2048
*Sqrt[x*(3*I + 4*x)])
```

Maple [A] time = 0.01, size = 51, normalized size = 0.7

$$\frac{3\,i+8\,x}{32}\left(3\,ix+4\,x^{2}\right)^{\frac{3}{2}}+\frac{81\,i+216\,x}{1024}\sqrt{3\,ix+4\,x^{2}}+\frac{243}{4096}\mathit{Arcsinh}\left(\frac{8\,x}{3}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*I*x+4*x^2)^(3/2),x)
```

```
[Out] 1/32*(3*I+8*x)*(3*I*x+4*x^2)^(3/2)+27/1024*(3*I+8*x)*(3*I*x+4*x^2
)^(1/2)+243/4096*arcsinh(8/3*x+I)
```

Maxima [A] time = 0.793266, size = 103, normalized size = 1.49

$$\frac{1}{4}\left(4\,x^{2}+3i\,x\right)^{\frac{3}{2}}x+\frac{3}{32}i\left(4\,x^{2}+3i\,x\right)^{\frac{3}{2}}+\frac{27}{128}\sqrt{4\,x^{2}+3i\,x}x+\frac{81}{1024}i\sqrt{4\,x^{2}+3i\,x}+\frac{243}{4096}\log\left(8\,x+4\sqrt{4\,x^{2}+3i\,x}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*x^2 + 3*I*x)^(3/2)*x + 3/32*I*(4*x^2 + 3*I*x)^(3/2) + 27/1
28*sqrt(4*x^2 + 3*I*x)*x + 81/1024*I*sqrt(4*x^2 + 3*I*x) + 243/40
96*log(8*x + 4*sqrt(4*x^2 + 3*I*x) + 3*I)
```

Fricas [A] time = 0.219453, size = 278, normalized size = 4.03

$$-\frac{2147483648\,x^{8}+6442450944i\,x^{7}-7247757312\,x^{6}-3623878656i\,x^{5}+623738880\,x^{4}-83607552i\,x^{3}+33219072\,x^{2}+\left(63\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -(2147483648*x^8 + 6442450944*I*x^7 - 7247757312*x^6 - 3623878656
*I*x^5 + 623738880*x^4 - 83607552*I*x^3 + 33219072*x^2 + (6370099
2*x^4 + 95551488*I*x^3 - 44789760*x^2 - (31850496*x^3 + 35831808*
I*x^2 - 11197440*x - 839808*I)*sqrt(4*x^2 + 3*I*x) - 6718464*I*x
+ 157464)*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I) - (1073741824*x
^7 + 2818572288*I*x^6 - 2642411520*x^5 - 990904320*I*x^4 + 650280
96*x^3 - 38320128*I*x^2 + 5505408*x - 34992*I)*sqrt(4*x^2 + 3*I*x
) + 1399680*I*x + 45927)/(1073741824*x^4 + 1610612736*I*x^3 - 754
974720*x^2 - (536870912*x^3 + 603979776*I*x^2 - 188743680*x - 141
55776*I)*sqrt(4*x^2 + 3*I*x) - 113246208*I*x + 2654208)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \left(4x^2 + 3ix\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*I*x+4*x**2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(3/2), x)
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int \left(4\,x^2 + 3i\,x\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((4*x^2 + 3*I*x)^(3/2), x)
```

3.5 $\int \sqrt{3ix+4x^2}\,dx$

Optimal. Leaf size=43

$$\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)+\frac{9}{64}i\sin^{-1}\left(1-\frac{8ix}{3}\right)$$

```
[Out] ((3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/16 + ((9*I)/64)*ArcSin[1 - ((
8*I)/3)*x]
```

Rubi [A] time = 0.024702, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2

$$\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)+\frac{9}{64}i\sin^{-1}\left(1-\frac{8ix}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[(3*I)*x + 4*x^2],x]
```

```
[Out] ((3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/16 + ((9*I)/64)*ArcSin[1 - ((
8*I)/3)*x]
```

Rubi in Sympy [A] time = 1.82941, size = 32, normalized size = 0.74

$$\frac{(8x+3i)\sqrt{4x^2+3ix}}{16}+\frac{9\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*I*x+4*x**2)**(1/2),x)
```

```
[Out] (8*x + 3*I)*sqrt(4*x**2 + 3*I*x)/16 + 9*asinh(8*x/3 + I)/64
```

Mathematica [A] time = 0.0541475, size = 62, normalized size = 1.44

$$\frac{1}{32}\sqrt{x(4x+3i)}\left(16x+\frac{9\log\left(2\sqrt{x}+\sqrt{4x+3i}\right)}{\sqrt{4x+3i}\sqrt{x}}+6i\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[(3*I)*x + 4*x^2],x]
```

```
[Out] (Sqrt[x*(3*I + 4*x)]*(6*I + 16*x + (9*Log[2*Sqrt[x] + Sqrt[3*I +
4*x]])/(Sqrt[x]*Sqrt[3*I + 4*x])))/32
```

Maple [A] time = 0.01, size = 31, normalized size = 0.7

$$\frac{3\,i+8\,x}{16}\sqrt{3\,ix+4\,x^{2}}+\frac{9}{64}Arcsinh\left(\frac{8\,x}{3}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*I*x+4*x^2)^(1/2),x)
```

```
[Out] 1/16*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)+9/64*arcsinh(8/3*x+I)
```

Maxima [A] time = 0.806609, size = 66, normalized size = 1.53

$$\frac{1}{2}\sqrt{4\,x^{2}+3i\,x}x+\frac{3}{16}i\sqrt{4\,x^{2}+3i\,x}+\frac{9}{64}\log\left(8\,x+4\sqrt{4\,x^{2}+3i\,x}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(4*x^2 + 3*I*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(4*x^2 + 3*I*x)*x + 3/16*I*sqrt(4*x^2 + 3*I*x) + 9/64*log
(8*x + 4*sqrt(4*x^2 + 3*I*x) + 3*I)
```

Fricas [A] time = 0.217779, size = 170, normalized size = 3.95

$$-\frac{32768\,x^{4}+49152i\,x^{3}-21888\,x^{2}+\left(4608\,x^{2}-\sqrt{4\,x^{2}+3i\,x}(2304\,x+864i)+3456i\,x-324\right)\log\left(-2\,x+\sqrt{4\,x^{2}+3i\,x}-\frac{3}{4}i\right)}{32768\,x^{2}-\sqrt{4\,x^{2}+3i\,x}(16384\,x+6144i)+24576i\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(4*x^2 + 3*I*x),x, algorithm="fricas")
```

```
[Out] -(32768*x^4 + 49152*I*x^3 - 21888*x^2 + (4608*x^2 - sqrt(4*x^2 +
3*I*x)*(2304*x + 864*I) + 3456*I*x - 324)*log(-2*x + sqrt(4*x^2 +
 3*I*x) - 3/4*I) - (16384*x^3 + 18432*I*x^2 - 5184*x - 216*I)*sqr
t(4*x^2 + 3*I*x) - 2592*I*x - 81)/(32768*x^2 - sqrt(4*x^2 + 3*I*x
)*(16384*x + 6144*I) + 24576*I*x - 2304)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4x^2 + 3ix}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*I*x+4*x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(4*x**2 + 3*I*x), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{4\,x^2 + 3i\,x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(4*x^2 + 3*I*x),x, algorithm="giac")
```

```
[Out] integrate(sqrt(4*x^2 + 3*I*x), x)
```

3.6 $\int \left(3x-4x^2\right)^{7/2}\,dx$

Optimal. Leaf size=101

$$-\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}-\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}-\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072}-\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304}-\frac{229635\sin^{-1}\left(1-\frac{8x}{3}\right)}{16777216}$$

```
[Out] (-25515*(3 - 8*x)*Sqrt[3*x - 4*x^2])/4194304 - (945*(3 - 8*x)*(3*
x - 4*x^2)^(3/2))/131072 - (21*(3 - 8*x)*(3*x - 4*x^2)^(5/2))/204
8 - ((3 - 8*x)*(3*x - 4*x^2)^(7/2))/64 - (229635*ArcSin[1 - (8*x)
/3])/16777216
```

Rubi [A] time = 0.0641755, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{1}{64}(3-8x)\left(3x-4x^2\right)^{7/2}-\frac{21(3-8x)\left(3x-4x^2\right)^{5/2}}{2048}-\frac{945(3-8x)\left(3x-4x^2\right)^{3/2}}{131072}-\frac{25515(3-8x)\sqrt{3x-4x^2}}{4194304}-\frac{229635\sin^{-1}\left(1-\frac{8x}{3}\right)}{16777216}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*x - 4*x^2)^(7/2),x]
```

```
[Out] (-25515*(3 - 8*x)*Sqrt[3*x - 4*x^2])/4194304 - (945*(3 - 8*x)*(3*
x - 4*x^2)^(3/2))/131072 - (21*(3 - 8*x)*(3*x - 4*x^2)^(5/2))/204
8 - ((3 - 8*x)*(3*x - 4*x^2)^(7/2))/64 - (229635*ArcSin[1 - (8*x)
/3])/16777216
```

Rubi in Sympy [A] time = 3.57857, size = 90, normalized size = 0.89

$$-\frac{(-8x+3)\left(-4x^2+3x\right)^{\frac{7}{2}}}{64}-\frac{21\left(-8x+3\right)\left(-4x^2+3x\right)^{\frac{5}{2}}}{2048}-\frac{945\left(-8x+3\right)\left(-4x^2+3x\right)^{\frac{3}{2}}}{131072}-\frac{25515\left(-8x+3\right)\sqrt{-4x^2+3x}}{4194304}+\frac{229635\,\mathrm{asin}\left(\frac{8x}{3}-1\right)}{16777216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-4*x**2+3*x)**(7/2),x)
```

```
[Out] -(-8*x + 3)*(-4*x**2 + 3*x)**(7/2)/64 - 21*(-8*x + 3)*(-4*x**2 +
3*x)**(5/2)/2048 - 945*(-8*x + 3)*(-4*x**2 + 3*x)**(3/2)/131072 -
 25515*(-8*x + 3)*sqrt(-4*x**2 + 3*x)/4194304 + 229635*asin(8*x/3
 - 1)/16777216
```

Mathematica [A] time = 0.110024, size = 102, normalized size = 1.01

$$\frac{\sqrt{x}\sqrt{4x-3}\left(2\sqrt{x}\sqrt{4x-3}\left(33554432x^7-88080384x^6+79429632x^5-25067520x^4+82944x^3+72576x^2+68040x+76545\right.\right.}{8388608\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*x - 4*x^2)^(7/2),x]
```

```
[Out] (Sqrt[x]*Sqrt[-3 + 4*x]*(2*Sqrt[x]*Sqrt[-3 + 4*x]*(76545 + 68040*
x + 72576*x^2 + 82944*x^3 - 25067520*x^4 + 79429632*x^5 - 8808038
4*x^6 + 33554432*x^7) + 229635*Log[2*Sqrt[x] + Sqrt[-3 + 4*x]]))/
(8388608*Sqrt[-(x*(-3 + 4*x))])
```

Maple [A] time = 0.005, size = 82, normalized size = 0.8

$$-\frac{2835-7560\,x}{131072}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}-\frac{63-168\,x}{2048}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}-\frac{3-8\,x}{64}\left(-4\,x^2+3\,x\right)^{\frac{7}{2}}+\frac{229635}{16777216}\arcsin\left(-1+\frac{8\,x}{3}\right)-\frac{76545-204120\,x}{4194304}\sqrt{-4\,x^2+3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-4*x^2+3*x)^(7/2),x)
```

```
[Out] -945/131072*(3-8*x)*(-4*x^2+3*x)^(3/2)-21/2048*(3-8*x)*(-4*x^2+3*
x)^(5/2)-1/64*(3-8*x)*(-4*x^2+3*x)^(7/2)+229635/16777216*arcsin(-
1+8/3*x)-25515/4194304*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

Maxima [A] time = 0.789892, size = 158, normalized size = 1.56

$$\frac{1}{8}\left(-4\,x^2+3\,x\right)^{\frac{7}{2}}x-\frac{3}{64}\left(-4\,x^2+3\,x\right)^{\frac{7}{2}}+\frac{21}{256}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}x-\frac{63}{2048}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}+\frac{945}{16384}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}x-\frac{2835}{131072}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}+\frac{25515}{524288}\sqrt{-4\,x^2+3\,x}x-\frac{76545}{4194304}\sqrt{-4\,x^2+3\,x}-\frac{229635}{16777216}\arcsin\left(-\frac{8}{3}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/8*(-4*x^2 + 3*x)^(7/2)*x - 3/64*(-4*x^2 + 3*x)^(7/2) + 21/256*(
-4*x^2 + 3*x)^(5/2)*x - 63/2048*(-4*x^2 + 3*x)^(5/2) + 945/16384*
(-4*x^2 + 3*x)^(3/2)*x - 2835/131072*(-4*x^2 + 3*x)^(3/2) + 25515
/524288*sqrt(-4*x^2 + 3*x)*x - 76545/4194304*sqrt(-4*x^2 + 3*x) -
 229635/16777216*arcsin(-8/3*x + 1)
```

Fricas [A] time = 0.219629, size = 92, normalized size = 0.91

$$-\frac{1}{4194304}\left(33554432\,x^{7}-88080384\,x^{6}+79429632\,x^{5}-25067520\,x^{4}+82944\,x^{3}+72576\,x^{2}+68040\,x+76545\right)\sqrt{-4\,x^{2}+3\,x}-\frac{229635}{8388608}\arctan\left(\frac{\sqrt{-4\,x^{2}+3\,x}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/4194304*(33554432*x^7 - 88080384*x^6 + 79429632*x^5 - 25067520
*x^4 + 82944*x^3 + 72576*x^2 + 68040*x + 76545)*sqrt(-4*x^2 + 3*x
) - 229635/8388608*arctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-4x^{2}+3x\right)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x**2+3*x)**(7/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(7/2), x)
```

<u>GIAC/XCAS</u> [A] time = 0.215052, size = 77, normalized size = 0.76

$$-\frac{1}{4194304}\,(8\,(16\,(8\,(32\,(8\,(16\,(8\,x-21)x+303)x-765)x+81)x+567)x+8505)x+76545)\sqrt{-4\,x^2+3\,x}+\frac{229635}{16777216}\,\arcsin\left(\frac{8}{3}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(7/2),x, algorithm="giac")
```

```
[Out] -1/4194304*(8*(16*(8*(32*(8*(16*(8*x - 21)*x + 303)*x - 765)*x +
81)*x + 567)*x + 8505)*x + 76545)*sqrt(-4*x^2 + 3*x) + 229635/167
77216*arcsin(8/3*x - 1)
```

3.7 $\int \left(3x-4x^2\right)^{5/2}\,dx$

Optimal. Leaf size=79

$$-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}-\frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024}-\frac{405(3-8x)\sqrt{3x-4x^2}}{32768}-\frac{3645\sin^{-1}\left(1-\frac{8x}{3}\right)}{131072}$$

```
[Out] (-405*(3 - 8*x)*Sqrt[3*x - 4*x^2])/32768 - (15*(3 - 8*x)*(3*x - 4
*x^2)^(3/2))/1024 - ((3 - 8*x)*(3*x - 4*x^2)^(5/2))/48 - (3645*Ar
cSin[1 - (8*x)/3])/131072
```

Rubi [A] time = 0.0469482, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}-\frac{15(3-8x)\left(3x-4x^2\right)^{3/2}}{1024}-\frac{405(3-8x)\sqrt{3x-4x^2}}{32768}-\frac{3645\sin^{-1}\left(1-\frac{8x}{3}\right)}{131072}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*x - 4*x^2)^(5/2),x]
```

```
[Out] (-405*(3 - 8*x)*Sqrt[3*x - 4*x^2])/32768 - (15*(3 - 8*x)*(3*x - 4
*x^2)^(3/2))/1024 - ((3 - 8*x)*(3*x - 4*x^2)^(5/2))/48 - (3645*Ar
cSin[1 - (8*x)/3])/131072
```

Rubi in Sympy [A] time = 2.81776, size = 70, normalized size = 0.89

$$-\frac{(-8x+3)\left(-4x^2+3x\right)^{\frac{5}{2}}}{48}-\frac{15\,(-8x+3)\left(-4x^2+3x\right)^{\frac{3}{2}}}{1024}-\frac{405\,(-8x+3)\sqrt{-4x^2+3x}}{32768}+\frac{3645\,\mathrm{asin}\left(\frac{8x}{3}-1\right)}{131072}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-4*x**2+3*x)**(5/2),x)
```

```
[Out] -(-8*x + 3)*(-4*x**2 + 3*x)**(5/2)/48 - 15*(-8*x + 3)*(-4*x**2 +
3*x)**(3/2)/1024 - 405*(-8*x + 3)*sqrt(-4*x**2 + 3*x)/32768 + 364
5*asin(8*x/3 - 1)/131072
```

Mathematica [A] time = 0.0610432, size = 92, normalized size = 1.16

$$\frac{\sqrt{-x(4x-3)}\left(2\sqrt{x}\sqrt{4x-3}\left(262144x^5-491520x^4+248832x^3-3456x^2-3240x-3645\right)-10935\log\left(2\sqrt{x}+\sqrt{4x-3}\right)\right)}{196608\sqrt{x}\sqrt{4x-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*x - 4*x^2)^(5/2),x]
```

```
[Out] (Sqrt[-(x*(-3 + 4*x))]*(2*Sqrt[x]*Sqrt[-3 + 4*x]*(-3645 - 3240*x
- 3456*x^2 + 248832*x^3 - 491520*x^4 + 262144*x^5) - 10935*Log[2*
Sqrt[x] + Sqrt[-3 + 4*x]]))/(196608*Sqrt[x]*Sqrt[-3 + 4*x])
```

Maple [A] time = 0.006, size = 64, normalized size = 0.8

$$-\frac{45-120\,x}{1024}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}-\frac{3-8\,x}{48}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}+\frac{3645}{131072}\arcsin\left(-1+\frac{8\,x}{3}\right)-\frac{1215-3240\,x}{32768}\sqrt{-4\,x^2+3\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-4*x^2+3*x)^(5/2),x)
```

```
[Out] -15/1024*(3-8*x)*(-4*x^2+3*x)^(3/2)-1/48*(3-8*x)*(-4*x^2+3*x)^(5/
2)+3645/131072*arcsin(-1+8/3*x)-405/32768*(3-8*x)*(-4*x^2+3*x)^(1
/2)
```

Maxima [A] time = 0.797944, size = 122, normalized size = 1.54

$$\frac{1}{6}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}x-\frac{1}{16}\left(-4\,x^2+3\,x\right)^{\frac{5}{2}}+\frac{15}{128}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}x-\frac{45}{1024}\left(-4\,x^2+3\,x\right)^{\frac{3}{2}}+\frac{405}{4096}\sqrt{-4\,x^2+3\,x}x-\frac{1215}{32768}\sqrt{-4\,x^2+3\,x}-\frac{3645}{131072}\arcsin\left(-\frac{8}{3}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(-4*x^2 + 3*x)^(5/2)*x - 1/16*(-4*x^2 + 3*x)^(5/2) + 15/128*(
-4*x^2 + 3*x)^(3/2)*x - 45/1024*(-4*x^2 + 3*x)^(3/2) + 405/4096*s
```

```
qrt(-4*x^2 + 3*x)*x - 1215/32768*sqrt(-4*x^2 + 3*x) - 3645/131072
*arcsin(-8/3*x + 1)
```

Fricas [A] time = 0.216161, size = 78, normalized size = 0.99

$$\frac{1}{98304}\left(262144\,x^5-491520\,x^4+248832\,x^3-3456\,x^2-3240\,x-3645\right)\sqrt{-4\,x^2+3\,x}-\frac{3645}{65536}\arctan\left(\frac{\sqrt{-4\,x^2+3\,x}}{2\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/98304*(262144*x^5 - 491520*x^4 + 248832*x^3 - 3456*x^2 - 3240*x
 - 3645)*sqrt(-4*x^2 + 3*x) - 3645/65536*arctan(1/2*sqrt(-4*x^2 +
 3*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(-4x^2+3x\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x**2+3*x)**(5/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(5/2), x)
```

GIAC/XCAS [A] time = 0.213834, size = 63, normalized size = 0.8

$$\frac{1}{98304}\left(8\left(16\left(8\left(32\left(8\,x-15\right)x+243\right)x-27\right)x-405\right)x-3645\right)\sqrt{-4\,x^2+3\,x}+\frac{3645}{131072}\arcsin\left(\frac{8}{3}\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/98304*(8*(16*(8*(32*(8*x - 15)*x + 243)*x - 27)*x - 405)*x - 36
45)*sqrt(-4*x^2 + 3*x) + 3645/131072*arcsin(8/3*x - 1)
```

3.8 $\int \left(3x-4x^2\right)^{3/2} \, dx$

Optimal. Leaf size=57

$$-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}-\frac{27(3-8x)\sqrt{3x-4x^2}}{1024}-\frac{243\sin^{-1}\left(1-\frac{8x}{3}\right)}{4096}$$

```
[Out] (-27*(3 - 8*x)*Sqrt[3*x - 4*x^2])/1024 - ((3 - 8*x)*(3*x - 4*x^2)
^(3/2))/32 - (243*ArcSin[1 - (8*x)/3])/4096
```

Rubi [A] time = 0.0331173, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}-\frac{27(3-8x)\sqrt{3x-4x^2}}{1024}-\frac{243\sin^{-1}\left(1-\frac{8x}{3}\right)}{4096}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*x - 4*x^2)^(3/2),x]
```

```
[Out] (-27*(3 - 8*x)*Sqrt[3*x - 4*x^2])/1024 - ((3 - 8*x)*(3*x - 4*x^2)
^(3/2))/32 - (243*ArcSin[1 - (8*x)/3])/4096
```

Rubi in Sympy [A] time = 2.18366, size = 49, normalized size = 0.86

$$-\frac{(-8x+3)\left(-4x^2+3x\right)^{\frac{3}{2}}}{32}-\frac{27\left(-8x+3\right)\sqrt{-4x^2+3x}}{1024}+\frac{243\,\mathrm{asin}\left(\frac{8x}{3}-1\right)}{4096}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-4*x**2+3*x)**(3/2),x)
```

```
[Out] -(-8*x + 3)*(-4*x**2 + 3*x)**(3/2)/32 - 27*(-8*x + 3)*sqrt(-4*x**
2 + 3*x)/1024 + 243*asin(8*x/3 - 1)/4096
```

Mathematica [A] time = 0.0714304, size = 74, normalized size = 1.3

$$\frac{2x\left(4096x^4-7680x^3+3744x^2+108x-243\right)+243\sqrt{x}\sqrt{4x-3}\log\left(2\sqrt{x}+\sqrt{4x-3}\right)}{2048\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*x - 4*x^2)^(3/2),x]
```

```
[Out] (2*x*(-243 + 108*x + 3744*x^2 - 7680*x^3 + 4096*x^4) + 243*Sqrt[x
]*Sqrt[-3 + 4*x]*Log[2*Sqrt[x] + Sqrt[-3 + 4*x]])/(2048*Sqrt[-(x*
(-3 + 4*x))])
```

Maple [A] time = 0.004, size = 46, normalized size = 0.8

$$-\frac{3-8x}{32}\left(-4x^{2}+3x\right)^{\frac{3}{2}}+\frac{243}{4096}\arcsin\left(-1+\frac{8x}{3}\right)-\frac{81-216x}{1024}\sqrt{-4x^{2}+3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-4*x^2+3*x)^(3/2),x)
```

```
[Out] -1/32*(3-8*x)*(-4*x^2+3*x)^(3/2)+243/4096*arcsin(-1+8/3*x)-27/102
4*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

Maxima [A] time = 0.793911, size = 85, normalized size = 1.49

$$\frac{1}{4}\left(-4x^{2}+3x\right)^{\frac{3}{2}}x-\frac{3}{32}\left(-4x^{2}+3x\right)^{\frac{3}{2}}+\frac{27}{128}\sqrt{-4x^{2}+3x}x-\frac{81}{1024}\sqrt{-4x^{2}+3x}-\frac{243}{4096}\arcsin\left(-\frac{8}{3}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-4*x^2 + 3*x)^(3/2)*x - 3/32*(-4*x^2 + 3*x)^(3/2) + 27/128*s
qrt(-4*x^2 + 3*x)*x - 81/1024*sqrt(-4*x^2 + 3*x) - 243/4096*arcsi
n(-8/3*x + 1)
```

Fricas [A] time = 0.210289, size = 65, normalized size = 1.14

$$-\frac{1}{1024}\left(1024x^{3}-1152x^{2}+72x+81\right)\sqrt{-4x^{2}+3x}-\frac{243}{2048}\arctan\left(\frac{\sqrt{-4x^{2}+3x}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/1024*(1024*x^3 - 1152*x^2 + 72*x + 81)*sqrt(-4*x^2 + 3*x) - 24
3/2048*arctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-4x^2 + 3x\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x**2+3*x)**(3/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(3/2), x)
```

GIAC/XCAS [A] time = 0.216115, size = 50, normalized size = 0.88

$$-\frac{1}{1024}\,(8\,(16\,(8\,x - 9)x + 9)x + 81)\sqrt{-4\,x^2 + 3\,x} + \frac{243}{4096}\,\arcsin\left(\frac{8}{3}\,x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/1024*(8*(16*(8*x - 9)*x + 9)*x + 81)*sqrt(-4*x^2 + 3*x) + 243/
4096*arcsin(8/3*x - 1)
```

3.9 $\int \sqrt{3x-4x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)-\frac{9}{64}\sin^{-1}\left(1-\frac{8x}{3}\right)$$

```
[Out] -((3 - 8*x)*Sqrt[3*x - 4*x^2])/16 - (9*ArcSin[1 - (8*x)/3])/64
```

Rubi [A] time = 0.0227386, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)-\frac{9}{64}\sin^{-1}\left(1-\frac{8x}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[3*x - 4*x^2],x]
```

```
[Out] -((3 - 8*x)*Sqrt[3*x - 4*x^2])/16 - (9*ArcSin[1 - (8*x)/3])/64
```

Rubi in Sympy [A] time = 1.7257, size = 29, normalized size = 0.83

$$-\frac{(-8x+3)\sqrt{-4x^2+3x}}{16}+\frac{9\,\mathrm{asin}\left(\frac{8x}{3}-1\right)}{64}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-4*x**2+3*x)**(1/2),x)
```

```
[Out] -(-8*x + 3)*sqrt(-4*x**2 + 3*x)/16 + 9*asin(8*x/3 - 1)/64
```

Mathematica [B] time = 0.0331512, size = 72, normalized size = 2.06

$$\frac{\sqrt{-x(4x-3)}\left(2\sqrt{x}\sqrt{4x-3}(8x-3)-9\log\left(2\sqrt{x}+\sqrt{4x-3}\right)\right)}{32\sqrt{x}\sqrt{4x-3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[3*x - 4*x^2],x]
```

```
[Out] (Sqrt[-(x*(-3 + 4*x))]*(2*Sqrt[x]*Sqrt[-3 + 4*x]*(-3 + 8*x) - 9*L
og[2*Sqrt[x] + Sqrt[-3 + 4*x]]))/(32*Sqrt[x]*Sqrt[-3 + 4*x])
```

Maple [A] time = 0.004, size = 28, normalized size = 0.8

$$\frac{9}{64}\arcsin\left(-1+\frac{8x}{3}\right)-\frac{3-8x}{16}\sqrt{-4x^2+3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-4*x^2+3*x)^(1/2),x)
```

```
[Out] 9/64*arcsin(-1+8/3*x)-1/16*(3-8*x)*(-4*x^2+3*x)^(1/2)
```

Maxima [A] time = 0.803727, size = 49, normalized size = 1.4

$$\frac{1}{2}\sqrt{-4x^2+3x}x-\frac{3}{16}\sqrt{-4x^2+3x}-\frac{9}{64}\arcsin\left(-\frac{8}{3}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-4*x^2 + 3*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-4*x^2 + 3*x)*x - 3/16*sqrt(-4*x^2 + 3*x) - 9/64*arcsin(
-8/3*x + 1)
```

Fricas [A] time = 0.21152, size = 51, normalized size = 1.46

$$\frac{1}{16}\sqrt{-4x^2+3x}(8x-3)-\frac{9}{32}\arctan\left(\frac{\sqrt{-4x^2+3x}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-4*x^2 + 3*x),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) - 9/32*arctan(1/2*sqrt(-4*x^2 +
 3*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-4x^2 + 3x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x**2+3*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-4*x**2 + 3*x), x)
```

GIAC/XCAS [A] time = 0.211361, size = 36, normalized size = 1.03

$$\frac{1}{16}\sqrt{-4\,x^2 + 3\,x}(8\,x - 3) + \frac{9}{64}\,\arcsin\left(\frac{8}{3}\,x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-4*x^2 + 3*x),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) + 9/64*arcsin(8/3*x - 1)
```

3.10 $\int \sqrt{6x - x^2}\, dx$

Optimal. Leaf size=35

$$-\frac{1}{2}\sqrt{6x-x^2}(3-x)-\frac{9}{2}\sin^{-1}\left(1-\frac{x}{3}\right)$$

```
[Out] -((3 - x)*Sqrt[6*x - x^2])/2 - (9*ArcSin[1 - x/3])/2
```

Rubi [A] time = 0.0244905, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{1}{2}\sqrt{6x-x^2}(3-x)-\frac{9}{2}\sin^{-1}\left(1-\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[6*x - x^2],x]

[Out] -((3 - x)*Sqrt[6*x - x^2])/2 - (9*ArcSin[1 - x/3])/2
```

Rubi in Sympy [A] time = 1.68944, size = 26, normalized size = 0.74

$$-\frac{(-2x+6)\sqrt{-x^2+6x}}{4}+\frac{9\,\mathrm{asin}\left(\frac{x}{3}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+6*x)**(1/2),x)

[Out] -(-2*x + 6)*sqrt(-x**2 + 6*x)/4 + 9*asin(x/3 - 1)/2
```

Mathematica [A] time = 0.0427161, size = 45, normalized size = 1.29

$$\frac{1}{2}\sqrt{-(x-6)x}\left(x-\frac{18\log\left(\sqrt{x-6}+\sqrt{x}\right)}{\sqrt{x-6}\sqrt{x}}-3\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[6*x - x^2],x]
```

```
[Out] (Sqrt[-((-6 + x)*x)]*(-3 + x - (18*Log[Sqrt[-6 + x] + Sqrt[x]])/(
Sqrt[-6 + x]*Sqrt[x])))/2
```

Maple [A] time = 0.005, size = 28, normalized size = 0.8

$$-\frac{-2\,x+6}{4}\sqrt{-x^2+6\,x}+\frac{9}{2}\arcsin\left(-1+\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+6*x)^(1/2),x)
```

```
[Out] -1/4*(-2*x+6)*(-x^2+6*x)^(1/2)+9/2*arcsin(-1+1/3*x)
```

Maxima [A] time = 0.798836, size = 49, normalized size = 1.4

$$\frac{1}{2}\,\sqrt{-x^2+6\,x}x-\frac{3}{2}\,\sqrt{-x^2+6\,x}-\frac{9}{2}\,\arcsin\left(-\frac{1}{3}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 6*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-x^2 + 6*x)*x - 3/2*sqrt(-x^2 + 6*x) - 9/2*arcsin(-1/3*x
 + 1)
```

Fricas [A] time = 0.213235, size = 47, normalized size = 1.34

$$\frac{1}{2}\,\sqrt{-x^2+6\,x}(x-3)-9\,\arctan\left(\frac{\sqrt{-x^2+6\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 6*x),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(-x^2 + 6*x)*(x - 3) - 9*arctan(sqrt(-x^2 + 6*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-x^2 + 6x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+6*x)**(1/2),x)

[Out] Integral(sqrt(-x**2 + 6*x), x)
```

GIAC/XCAS [A] time = 0.208856, size = 34, normalized size = 0.97

$$\frac{1}{2}\sqrt{-x^2 + 6\,x}(x-3) + \frac{9}{2}\,\arcsin\left(\frac{1}{3}\,x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-x^2 + 6*x),x, algorithm="giac")

[Out] 1/2*sqrt(-x^2 + 6*x)*(x - 3) + 9/2*arcsin(1/3*x - 1)
```

3.11 $\int \sqrt{5x-9x^2}\,dx$

Optimal. Leaf size=35

$$-\frac{1}{36}\sqrt{5x-9x^2}(5-18x)-\frac{25}{216}\sin^{-1}\left(1-\frac{18x}{5}\right)$$

```
[Out] -((5 - 18*x)*Sqrt[5*x - 9*x^2])/36 - (25*ArcSin[1 - (18*x)/5])/21
6
```

Rubi [A] time = 0.0239242, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$-\frac{1}{36}\sqrt{5x-9x^2}(5-18x)-\frac{25}{216}\sin^{-1}\left(1-\frac{18x}{5}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[5*x - 9*x^2],x]
```

```
[Out] -((5 - 18*x)*Sqrt[5*x - 9*x^2])/36 - (25*ArcSin[1 - (18*x)/5])/21
6
```

Rubi in Sympy [A] time = 1.72949, size = 29, normalized size = 0.83

$$-\frac{(-18x+5)\sqrt{-9x^2+5x}}{36}+\frac{25\,\mathrm{asin}\left(\frac{18x}{5}-1\right)}{216}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-9*x**2+5*x)**(1/2),x)
```

```
[Out] -(-18*x + 5)*sqrt(-9*x**2 + 5*x)/36 + 25*asin(18*x/5 - 1)/216
```

Mathematica [B] time = 0.0387896, size = 72, normalized size = 2.06

$$\frac{\sqrt{-x(9x-5)}\left(3\sqrt{x}\sqrt{9x-5}(18x-5)-25\log\left(3\sqrt{x}+\sqrt{9x-5}\right)\right)}{108\sqrt{x}\sqrt{9x-5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[5*x - 9*x^2],x]
```

```
[Out] (Sqrt[-(x*(-5 + 9*x))]*(3*Sqrt[x]*Sqrt[-5 + 9*x]*(-5 + 18*x) - 25
*Log[3*Sqrt[x] + Sqrt[-5 + 9*x]]))/(108*Sqrt[x]*Sqrt[-5 + 9*x])
```

Maple [A] time = 0.005, size = 28, normalized size = 0.8

$$\frac{25}{216}\arcsin\left(-1+\frac{18\,x}{5}\right)-\frac{5-18\,x}{36}\sqrt{-9\,x^{2}+5\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-9*x^2+5*x)^(1/2),x)
```

```
[Out] 25/216*arcsin(-1+18/5*x)-1/36*(5-18*x)*(-9*x^2+5*x)^(1/2)
```

Maxima [A] time = 0.851527, size = 49, normalized size = 1.4

$$\frac{1}{2}\sqrt{-9\,x^{2}+5\,x}x-\frac{5}{36}\sqrt{-9\,x^{2}+5\,x}-\frac{25}{216}\arcsin\left(-\frac{18}{5}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-9*x^2 + 5*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-9*x^2 + 5*x)*x - 5/36*sqrt(-9*x^2 + 5*x) - 25/216*arcsi
n(-18/5*x + 1)
```

Fricas [A] time = 0.211145, size = 51, normalized size = 1.46

$$\frac{1}{36}\sqrt{-9\,x^{2}+5\,x}(18\,x-5)-\frac{25}{108}\arctan\left(\frac{\sqrt{-9\,x^{2}+5\,x}}{3\,x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-9*x^2 + 5*x),x, algorithm="fricas")
```

```
[Out] 1/36*sqrt(-9*x^2 + 5*x)*(18*x - 5) - 25/108*arctan(1/3*sqrt(-9*x^
2 + 5*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-9x^2 + 5x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-9*x**2+5*x)**(1/2),x)
```

```
[Out] Integral(sqrt(-9*x**2 + 5*x), x)
```

GIAC/XCAS [A] time = 0.21081, size = 36, normalized size = 1.03

$$\frac{1}{36}\sqrt{-9\,x^2 + 5\,x}(18\,x - 5) + \frac{25}{216}\,\arcsin\left(\frac{18}{5}\,x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-9*x^2 + 5*x),x, algorithm="giac")
```

```
[Out] 1/36*sqrt(-9*x^2 + 5*x)*(18*x - 5) + 25/216*arcsin(18/5*x - 1)
```

3.12 $\int \left(x-x^2\right)^{3/2} \, dx$

Optimal. Leaf size=51

$$-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}-\frac{3}{64}(1-2x)\sqrt{x-x^2}-\frac{3}{128}\sin^{-1}(1-2x)$$

```
[Out] (-3*(1 - 2*x)*Sqrt[x - x^2])/64 - ((1 - 2*x)*(x - x^2)^(3/2))/8 -
 (3*ArcSin[1 - 2*x])/128
```

Rubi [A] time = 0.0265052, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}-\frac{3}{64}(1-2x)\sqrt{x-x^2}-\frac{3}{128}\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

```
[In]  Int[(x - x^2)^(3/2),x]
```

```
[Out] (-3*(1 - 2*x)*Sqrt[x - x^2])/64 - ((1 - 2*x)*(x - x^2)^(3/2))/8 -
 (3*ArcSin[1 - 2*x])/128
```

Rubi in Sympy [A] time = 1.89413, size = 41, normalized size = 0.8

$$-\frac{(-2x+1)\left(-x^2+x\right)^{\frac{3}{2}}}{8}-\frac{3(-2x+1)\sqrt{-x^2+x}}{64}+\frac{3\operatorname{asin}(2x-1)}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+x)**(3/2),x)
```

```
[Out] -(-2*x + 1)*(-x**2 + x)**(3/2)/8 - 3*(-2*x + 1)*sqrt(-x**2 + x)/6
4 + 3*asin(2*x - 1)/128
```

Mathematica [A] time = 0.0688677, size = 63, normalized size = 1.24

$$\frac{x\left(16x^4-40x^3+26x^2+x-3\right)+3\sqrt{x-1}\sqrt{x}\log\left(\sqrt{x-1}+\sqrt{x}\right)}{64\sqrt{-(x-1)x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(x - x^2)^(3/2),x]
```

```
[Out] (x*(-3 + x + 26*x^2 - 40*x^3 + 16*x^4) + 3*Sqrt[-1 + x]*Sqrt[x]*L
og[Sqrt[-1 + x] + Sqrt[x]])/(64*Sqrt[-((-1 + x)*x)])
```

Maple [A] time = 0.005, size = 42, normalized size = 0.8

$$-\frac{1-2x}{8}\left(-x^2+x\right)^{\frac{3}{2}}+\frac{3\arcsin\left(2x-1\right)}{128}-\frac{-6x+3}{64}\sqrt{-x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+x)^(3/2),x)
```

```
[Out] -1/8*(1-2*x)*(-x^2+x)^(3/2)+3/128*arcsin(2*x-1)-3/64*(1-2*x)*(-x^
2+x)^(1/2)
```

Maxima [A] time = 0.795012, size = 74, normalized size = 1.45

$$\frac{1}{4}\left(-x^2+x\right)^{\frac{3}{2}}x-\frac{1}{8}\left(-x^2+x\right)^{\frac{3}{2}}+\frac{3}{32}\sqrt{-x^2+x}x-\frac{3}{64}\sqrt{-x^2+x}+\frac{3}{128}\arcsin\left(2x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + x)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(-x^2 + x)^(3/2)*x - 1/8*(-x^2 + x)^(3/2) + 3/32*sqrt(-x^2 +
x)*x - 3/64*sqrt(-x^2 + x) + 3/128*arcsin(2*x - 1)
```

Fricas [A] time = 0.230842, size = 58, normalized size = 1.14

$$-\frac{1}{64}\left(16x^3-24x^2+2x+3\right)\sqrt{-x^2+x}-\frac{3}{64}\arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + x)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/64*(16*x^3 - 24*x^2 + 2*x + 3)*sqrt(-x^2 + x) - 3/64*arctan(sq
rt(-x^2 + x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^2+x\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+x)**(3/2),x)
```

```
[Out] Integral((-x**2 + x)**(3/2), x)
```

GIAC/XCAS [A] time = 0.212925, size = 47, normalized size = 0.92

$$-\frac{1}{64}\,(2\,(4\,(2\,x-3)x+1)x+3)\sqrt{-x^2+x}+\frac{3}{128}\,\arcsin\left(2\,x-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + x)^(3/2),x, algorithm="giac")
```

```
[Out] -1/64*(2*(4*(2*x - 3)*x + 1)*x + 3)*sqrt(-x^2 + x) + 3/128*arcsin
(2*x - 1)
```

3.13 $\int \sqrt{4x+x^2}\,dx$

Optimal. Leaf size=35

$$\frac{1}{2}(x+2)\sqrt{x^2+4x}-4\tanh^{-1}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

```
[Out] ((2 + x)*Sqrt[4*x + x^2])/2 - 4*ArcTanh[x/Sqrt[4*x + x^2]]
```

Rubi [A] time = 0.0195852, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{1}{2}(x+2)\sqrt{x^2+4x}-4\tanh^{-1}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[4*x + x^2],x]
```

```
[Out] ((2 + x)*Sqrt[4*x + x^2])/2 - 4*ArcTanh[x/Sqrt[4*x + x^2]]
```

Rubi in Sympy [A] time = 1.47816, size = 31, normalized size = 0.89

$$\frac{(2x+4)\sqrt{x^2+4x}}{4}-4\,\mathrm{atanh}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+4*x)**(1/2),x)
```

```
[Out] (2*x + 4)*sqrt(x**2 + 4*x)/4 - 4*atanh(x/sqrt(x**2 + 4*x))
```

Mathematica [A] time = 0.0432707, size = 40, normalized size = 1.14

$$\frac{1}{2}\sqrt{x(x+4)}\left(x-\frac{8\sinh^{-1}\left(\frac{\sqrt{x}}{2}\right)}{\sqrt{x+4}\sqrt{x}}+2\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[4*x + x^2],x]
```

```
[Out] (Sqrt[x*(4 + x)]*(2 + x - (8*ArcSinh[Sqrt[x]/2])/(Sqrt[x]*Sqrt[4
+ x])))/2
```

Maple [A] time = 0.005, size = 33, normalized size = 0.9

$$\frac{2x+4}{4}\sqrt{x^2+4x}-2\ln\left(2+x+\sqrt{x^2+4x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+4*x)^(1/2),x)
```

```
[Out] 1/4*(2*x+4)*(x^2+4*x)^(1/2)-2*ln(2+x+(x^2+4*x)^(1/2))
```

Maxima [A] time = 0.749732, size = 55, normalized size = 1.57

$$\frac{1}{2}\sqrt{x^2+4x}x+\sqrt{x^2+4x}-2\log\left(2x+2\sqrt{x^2+4x}+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 4*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 + 4*x)*x + sqrt(x^2 + 4*x) - 2*log(2*x + 2*sqrt(x^2
+ 4*x) + 4)
```

Fricas [A] time = 0.231899, size = 144, normalized size = 4.11

$$-\frac{x^4+8x^3+19x^2-4\left(x^2-\sqrt{x^2+4x}(x+2)+4x+2\right)\log\left(-x+\sqrt{x^2+4x}-2\right)-\left(x^3+6x^2+9x+2\right)\sqrt{x^2+4x}+12x-}{2\left(x^2-\sqrt{x^2+4x}(x+2)+4x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 4*x),x, algorithm="fricas")
```

```
[Out] -1/2*(x^4 + 8*x^3 + 19*x^2 - 4*(x^2 - sqrt(x^2 + 4*x)*(x + 2) + 4
*x + 2)*log(-x + sqrt(x^2 + 4*x) - 2) - (x^3 + 6*x^2 + 9*x + 2)*s
```

```
qrt(x^2 + 4*x) + 12*x - 2)/(x^2 - sqrt(x^2 + 4*x)*(x + 2) + 4*x +
 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2+4x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+4*x)**(1/2),x)

[Out] Integral(sqrt(x**2 + 4*x), x)
```

GIAC/XCAS [A] time = 0.210889, size = 45, normalized size = 1.29

$$\frac{1}{2}\sqrt{x^2+4\,x}(x+2)+2\ln\left(\left|-x+\sqrt{x^2+4\,x}-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 + 4*x),x, algorithm="giac")

[Out] 1/2*sqrt(x^2 + 4*x)*(x + 2) + 2*ln(abs(-x + sqrt(x^2 + 4*x) - 2))
```

3.14 $\int \sqrt{-8x + x^2}\, dx$

Optimal. Leaf size=37

$$-\frac{1}{2}\sqrt{x^2-8x}(4-x) - 16\tanh^{-1}\left(\frac{x}{\sqrt{x^2-8x}}\right)$$

```
[Out] -((4 - x)*Sqrt[-8*x + x^2])/2 - 16*ArcTanh[x/Sqrt[-8*x + x^2]]
```

Rubi [A] time = 0.0197001, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$-\frac{1}{2}\sqrt{x^2-8x}(4-x) - 16\tanh^{-1}\left(\frac{x}{\sqrt{x^2-8x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-8*x + x^2],x]
```

```
[Out] -((4 - x)*Sqrt[-8*x + x^2])/2 - 16*ArcTanh[x/Sqrt[-8*x + x^2]]
```

Rubi in Sympy [A] time = 1.48839, size = 32, normalized size = 0.86

$$-\frac{(-2x+8)\sqrt{x^2-8x}}{4} - 16\,\text{atanh}\left(\frac{x}{\sqrt{x^2-8x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-8*x)**(1/2),x)
```

```
[Out] -(-2*x + 8)*sqrt(x**2 - 8*x)/4 - 16*atanh(x/sqrt(x**2 - 8*x))
```

Mathematica [A] time = 0.0440485, size = 44, normalized size = 1.19

$$\frac{1}{2}\sqrt{(x-8)x}\left(x - \frac{32\log\left(\sqrt{x-8}+\sqrt{x}\right)}{\sqrt{x-8}\sqrt{x}} - 4\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-8*x + x^2],x]
```

```
[Out] (Sqrt[(-8 + x)*x]*(-4 + x - (32*Log[Sqrt[-8 + x] + Sqrt[x]])/(Sqr
t[-8 + x]*Sqrt[x])))/2
```

Maple [A] time = 0.005, size = 33, normalized size = 0.9

$$\frac{2x-8}{4}\sqrt{x^2-8x}-8\ln\left(x-4+\sqrt{x^2-8x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-8*x)^(1/2),x)
```

```
[Out] 1/4*(2*x-8)*(x^2-8*x)^(1/2)-8*ln(x-4+(x^2-8*x)^(1/2))
```

Maxima [A] time = 0.726829, size = 58, normalized size = 1.57

$$\frac{1}{2}\sqrt{x^2-8x}x-2\sqrt{x^2-8x}-8\log\left(2x+2\sqrt{x^2-8x}-8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - 8*x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 - 8*x)*x - 2*sqrt(x^2 - 8*x) - 8*log(2*x + 2*sqrt(x^
2 - 8*x) - 8)
```

Fricas [A] time = 0.214963, size = 144, normalized size = 3.89

$$-\frac{x^4-16x^3+76x^2-16\left(x^2-\sqrt{x^2-8x}(x-4)-8x+8\right)\log\left(-x+\sqrt{x^2-8x}+4\right)-\left(x^3-12x^2+36x-16\right)\sqrt{x^2-8x}}{2\left(x^2-\sqrt{x^2-8x}(x-4)-8x+8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - 8*x),x, algorithm="fricas")
```

```
[Out] -1/2*(x^4 - 16*x^3 + 76*x^2 - 16*(x^2 - sqrt(x^2 - 8*x)*(x - 4) -
 8*x + 8)*log(-x + sqrt(x^2 - 8*x) + 4) - (x^3 - 12*x^2 + 36*x -
```

```
16)*sqrt(x^2 - 8*x) - 96*x - 32)/(x^2 - sqrt(x^2 - 8*x)*(x - 4) -
 8*x + 8)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2 - 8x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-8*x)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 - 8*x), x)
```

GIAC/XCAS [A] time = 0.210566, size = 45, normalized size = 1.22

$$\frac{1}{2}\sqrt{x^2 - 8\,x}(x - 4) + 8\ln\left(\left|-x + \sqrt{x^2 - 8\,x} + 4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - 8*x),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2 - 8*x)*(x - 4) + 8*ln(abs(-x + sqrt(x^2 - 8*x) + 4))
```

3.15 $\int \sqrt{-x + x^2}\, dx$

Optimal. Leaf size=39

$$-\frac{1}{4}\sqrt{x^2 - x}(1 - 2x) - \frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x^2 - x}}\right)$$

```
[Out] -((1 - 2*x)*Sqrt[-x + x^2])/4 - ArcTanh[x/Sqrt[-x + x^2]]/4
```

Rubi [A] time = 0.0197663, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$-\frac{1}{4}\sqrt{x^2 - x}(1 - 2x) - \frac{1}{4}\tanh^{-1}\left(\frac{x}{\sqrt{x^2 - x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-x + x^2],x]

[Out] -((1 - 2*x)*Sqrt[-x + x^2])/4 - ArcTanh[x/Sqrt[-x + x^2]]/4
```

Rubi in Sympy [A] time = 1.48196, size = 29, normalized size = 0.74

$$-\frac{(-2x + 1)\sqrt{x^2 - x}}{4} - \frac{\operatorname{atanh}\left(\frac{x}{\sqrt{x^2 - x}}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2-x)**(1/2),x)

[Out] -(-2*x + 1)*sqrt(x**2 - x)/4 - atanh(x/sqrt(x**2 - x))/4
```

Mathematica [A] time = 0.0447864, size = 46, normalized size = 1.18

$$\frac{1}{4}\sqrt{(x-1)x}\left(2x - \frac{\log\left(\sqrt{x-1} + \sqrt{x}\right)}{\sqrt{x-1}\sqrt{x}} - 1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-x + x^2],x]
```

```
[Out] (Sqrt[(-1 + x)*x]*(-1 + 2*x - Log[Sqrt[-1 + x] + Sqrt[x]]/(Sqrt[-
1 + x]*Sqrt[x])))/4
```

Maple [A] time = 0.005, size = 33, normalized size = 0.9

$$\frac{2x-1}{4}\sqrt{x^2-x}-\frac{1}{8}\ln\left(-\frac{1}{2}+x+\sqrt{x^2-x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2-x)^(1/2),x)
```

```
[Out] 1/4*(2*x-1)*(x^2-x)^(1/2)-1/8*ln(-1/2+x+(x^2-x)^(1/2))
```

Maxima [A] time = 0.716423, size = 58, normalized size = 1.49

$$\frac{1}{2}\sqrt{x^2-x}x-\frac{1}{4}\sqrt{x^2-x}-\frac{1}{8}\log\left(2x+2\sqrt{x^2-x}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - x),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^2 - x)*x - 1/4*sqrt(x^2 - x) - 1/8*log(2*x + 2*sqrt(x^
2 - x) - 1)
```

Fricas [A] time = 0.216311, size = 163, normalized size = 4.18

$$-\frac{128x^4-256x^3+152x^2-4\left(8x^2-4\sqrt{x^2-x}(2x-1)-8x+1\right)\log\left(-2x+2\sqrt{x^2-x}+1\right)-4\left(32x^3-48x^2+18x-\right.}{32\left(8x^2-4\sqrt{x^2-x}(2x-1)-8x+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - x),x, algorithm="fricas")
```

```
[Out] -1/32*(128*x^4 - 256*x^3 + 152*x^2 - 4*(8*x^2 - 4*sqrt(x^2 - x)*(
2*x - 1) - 8*x + 1)*log(-2*x + 2*sqrt(x^2 - x) + 1) - 4*(32*x^3 -
```

```
48*x^2 + 18*x - 1)*sqrt(x^2 - x) - 24*x - 1)/(8*x^2 - 4*sqrt(x^2
- x)*(2*x - 1) - 8*x + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2 - x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2-x)**(1/2),x)
```

```
[Out] Integral(sqrt(x**2 - x), x)
```

GIAC/XCAS [A] time = 0.210647, size = 50, normalized size = 1.28

$$\frac{1}{4}\sqrt{x^2 - x}(2x - 1) + \frac{1}{8}\ln\left(\left|-2x + 2\sqrt{x^2 - x} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(x^2 - x),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^2 - x)*(2*x - 1) + 1/8*ln(abs(-2*x + 2*sqrt(x^2 - x) +
 1))
```

3.16 $\int \frac{1}{(bx+cx^2)^{7/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{256c^2(b+2cx)}{15b^6\sqrt{bx+cx^2}}+\frac{32c(b+2cx)}{15b^4\,(bx+cx^2)^{3/2}}-\frac{2(b+2cx)}{5b^2\,(bx+cx^2)^{5/2}}$$

```
[Out] (-2*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/2)) + (32*c*(b + 2*c*x))
/(15*b^4*(b*x + c*x^2)^(3/2)) - (256*c^2*(b + 2*c*x))/(15*b^6*Sqr
t[b*x + c*x^2])
```

Rubi [A] time = 0.0519505, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{256c^2(b+2cx)}{15b^6\sqrt{bx+cx^2}}+\frac{32c(b+2cx)}{15b^4\,(bx+cx^2)^{3/2}}-\frac{2(b+2cx)}{5b^2\,(bx+cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)^(-7/2),x]
```

```
[Out] (-2*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/2)) + (32*c*(b + 2*c*x))
/(15*b^4*(b*x + c*x^2)^(3/2)) - (256*c^2*(b + 2*c*x))/(15*b^6*Sqr
t[b*x + c*x^2])
```

Rubi in Sympy [A] time = 4.94692, size = 82, normalized size = 0.99

$$-\frac{2\,(b+2cx)}{5b^2\,(bx+cx^2)^{\frac{5}{2}}}+\frac{32c\,(b+2cx)}{15b^4\,(bx+cx^2)^{\frac{3}{2}}}-\frac{128c^2\,(2b+4cx)}{15b^6\sqrt{bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+b*x)**(7/2),x)
```

```
[Out] -2*(b + 2*c*x)/(5*b**2*(b*x + c*x**2)**(5/2)) + 32*c*(b + 2*c*x)/
(15*b**4*(b*x + c*x**2)**(3/2)) - 128*c**2*(2*b + 4*c*x)/(15*b**6
*sqrt(b*x + c*x**2))
```

Mathematica [A] time = 0.06572, size = 70, normalized size = 0.84

$$-\frac{2\left(3b^5-10b^4cx+80b^3c^2x^2+480b^2c^3x^3+640bc^4x^4+256c^5x^5\right)}{15b^6(x(b+cx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(-7/2),x]
```

```
[Out] (-2*(3*b^5 - 10*b^4*c*x + 80*b^3*c^2*x^2 + 480*b^2*c^3*x^3 + 640*
b*c^4*x^4 + 256*c^5*x^5))/(15*b^6*(x*(b + c*x))^(5/2))
```

Maple [A] time = 0.006, size = 75, normalized size = 0.9

$$-\frac{2\,x\,(cx+b)\left(256\,c^5x^5+640\,c^4x^4b+480\,c^3x^3b^2+80\,c^2x^2b^3-10\,cxb^4+3\,b^5\right)}{15\,b^6}\left(cx^2+bx\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+b*x)^(7/2),x)
```

```
[Out] -2/15*x*(c*x+b)*(256*c^5*x^5+640*b*c^4*x^4+480*b^2*c^3*x^3+80*b^3
*c^2*x^2-10*b^4*c*x+3*b^5)/b^6/(c*x^2+b*x)^(7/2)
```

Maxima [A] time = 0.738817, size = 150, normalized size = 1.81

$$-\frac{4\,cx}{5\,(cx^2+bx)^{\frac{5}{2}}b^2}+\frac{64\,c^2x}{15\,(cx^2+bx)^{\frac{3}{2}}b^4}-\frac{512\,c^3x}{15\,\sqrt{cx^2+bx}b^6}-\frac{2}{5\,(cx^2+bx)^{\frac{5}{2}}b}+\frac{32\,c}{15\,(cx^2+bx)^{\frac{3}{2}}b^3}-\frac{256\,c^2}{15\,\sqrt{cx^2+bx}b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-7/2),x, algorithm="maxima")
```

```
[Out] -4/5*c*x/((c*x^2 + b*x)^(5/2)*b^2) + 64/15*c^2*x/((c*x^2 + b*x)^(
3/2)*b^4) - 512/15*c^3*x/(sqrt(c*x^2 + b*x)*b^6) - 2/5/((c*x^2 +
b*x)^(5/2)*b) + 32/15*c/((c*x^2 + b*x)^(3/2)*b^3) - 256/15*c^2/(s
qrt(c*x^2 + b*x)*b^5)
```

Fricas [A] time = 0.216603, size = 127, normalized size = 1.53

$$-\frac{2\left(256\,c^5x^5+640\,bc^4x^4+480\,b^2c^3x^3+80\,b^3c^2x^2-10\,b^4cx+3\,b^5\right)}{15\,(b^6c^2x^4+2\,b^7cx^3+b^8x^2)\sqrt{cx^2+bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-7/2),x, algorithm="fricas")
```

```
[Out] -2/15*(256*c^5*x^5 + 640*b*c^4*x^4 + 480*b^2*c^3*x^3 + 80*b^3*c^2
*x^2 - 10*b^4*c*x + 3*b^5)/((b^6*c^2*x^4 + 2*b^7*c*x^3 + b^8*x^2)
*sqrt(c*x^2 + b*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(bx+cx^2)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+b*x)**(7/2),x)
```

```
[Out] Integral((b*x + c*x**2)**(-7/2), x)
```

GIAC/XCAS [A] time = 0.222061, size = 100, normalized size = 1.2

$$-\frac{2\left(2\left(8\left(2\left(4\,x\left(\frac{2\,c^5x}{b^6}+\frac{5\,c^4}{b^5}\right)+\frac{15\,c^3}{b^4}\right)x+\frac{5\,c^2}{b^3}\right)x-\frac{5\,c}{b^2}\right)x+\frac{3}{b}\right)}{15\,(cx^2+bx)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-7/2),x, algorithm="giac")
```

```
[Out] -2/15*(2*(8*(2*(4*x*(2*c^5*x/b^6 + 5*c^4/b^5) + 15*c^3/b^4)*x + 5
*c^2/b^3)*x - 5*c/b^2)*x + 3/b)/(c*x^2 + b*x)^(5/2)
```

3.17 $\int \frac{1}{\sqrt{3ix+4x^2}}\,dx$

Optimal. Leaf size=16

$$\frac{1}{2}i\sin^{-1}\left(1-\frac{8ix}{3}\right)$$

```
[Out] (I/2)*ArcSin[1 - ((8*I)/3)*x]
```

Rubi [A] time = 0.0159438, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{1}{2}i\sin^{-1}\left(1-\frac{8ix}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(3*I)*x + 4*x^2],x]

[Out] (I/2)*ArcSin[1 - ((8*I)/3)*x]
```

Rubi in Sympy [A] time = 1.36067, size = 8, normalized size = 0.5

$$\frac{\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*I*x+4*x**2)**(1/2),x)

[Out] asinh(8*x/3 + I)/2
```

Mathematica [B] time = 0.0178141, size = 50, normalized size = 3.12

$$\frac{\sqrt{x}\sqrt{4x+3i}\log\left(2\sqrt{x}+\sqrt{4x+3i}\right)}{\sqrt{x(4x+3i)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(3*I)*x + 4*x^2],x]
```

```
[Out] (Sqrt[x]*Sqrt[3*I + 4*x]*Log[2*Sqrt[x] + Sqrt[3*I + 4*x]])/Sqrt[x
*(3*I + 4*x)]
```

Maple [A] time = 0.01, size = 10, normalized size = 0.6

$$\frac{1}{2}\mathit{Arcsinh}\left(\frac{8x}{3}+i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*I*x+4*x^2)^(1/2),x)
```

```
[Out] 1/2*arcsinh(8/3*x+I)
```

Maxima [A] time = 0.792402, size = 28, normalized size = 1.75

$$\frac{1}{2}\log\left(8x+4\sqrt{4x^2+3ix}+3i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(4*x^2 + 3*I*x),x, algorithm="maxima")
```

```
[Out] 1/2*log(8*x + 4*sqrt(4*x^2 + 3*I*x) + 3*I)
```

Fricas [A] time = 0.212793, size = 26, normalized size = 1.62

$$-\frac{1}{2}\log\left(-2x+\sqrt{4x^2+3ix}-\frac{3}{4}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(4*x^2 + 3*I*x),x, algorithm="fricas")
```

```
[Out] -1/2*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{4x^2 + 3ix}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*I*x+4*x**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(4*x**2 + 3*I*x), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(4*x^2 + 3*I*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.18 $\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx$

Optimal. Leaf size=26

$$\frac{2(8x+3i)}{9\sqrt{4x^2+3ix}}$$

```
[Out] (2*(3*I + 8*x))/(9*Sqrt[(3*I)*x + 4*x^2])
```

Rubi [A] time = 0.0101403, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{2(8x+3i)}{9\sqrt{4x^2+3ix}}$$

Antiderivative was successfully verified.

```
[In] Int[((3*I)*x + 4*x^2)^(-3/2),x]
```

```
[Out] (2*(3*I + 8*x))/(9*Sqrt[(3*I)*x + 4*x^2])
```

Rubi in Sympy [A] time = 1.27876, size = 20, normalized size = 0.77

$$\frac{16x+6i}{9\sqrt{4x^2+3ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(3*I*x+4*x**2)**(3/2),x)
```

```
[Out] (16*x + 6*I)/(9*sqrt(4*x**2 + 3*I*x))
```

Mathematica [A] time = 0.0153368, size = 24, normalized size = 0.92

$$\frac{2(8x+3i)}{9\sqrt{x(4x+3i)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((3*I)*x + 4*x^2)^(-3/2),x]
```

```
[Out] (2*(3*I + 8*x))/(9*Sqrt[x*(3*I + 4*x)])
```

Maple [A] time = 0.01, size = 21, normalized size = 0.8

$$\frac{6\,i+16\,x}{9}\frac{1}{\sqrt{3\,ix+4\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*I*x+4*x^2)^(3/2),x)
```

```
[Out] 2/9*(3*I+8*x)/(3*I*x+4*x^2)^(1/2)
```

Maxima [A] time = 0.711006, size = 38, normalized size = 1.46

$$\frac{16\,x}{9\,\sqrt{4\,x^{2}+3i\,x}}+\frac{2i}{3\,\sqrt{4\,x^{2}+3i\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(-3/2),x, algorithm="maxima")
```

```
[Out] 16/9*x/sqrt(4*x^2 + 3*I*x) + 2/3*I/sqrt(4*x^2 + 3*I*x)
```

Fricas [A] time = 0.212089, size = 42, normalized size = 1.62

$$\frac{2}{16\,x^{2}-\sqrt{4\,x^{2}+3i\,x}(8\,x+3i)+12i\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(-3/2),x, algorithm="fricas")
```

```
[Out] 2/(16*x^2 - sqrt(4*x^2 + 3*I*x)*(8*x + 3*I) + 12*I*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(4x^2+3ix)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*I*x+4*x**2)**(3/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(-3/2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(-3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.19 $\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx$

Optimal. Leaf size=53

$$\frac{64(8x+3i)}{243\sqrt{4x^2+3ix}}+\frac{2(8x+3i)}{27\,(4x^2+3ix)^{3/2}}$$

```
[Out] (2*(3*I + 8*x))/(27*((3*I)*x + 4*x^2)^(3/2)) + (64*(3*I + 8*x))/(
243*Sqrt[(3*I)*x + 4*x^2])
```

Rubi [A] time = 0.0223492, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{64(8x+3i)}{243\sqrt{4x^2+3ix}}+\frac{2(8x+3i)}{27\,(4x^2+3ix)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((3*I)*x + 4*x^2)^(-5/2),x]
```

```
[Out] (2*(3*I + 8*x))/(27*((3*I)*x + 4*x^2)^(3/2)) + (64*(3*I + 8*x))/(
243*Sqrt[(3*I)*x + 4*x^2])
```

Rubi in Sympy [A] time = 1.8415, size = 42, normalized size = 0.79

$$\frac{2\,(8x+3i)}{27\,(4x^2+3ix)^{\frac{3}{2}}}+\frac{32\,(16x+6i)}{243\sqrt{4x^2+3ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*I*x+4*x**2)**(5/2),x)
```

```
[Out] 2*(8*x + 3*I)/(27*(4*x**2 + 3*I*x)**(3/2)) + 32*(16*x + 6*I)/(243
*sqrt(4*x**2 + 3*I*x))
```

Mathematica [A] time = 0.0244211, size = 36, normalized size = 0.68

$$\frac{2048x^3+2304ix^2-432x+54i}{243(x(4x+3i))^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((3*I)*x + 4*x^2)^(-5/2),x]
```

```
[Out] (54*I - 432*x + (2304*I)*x^2 + 2048*x^3)/(243*(x*(3*I + 4*x))^(3/
2))
```

Maple [A] time = 0.01, size = 42, normalized size = 0.8

$$\frac{6\,i+16\,x}{27}\left(3\,ix+4\,x^{2}\right)^{-\frac{3}{2}}+\frac{192\,i+512\,x}{243}\frac{1}{\sqrt{3\,ix+4\,x^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*I*x+4*x^2)^(5/2),x)
```

```
[Out] 2/27*(3*I+8*x)/(3*I*x+4*x^2)^(3/2)+64/243*(3*I+8*x)/(3*I*x+4*x^2)
^(1/2)
```

Maxima [A] time = 0.694289, size = 74, normalized size = 1.4

$$\frac{512\,x}{243\,\sqrt{4\,x^{2}+3i\,x}}+\frac{64i}{81\,\sqrt{4\,x^{2}+3i\,x}}+\frac{16\,x}{27\,(4\,x^{2}+3i\,x)^{\frac{3}{2}}}+\frac{2i}{9\,(4\,x^{2}+3i\,x)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(-5/2),x, algorithm="maxima")
```

```
[Out] 512/243*x/sqrt(4*x^2 + 3*I*x) + 64/81*I/sqrt(4*x^2 + 3*I*x) + 16/
27*x/(4*x^2 + 3*I*x)^(3/2) + 2/9*I/(4*x^2 + 3*I*x)^(3/2)
```

Fricas [A] time = 0.222867, size = 127, normalized size = 2.4

$$\frac{128\,x^{2}-\sqrt{4\,x^{2}+3i\,x}(64\,x+24i)+96i\,x-6}{16384\,x^{6}+36864i\,x^{5}-29376\,x^{4}-9504i\,x^{3}+972\,x^{2}-\left(8192\,x^{5}+15360i\,x^{4}-9504\,x^{3}-2052i\,x^{2}+81\,x\right)\sqrt{4\,x^{2}+3i\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(-5/2),x, algorithm="fricas")
```

```
[Out] (128*x^2 - sqrt(4*x^2 + 3*I*x)*(64*x + 24*I) + 96*I*x - 6)/(16384
*x^6 + 36864*I*x^5 - 29376*x^4 - 9504*I*x^3 + 972*x^2 - (8192*x^5
 + 15360*I*x^4 - 9504*x^3 - 2052*I*x^2 + 81*x)*sqrt(4*x^2 + 3*I*x
))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(4x^2 + 3ix)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*I*x+4*x**2)**(5/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(-5/2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(-5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.20 $\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx$

Optimal. Leaf size=79

$$\frac{4096(8x+3i)}{10935\sqrt{4x^2+3ix}}+\frac{128(8x+3i)}{1215\,(4x^2+3ix)^{3/2}}+\frac{2(8x+3i)}{45\,(4x^2+3ix)^{5/2}}$$

```
[Out] (2*(3*I + 8*x))/(45*((3*I)*x + 4*x^2)^(5/2)) + (128*(3*I + 8*x))/
(1215*((3*I)*x + 4*x^2)^(3/2)) + (4096*(3*I + 8*x))/(10935*Sqrt[(
3*I)*x + 4*x^2])
```

Rubi [A] time = 0.0353751, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{4096(8x+3i)}{10935\sqrt{4x^2+3ix}}+\frac{128(8x+3i)}{1215\,(4x^2+3ix)^{3/2}}+\frac{2(8x+3i)}{45\,(4x^2+3ix)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[((3*I)*x + 4*x^2)^(-7/2),x]
```

```
[Out] (2*(3*I + 8*x))/(45*((3*I)*x + 4*x^2)^(5/2)) + (128*(3*I + 8*x))/
(1215*((3*I)*x + 4*x^2)^(3/2)) + (4096*(3*I + 8*x))/(10935*Sqrt[(
3*I)*x + 4*x^2])
```

Rubi in Sympy [A] time = 2.58007, size = 65, normalized size = 0.82

$$\frac{128\,(8x+3i)}{1215\,(4x^2+3ix)^{\frac{3}{2}}}+\frac{2\,(8x+3i)}{45\,(4x^2+3ix)^{\frac{5}{2}}}+\frac{2048\,(16x+6i)}{10935\sqrt{4x^2+3ix}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*I*x+4*x**2)**(7/2),x)
```

```
[Out] 128*(8*x + 3*I)/(1215*(4*x**2 + 3*I*x)**(3/2)) + 2*(8*x + 3*I)/(4
5*(4*x**2 + 3*I*x)**(5/2)) + 2048*(16*x + 6*I)/(10935*sqrt(4*x**2
 + 3*I*x))
```

Mathematica [A] time = 0.0322578, size = 48, normalized size = 0.61

$$\frac{524288x^5 + 983040ix^4 - 552960x^3 - 69120ix^2 - 6480x + 1458i}{10935(x(4x + 3i))^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((3*I)*x + 4*x^2)^(-7/2),x]
```

```
[Out] (1458*I - 6480*x - (69120*I)*x^2 - 552960*x^3 + (983040*I)*x^4 +
524288*x^5)/(10935*(x*(3*I + 4*x))^(5/2))
```

Maple [A] time = 0.012, size = 62, normalized size = 0.8

$$\frac{6\,i + 16\,x}{45}\left(3\,ix + 4\,x^2\right)^{-\frac{5}{2}} + \frac{384\,i + 1024\,x}{1215}\left(3\,ix + 4\,x^2\right)^{-\frac{3}{2}} + \frac{12288\,i + 32768\,x}{10935}\frac{1}{\sqrt{3\,ix + 4\,x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*I*x+4*x^2)^(7/2),x)
```

```
[Out] 2/45*(3*I+8*x)/(3*I*x+4*x^2)^(5/2)+128/1215*(3*I+8*x)/(3*I*x+4*x^
2)^(3/2)+4096/10935*(3*I+8*x)/(3*I*x+4*x^2)^(1/2)
```

Maxima [A] time = 0.69541, size = 111, normalized size = 1.41

$$\frac{32768\,x}{10935\,\sqrt{4\,x^2 + 3i\,x}} + \frac{4096i}{3645\,\sqrt{4\,x^2 + 3i\,x}} + \frac{1024\,x}{1215\,(4\,x^2 + 3i\,x)^{\frac{3}{2}}} + \frac{128i}{405\,(4\,x^2 + 3i\,x)^{\frac{3}{2}}} + \frac{16\,x}{45\,(4\,x^2 + 3i\,x)^{\frac{5}{2}}} + \frac{2i}{15\,(4\,x^2 + 3i\,x)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(-7/2),x, algorithm="maxima")
```

```
[Out] 32768/10935*x/sqrt(4*x^2 + 3*I*x) + 4096/3645*I/sqrt(4*x^2 + 3*I*
x) + 1024/1215*x/(4*x^2 + 3*I*x)^(3/2) + 128/405*I/(4*x^2 + 3*I*x
)^(3/2) + 16/45*x/(4*x^2 + 3*I*x)^(5/2) + 2/15*I/(4*x^2 + 3*I*x)^
(5/2)
```

Fricas [A] time = 0.225158, size = 197, normalized size = 2.49

$$\frac{327680\,x^4 + 491520i\,x^3 - 224640\,x^2 - \left(163840\right.}{251658240\,x^{10} + 943718400i\,x^9 - 1459814400\,x^8 - 1194393600i\,x^7 + 548985600\,x^6 + 137868480i\,x^5 - 16621200\,x^4 - 65610}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(-7/2),x, algorithm="fricas")
```

```
[Out] (327680*x^4 + 491520*I*x^3 - 224640*x^2 - (163840*x^3 + 184320*I*
x^2 - 54720*x - 3240*I)*sqrt(4*x^2 + 3*I*x) - 30240*I*x + 486)/(2
51658240*x^10 + 943718400*I*x^9 - 1459814400*x^8 - 1194393600*I*x
^7 + 548985600*x^6 + 137868480*I*x^5 - 16621200*x^4 - 656100*I*x^
3 - (125829120*x^9 + 424673280*I*x^8 - 579502080*x^7 - 406425600*
I*x^6 + 153187200*x^5 + 29218320*I*x^4 - 2274480*x^3 - 32805*I*x^
2)*sqrt(4*x^2 + 3*I*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(4x^2 + 3ix)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*I*x+4*x**2)**(7/2),x)
```

```
[Out] Integral((4*x**2 + 3*I*x)**(-7/2), x)
```

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 3*I*x)^(-7/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.21 $\int \frac{1}{\sqrt{3x-4x^2}} \, dx$

Optimal. Leaf size=12

$$-\frac{1}{2} \sin^{-1}\left(1-\frac{8x}{3}\right)$$

```
[Out] -ArcSin[1 - (8*x)/3]/2
```

Rubi [A] time = 0.0142831, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{1}{2} \sin^{-1}\left(1-\frac{8x}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3*x - 4*x^2],x]

[Out] -ArcSin[1 - (8*x)/3]/2
```

Rubi in Sympy [A] time = 1.35323, size = 8, normalized size = 0.67

$$\frac{\operatorname{asin}\left(\frac{8x}{3}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-4*x**2+3*x)**(1/2),x)

[Out] asin(8*x/3 - 1)/2
```

Mathematica [B] time = 0.0159947, size = 45, normalized size = 3.75

$$\frac{\sqrt{x}\sqrt{4x-3}\log\left(2\sqrt{x}+\sqrt{4x-3}\right)}{\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3*x - 4*x^2],x]
```

```
[Out] (Sqrt[x]*Sqrt[-3 + 4*x]*Log[2*Sqrt[x] + Sqrt[-3 + 4*x]])/Sqrt[-(x
*(-3 + 4*x))]
```

Maple [A] time = 0.005, size = 9, normalized size = 0.8

$$\frac{1}{2}\arcsin\left(-1+\frac{8x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-4*x^2+3*x)^(1/2),x)
```

```
[Out] 1/2*arcsin(-1+8/3*x)
```

Maxima [A] time = 0.778173, size = 11, normalized size = 0.92

$$-\frac{1}{2}\arcsin\left(-\frac{8}{3}x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-4*x^2 + 3*x),x, algorithm="maxima")
```

```
[Out] -1/2*arcsin(-8/3*x + 1)
```

Fricas [A] time = 0.215647, size = 26, normalized size = 2.17

$$-\arctan\left(\frac{\sqrt{-4x^2+3x}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-4*x^2 + 3*x),x, algorithm="fricas")
```

```
[Out] -arctan(1/2*sqrt(-4*x^2 + 3*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-4x^2 + 3x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-4*x**2+3*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-4*x**2 + 3*x), x)
```

GIAC/XCAS [A] time = 0.214183, size = 11, normalized size = 0.92

$$\frac{1}{2}\,\arcsin\left(\frac{8}{3}\,x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-4*x^2 + 3*x),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(8/3*x - 1)
```

3.22 $\int \frac{1}{(3x-4x^2)^{3/2}}\,dx$

Optimal. Leaf size=22

$$-\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

```
[Out] (-2*(3 - 8*x))/(9*Sqrt[3*x - 4*x^2])
```

Rubi [A] time = 0.00967405, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077

$$-\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*x - 4*x^2)^(-3/2),x]
```

```
[Out] (-2*(3 - 8*x))/(9*Sqrt[3*x - 4*x^2])
```

Rubi in Sympy [A] time = 1.18175, size = 19, normalized size = 0.86

$$-\frac{-16x+6}{9\sqrt{-4x^2+3x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-4*x**2+3*x)**(3/2),x)
```

```
[Out] -(-16*x + 6)/(9*sqrt(-4*x**2 + 3*x))
```

Mathematica [A] time = 0.0142296, size = 21, normalized size = 0.95

$$\frac{2(8x-3)}{9\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*x - 4*x^2)^(-3/2),x]

[Out] (2*(-3 + 8*x))/(9*Sqrt[-(x*(-3 + 4*x))])
```

Maple [A] time = 0.005, size = 25, normalized size = 1.1

$$-\frac{2\,x\,(4\,x-3)\,(-3+8\,x)}{9}\left(-4\,x^2+3\,x\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-4*x^2+3*x)^(3/2),x)

[Out] -2/9*x*(4*x-3)*(-3+8*x)/(-4*x^2+3*x)^(3/2)
```

Maxima [A] time = 0.698419, size = 38, normalized size = 1.73

$$\frac{16\,x}{9\,\sqrt{-4\,x^2+3\,x}}-\frac{2}{3\,\sqrt{-4\,x^2+3\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(-3/2),x, algorithm="maxima")

[Out] 16/9*x/sqrt(-4*x^2 + 3*x) - 2/3/sqrt(-4*x^2 + 3*x)
```

Fricas [A] time = 0.220754, size = 24, normalized size = 1.09

$$\frac{2\,(8\,x-3)}{9\,\sqrt{-4\,x^2+3\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(-3/2),x, algorithm="fricas")

[Out] 2/9*(8*x - 3)/sqrt(-4*x^2 + 3*x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-4x^2 + 3x)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-4*x**2+3*x)**(3/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(-3/2), x)
```

GIAC/XCAS [A] time = 0.217105, size = 39, normalized size = 1.77

$$-\frac{2\sqrt{-4\,x^2 + 3\,x}(8\,x - 3)}{9\,(4\,x^2 - 3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(-3/2),x, algorithm="giac")
```

```
[Out] -2/9*sqrt(-4*x^2 + 3*x)*(8*x - 3)/(4*x^2 - 3*x)
```

3.23 $\int \frac{1}{(3x-4x^2)^{5/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{64(3-8x)}{243\sqrt{3x-4x^2}}-\frac{2(3-8x)}{27\left(3x-4x^2\right)^{3/2}}$$

```
[Out] (-2*(3 - 8*x))/(27*(3*x - 4*x^2)^(3/2)) - (64*(3 - 8*x))/(243*Sqr
t[3*x - 4*x^2])
```

Rubi [A] time = 0.0202536, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{64(3-8x)}{243\sqrt{3x-4x^2}}-\frac{2(3-8x)}{27\left(3x-4x^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*x - 4*x^2)^(-5/2),x]
```

```
[Out] (-2*(3 - 8*x))/(27*(3*x - 4*x^2)^(3/2)) - (64*(3 - 8*x))/(243*Sqr
t[3*x - 4*x^2])
```

Rubi in Sympy [A] time = 1.61603, size = 37, normalized size = 0.82

$$-\frac{32\left(-16x+6\right)}{243\sqrt{-4x^2+3x}}-\frac{2\left(-8x+3\right)}{27\left(-4x^2+3x\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-4*x**2+3*x)**(5/2),x)
```

```
[Out] -32*(-16*x + 6)/(243*sqrt(-4*x**2 + 3*x)) - 2*(-8*x + 3)/(27*(-4*
x**2 + 3*x)**(3/2))
```

Mathematica [A] time = 0.0230932, size = 31, normalized size = 0.69

$$-\frac{2048x^3-2304x^2+432x+54}{243(-x(4x-3))^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*x - 4*x^2)^(-5/2),x]
```

```
[Out] -(54 + 432*x - 2304*x^2 + 2048*x^3)/(243*(-(x*(-3 + 4*x)))^(3/2))
```

Maple [A] time = 0.004, size = 35, normalized size = 0.8

$$\frac{2\,x\,(4\,x-3)\left(1024\,x^{3}-1152\,x^{2}+216\,x+27\right)}{243}\left(-4\,x^{2}+3\,x\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-4*x^2+3*x)^(5/2),x)
```

```
[Out] 2/243*x*(4*x-3)*(1024*x^3-1152*x^2+216*x+27)/(-4*x^2+3*x)^(5/2)
```

Maxima [A] time = 0.689655, size = 74, normalized size = 1.64

$$\frac{512\,x}{243\,\sqrt{-4\,x^{2}+3\,x}}-\frac{64}{81\,\sqrt{-4\,x^{2}+3\,x}}+\frac{16\,x}{27\,(-4\,x^{2}+3\,x)^{\frac{3}{2}}}-\frac{2}{9\,(-4\,x^{2}+3\,x)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(-5/2),x, algorithm="maxima")
```

```
[Out] 512/243*x/sqrt(-4*x^2 + 3*x) - 64/81/sqrt(-4*x^2 + 3*x) + 16/27*x
/(-4*x^2 + 3*x)^(3/2) - 2/9/(-4*x^2 + 3*x)^(3/2)
```

Fricas [A] time = 0.216312, size = 53, normalized size = 1.18

$$\frac{2\left(1024\,x^{3}-1152\,x^{2}+216\,x+27\right)}{243\,(4\,x^{2}-3\,x)\sqrt{-4\,x^{2}+3\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(-5/2),x, algorithm="fricas")
```

```
[Out] 2/243*(1024*x^3 - 1152*x^2 + 216*x + 27)/((4*x^2 - 3*x)*sqrt(-4*x
^2 + 3*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-4x^2+3x)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-4*x**2+3*x)**(5/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(-5/2), x)
```

GIAC/XCAS [A] time = 0.219204, size = 53, normalized size = 1.18

$$-\frac{2\,(8\,(16\,(8\,x-9)x+27)x+27)\sqrt{-4\,x^2+3\,x}}{243\,(4\,x^2-3\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(-5/2),x, algorithm="giac")
```

```
[Out] -2/243*(8*(16*(8*x - 9)*x + 27)*x + 27)*sqrt(-4*x^2 + 3*x)/(4*x^2
 - 3*x)^2
```

3.24 $\int \frac{1}{(3x-4x^2)^{7/2}}\,dx$

Optimal. Leaf size=67

$$-\frac{4096(3-8x)}{10935\sqrt{3x-4x^2}}-\frac{128(3-8x)}{1215\,(3x-4x^2)^{3/2}}-\frac{2(3-8x)}{45\,(3x-4x^2)^{5/2}}$$

```
[Out] (-2*(3 - 8*x))/(45*(3*x - 4*x^2)^(5/2)) - (128*(3 - 8*x))/(1215*(
3*x - 4*x^2)^(3/2)) - (4096*(3 - 8*x))/(10935*Sqrt[3*x - 4*x^2])
```

Rubi [A] time = 0.0330418, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$-\frac{4096(3-8x)}{10935\sqrt{3x-4x^2}}-\frac{128(3-8x)}{1215\,(3x-4x^2)^{3/2}}-\frac{2(3-8x)}{45\,(3x-4x^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(3*x - 4*x^2)^(-7/2),x]
```

```
[Out] (-2*(3 - 8*x))/(45*(3*x - 4*x^2)^(5/2)) - (128*(3 - 8*x))/(1215*(
3*x - 4*x^2)^(3/2)) - (4096*(3 - 8*x))/(10935*Sqrt[3*x - 4*x^2])
```

Rubi in Sympy [A] time = 2.15337, size = 56, normalized size = 0.84

$$-\frac{2048\,(-16x+6)}{10935\sqrt{-4x^2+3x}}-\frac{128\,(-8x+3)}{1215\,(-4x^2+3x)^{\frac{3}{2}}}-\frac{2\,(-8x+3)}{45\,(-4x^2+3x)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-4*x**2+3*x)**(7/2),x)
```

```
[Out] -2048*(-16*x + 6)/(10935*sqrt(-4*x**2 + 3*x)) - 128*(-8*x + 3)/(1
215*(-4*x**2 + 3*x)**(3/2)) - 2*(-8*x + 3)/(45*(-4*x**2 + 3*x)**(
5/2))
```

Mathematica [A] time = 0.0317378, size = 51, normalized size = 0.76

$$\frac{2\left(262144x^5-491520x^4+276480x^3-34560x^2-3240x-729\right)}{10935(3-4x)^2x^2\sqrt{-x(4x-3)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3*x - 4*x^2)^(-7/2),x]
```

```
[Out] (2*(-729 - 3240*x - 34560*x^2 + 276480*x^3 - 491520*x^4 + 262144*
x^5))/(10935*(3 - 4*x)^2*x^2*Sqrt[-(x*(-3 + 4*x))])
```

Maple [A]   time = 0.005, size = 45, normalized size = 0.7

$$-\frac{2\,x\,(4\,x-3)\left(262144\,x^{5}-491520\,x^{4}+276480\,x^{3}-34560\,x^{2}-3240\,x-729\right)}{10935}\left(-4\,x^{2}+3\,x\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-4*x^2+3*x)^(7/2),x)
```

```
[Out] -2/10935*x*(4*x-3)*(262144*x^5-491520*x^4+276480*x^3-34560*x^2-32
40*x-729)/(-4*x^2+3*x)^(7/2)
```

Maxima [A]   time = 0.703799, size = 111, normalized size = 1.66

$$\frac{32768\,x}{10935\,\sqrt{-4\,x^{2}+3\,x}}-\frac{4096}{3645\,\sqrt{-4\,x^{2}+3\,x}}+\frac{1024\,x}{1215\,(-4\,x^{2}+3\,x)^{\frac{3}{2}}}-\frac{128}{405\,(-4\,x^{2}+3\,x)^{\frac{3}{2}}}+\frac{16\,x}{45\,(-4\,x^{2}+3\,x)^{\frac{5}{2}}}-\frac{2}{15\,(-4\,x^{2}+3\,x)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(-7/2),x, algorithm="maxima")
```

```
[Out] 32768/10935*x/sqrt(-4*x^2 + 3*x) - 4096/3645/sqrt(-4*x^2 + 3*x) +
 1024/1215*x/(-4*x^2 + 3*x)^(3/2) - 128/405/(-4*x^2 + 3*x)^(3/2)
+ 16/45*x/(-4*x^2 + 3*x)^(5/2) - 2/15/(-4*x^2 + 3*x)^(5/2)
```

Fricas [A]   time = 0.225794, size = 76, normalized size = 1.13

$$\frac{2\left(262144\,x^{5}-491520\,x^{4}+276480\,x^{3}-34560\,x^{2}-3240\,x-729\right)}{10935\,(16\,x^{4}-24\,x^{3}+9\,x^{2})\sqrt{-4\,x^{2}+3\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(-7/2),x, algorithm="fricas")
```

```
[Out] 2/10935*(262144*x^5 - 491520*x^4 + 276480*x^3 - 34560*x^2 - 3240*
x - 729)/((16*x^4 - 24*x^3 + 9*x^2)*sqrt(-4*x^2 + 3*x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-4x^2 + 3x)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-4*x**2+3*x)**(7/2),x)
```

```
[Out] Integral((-4*x**2 + 3*x)**(-7/2), x)
```

GIAC/XCAS [A] time = 0.220974, size = 66, normalized size = 0.99

$$-\frac{2\,(8\,(32\,(8\,(16\,(8\,x - 15)x + 135)x - 135)x - 405)x - 729)\sqrt{-4\,x^2 + 3\,x}}{10935\,(4\,x^2 - 3\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 3*x)^(-7/2),x, algorithm="giac")
```

```
[Out] -2/10935*(8*(32*(8*(16*(8*x - 15)*x + 135)*x - 135)*x - 405)*x -
729)*sqrt(-4*x^2 + 3*x)/(4*x^2 - 3*x)^3
```

3.25 $\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin^{-1}(1-2bx)}{b}$$

```
[Out] -(ArcSin[1 - 2*b*x]/b)
```

Rubi [A] time = 0.019174, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$-\frac{\sin^{-1}(1-2bx)}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[b*x - b^2*x^2],x]

[Out] -(ArcSin[1 - 2*b*x]/b)
```

Rubi in Sympy [A] time = 3.31736, size = 8, normalized size = 0.67

$$\frac{\operatorname{asin}(2bx-1)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b**2*x**2+b*x)**(1/2),x)

[Out] asin(2*b*x - 1)/b
```

Mathematica [B] time = 0.0324972, size = 58, normalized size = 4.83

$$\frac{2\sqrt{x}\sqrt{bx-1}\log\left(b\sqrt{x}+\sqrt{b}\sqrt{bx-1}\right)}{\sqrt{b}\sqrt{-bx(bx-1)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[b*x - b^2*x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[-1 + b*x]*Log[b*Sqrt[x] + Sqrt[b]*Sqrt[-1 + b*x]]
)/(Sqrt[b]*Sqrt[-(b*x*(-1 + b*x))])
```

Maple [B] time = 0.009, size = 35, normalized size = 2.9

$$1\arctan\left(1\sqrt{b^2}\left(x-\frac{1}{2\,b}\right)\frac{1}{\sqrt{-b^2x^2+bx}}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b^2*x^2+b*x)^(1/2),x)
```

```
[Out] 1/(b^2)^(1/2)*arctan((b^2)^(1/2)*(x-1/2/b)/(-b^2*x^2+b*x)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-b^2*x^2 + b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.21612, size = 36, normalized size = 3.

$$-\frac{2\arctan\left(\frac{\sqrt{-b^2x^2+bx}}{bx}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-b^2*x^2 + b*x),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-b^2*x^2 + b*x)/(b*x))/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-b^2x^2 + bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b**2*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-b**2*x**2 + b*x), x)
```

GIAC/XCAS [A] time = 0.220862, size = 20, normalized size = 1.67

$$-\frac{\arcsin\left(-2\,bx + 1\right)\mathrm{sign}\left(b\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-b^2*x^2 + b*x),x, algorithm="giac")
```

```
[Out] -arcsin(-2*b*x + 1)*sign(b)/abs(b)
```

3.26 $\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx$

Optimal. Leaf size=24

$$\frac{2\tanh^{-1}\left(\frac{bx}{\sqrt{b^2x^2+bx}}\right)}{b}$$

```
[Out] (2*ArcTanh[(b*x)/Sqrt[b*x + b^2*x^2]])/b
```

Rubi [A] time = 0.0202914, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$\frac{2\tanh^{-1}\left(\frac{bx}{\sqrt{b^2x^2+bx}}\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[b*x + b^2*x^2],x]

[Out] (2*ArcTanh[(b*x)/Sqrt[b*x + b^2*x^2]])/b
```

Rubi in Sympy [A] time = 2.54073, size = 20, normalized size = 0.83

$$\frac{2\,\mathrm{atanh}\left(\frac{bx}{\sqrt{b^2x^2+bx}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b**2*x**2+b*x)**(1/2),x)

[Out] 2*atanh(b*x/sqrt(b**2*x**2 + b*x))/b
```

Mathematica [A] time = 0.0385247, size = 45, normalized size = 1.88

$$\frac{2\sqrt{x}\sqrt{bx+1}\sinh^{-1}\left(\sqrt{b}\sqrt{x}\right)}{\sqrt{b}\sqrt{bx(bx+1)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[b*x + b^2*x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[1 + b*x]*ArcSinh[Sqrt[b]*Sqrt[x]])/(Sqrt[b]*Sqrt[
b*x*(1 + b*x)])
```

Maple [A] time = 0.004, size = 37, normalized size = 1.5

$$1\ln\left(1\left(\frac{b}{2}+b^2x\right)\frac{1}{\sqrt{b^2}}+\sqrt{b^2x^2+bx}\right)\frac{1}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b^2*x^2+b*x)^(1/2),x)
```

```
[Out] ln((1/2*b+b^2*x)/(b^2)^(1/2)+(b^2*x^2+b*x)^(1/2))/(b^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b^2*x^2 + b*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.212474, size = 36, normalized size = 1.5

$$-\frac{\log\left(-2\,bx+2\,\sqrt{b^2x^2+bx}-1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b^2*x^2 + b*x),x, algorithm="fricas")
```

```
[Out] -log(-2*b*x + 2*sqrt(b^2*x^2 + b*x) - 1)/b
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b^2x^2 + bx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b**2*x**2+b*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(b**2*x**2 + b*x), x)
```

GIAC/XCAS [A] time = 0.222373, size = 49, normalized size = 2.04

$$-\frac{\ln\left(\left|-2\left(x|b| - \sqrt{b^2x^2 + bx}\right)|b| - b\right|\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(b^2*x^2 + b*x),x, algorithm="giac")
```

```
[Out] -ln(abs(-2*(x*abs(b) - sqrt(b^2*x^2 + b*x))*abs(b) - b))/abs(b)
```

3.27 $\int \frac{1}{\sqrt{6x-x^2}}\,dx$

Optimal. Leaf size=10

$$-\sin^{-1}\left(1-\frac{x}{3}\right)$$

```
[Out] -ArcSin[1 - x/3]
```

Rubi [A] time = 0.0152482, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\sin^{-1}\left(1-\frac{x}{3}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[6*x - x^2],x]

[Out] -ArcSin[1 - x/3]
```

Rubi in Sympy [A] time = 1.34171, size = 5, normalized size = 0.5

$$\operatorname{asin}\left(\frac{x}{3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2+6*x)**(1/2),x)

[Out] asin(x/3 - 1)
```

Mathematica [B] time = 0.0142168, size = 38, normalized size = 3.8

$$\frac{2\sqrt{x-6}\sqrt{x}\log\left(\sqrt{x-6}+\sqrt{x}\right)}{\sqrt{-(x-6)x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[6*x - x^2],x]
```

```
[Out] (2*Sqrt[-6 + x]*Sqrt[x]*Log[Sqrt[-6 + x] + Sqrt[x]])/Sqrt[-((-6 +
 x)*x)]
```

Maple [A] time = 0.005, size = 7, normalized size = 0.7

$$\arcsin\left(-1+\frac{x}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2+6*x)^(1/2),x)
```

```
[Out] arcsin(-1+1/3*x)
```

Maxima [A] time = 0.781981, size = 11, normalized size = 1.1

$$-\arcsin\left(-\frac{1}{3}\,x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + 6*x),x, algorithm="maxima")
```

```
[Out] -arcsin(-1/3*x + 1)
```

Fricas [A] time = 0.219675, size = 24, normalized size = 2.4

$$-2\,\arctan\left(\frac{\sqrt{-x^2+6\,x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + 6*x),x, algorithm="fricas")
```

```
[Out] -2*arctan(sqrt(-x^2 + 6*x)/x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-x^2 + 6x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2+6*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-x**2 + 6*x), x)
```

GIAC/XCAS [A] time = 0.212672, size = 8, normalized size = 0.8

$$\arcsin\left(\frac{1}{3}\, x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-x^2 + 6*x),x, algorithm="giac")
```

```
[Out] arcsin(1/3*x - 1)
```

3.28 $\int \frac{1}{\sqrt{4x+x^2}}\, dx$

Optimal. Leaf size=16

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

```
[Out] 2*ArcTanh[x/Sqrt[4*x + x^2]]
```

Rubi [A] time = 0.0113501, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[4*x + x^2],x]

[Out] 2*ArcTanh[x/Sqrt[4*x + x^2]]
```

Rubi in Sympy [A] time = 1.19149, size = 14, normalized size = 0.88

$$2\,\text{atanh}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+4*x)**(1/2),x)

[Out] 2*atanh(x/sqrt(x**2 + 4*x))
```

Mathematica [B] time = 0.0128217, size = 33, normalized size = 2.06

$$\frac{2\sqrt{x}\sqrt{x+4}\sinh^{-1}\left(\frac{\sqrt{x}}{2}\right)}{\sqrt{x(x+4)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[4*x + x^2],x]
```

```
[Out] (2*Sqrt[x]*Sqrt[4 + x]*ArcSinh[Sqrt[x]/2])/Sqrt[x*(4 + x)]
```

Maple [A] time = 0.004, size = 14, normalized size = 0.9

$$\ln\left(2 + x + \sqrt{x^2 + 4x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+4*x)^(1/2),x)
```

```
[Out] ln(2+x+(x^2+4*x)^(1/2))
```

Maxima [A] time = 0.728114, size = 23, normalized size = 1.44

$$\log\left(2x + 2\sqrt{x^2 + 4x} + 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 4*x),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 + 4*x) + 4)
```

Fricas [A] time = 0.212032, size = 23, normalized size = 1.44

$$-\log\left(-x + \sqrt{x^2 + 4x} - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 4*x),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 + 4*x) - 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2 + 4x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+4*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(x**2 + 4*x), x)
```

GIAC/XCAS [A] time = 0.212315, size = 24, normalized size = 1.5

$$-\ln\left(\left|-x + \sqrt{x^2 + 4\,x} - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 + 4*x),x, algorithm="giac")
```

```
[Out] -ln(abs(-x + sqrt(x^2 + 4*x) - 2))
```

3.29 $\int \frac{1}{\sqrt{-2x+x^2}}\,dx$

Optimal. Leaf size=16

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2-2x}}\right)$$

```
[Out] 2*ArcTanh[x/Sqrt[-2*x + x^2]]
```

Rubi [A] time = 0.0114016, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$2\tanh^{-1}\left(\frac{x}{\sqrt{x^2-2x}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2*x + x^2],x]
```

```
[Out] 2*ArcTanh[x/Sqrt[-2*x + x^2]]
```

Rubi in Sympy [A] time = 1.18585, size = 14, normalized size = 0.88

$$2\,\text{atanh}\left(\frac{x}{\sqrt{x^2-2x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2-2*x)**(1/2),x)
```

```
[Out] 2*atanh(x/sqrt(x**2 - 2*x))
```

Mathematica [B] time = 0.0140562, size = 37, normalized size = 2.31

$$\frac{2\sqrt{x-2}\sqrt{x}\log\left(\sqrt{x-2}+\sqrt{x}\right)}{\sqrt{(x-2)x}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2*x + x^2],x]
```

```
[Out] (2*Sqrt[-2 + x]*Sqrt[x]*Log[Sqrt[-2 + x] + Sqrt[x]])/Sqrt[(-2 + x
)*x]
```

Maple [A] time = 0.005, size = 14, normalized size = 0.9

$$\ln\left(x-1+\sqrt{x^2-2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2-2*x)^(1/2),x)
```

```
[Out] ln(x-1+(x^2-2*x)^(1/2))
```

Maxima [A] time = 0.703835, size = 23, normalized size = 1.44

$$\log\left(2x+2\sqrt{x^2-2x}-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 - 2*x),x, algorithm="maxima")
```

```
[Out] log(2*x + 2*sqrt(x^2 - 2*x) - 2)
```

Fricas [A] time = 0.211598, size = 23, normalized size = 1.44

$$-\log\left(-x+\sqrt{x^2-2x}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 - 2*x),x, algorithm="fricas")
```

```
[Out] -log(-x + sqrt(x^2 - 2*x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2 - 2x}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2-2*x)**(1/2),x)

[Out] Integral(1/sqrt(x**2 - 2*x), x)
```

GIAC/XCAS [A] time = 0.213045, size = 24, normalized size = 1.5

$$-\ln\left(\left|-x + \sqrt{x^2 - 2\,x} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(x^2 - 2*x),x, algorithm="giac")

[Out] -ln(abs(-x + sqrt(x^2 - 2*x) + 1))
```

3.30 $\int \left(bx + cx^2\right)^{4/3} \, dx$

Optimal. Leaf size=448

$$\frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}+\frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\left(bx+cx^2\right)^{4/3}}{55c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}+\frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\left(bx+cx^2\right)^{4/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)\middle|-7+4\sqrt{3}\right)}{55c(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3*(-((c*x*(b + c*x))/b^2))^(1/3)*(b + 2*c*x)*(b*x + c*x^2)^(4/3)
)/(55*c*(-((c*(b*x + c*x^2))/b^2))^(4/3)) + (3*(-((c*x*(b + c*x))
/b^2))^(4/3)*(b + 2*c*x)*(b*x + c*x^2)^(4/3))/(22*c*(-((c*(b*x +
c*x^2))/b^2))^(4/3)) + (2^(1/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(b*
x + c*x^2)^(4/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqr
t[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x
*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x)
)/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x
*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x)
)/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(55*c*(b + 2*c*x)*(-((c*(b*x +
c*x^2))/b^2))^(4/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^
(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)]
)
```

Rubi [A] time = 1.33759, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$

$$\frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}(b+2cx)\left(bx+cx^2\right)^{4/3}}{22c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}+\frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\left(bx+cx^2\right)^{4/3}}{55c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}$$

$$+\frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(bx+cx^2\right)^{4/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b}{b}}}{-2^{2/3}\sqrt[3]{-\frac{cx(b}{b}}}\right.\right.}{55c(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)^(4/3),x]
```

```
[Out] (3*(-((c*x*(b + c*x))/b^2))^(1/3)*(b + 2*c*x)*(b*x + c*x^2)^(4/3)
)/(55*c*(-((c*(b*x + c*x^2))/b^2))^(4/3)) + (3*(-((c*x*(b + c*x))
/b^2))^(4/3)*(b + 2*c*x)*(b*x + c*x^2)^(4/3))/(22*c*(-((c*(b*x +
c*x^2))/b^2))^(4/3)) + (2^(1/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(b*
x + c*x^2)^(4/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqr
t[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x
*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x)
)/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x
*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x)
)/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(55*c*(b + 2*c*x)*(-((c*(b*x +
c*x^2))/b^2))^(4/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^
(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)]
)
```

Rubi in Sympy [A] time = 41.3632, size = 398, normalized size = 0.89

$$\frac{\sqrt[3]{2}\cdot 3^{\frac{3}{4}}b^2\sqrt{\frac{\left(1-\frac{(-b-2cx)^2}{b^2}\right)^{\frac{2}{3}}+\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}+1}{\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\left(bx+cx^2\right)^{\frac{4}{3}}\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}+1\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{1-\frac{(-b-2c}{b^2}}}{-\sqrt[3]{1-\frac{(-b-2c}{b^2}}}\right.\right.}{55c\sqrt{\frac{\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-1}{\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-\sqrt{3}+1\right)^2}}\left(\frac{c(-bx-cx^2)}{b^2}\right)^{\frac{4}{3}}(b+2cx)}$$

$$+\frac{3\sqrt[3]{2}\left(1-\frac{(-b-2cx)^2}{b^2}\right)^{\frac{4}{3}}(b+2cx)\left(bx+cx^2\right)^{\frac{4}{3}}}{176c\left(\frac{c(-bx-cx^2)}{b^2}\right)^{\frac{4}{3}}}+\frac{3\sqrt[3]{2}\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}(b+2cx)\left(bx+cx^2\right)^{\frac{4}{3}}}{110c\left(\frac{c(-bx-cx^2)}{b^2}\right)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x)**(4/3),x)
```

```
[Out] 2**(1/3)*3**(3/4)*b**2*sqrt(((1 - (-b - 2*c*x)**2/b**2)**(2/3) +
(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)/(-(1 - (-b - 2*c*x)**2/b**
2)**(1/3) - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(b*x + c*x**2)**(
4/3)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)*elliptic_f(asin((-(
1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1 + sqrt(3))/(-(1 - (-b - 2*c*
x)**2/b**2)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/(55*c*sqrt(((
1 - (-b - 2*c*x)**2/b**2)**(1/3) - 1)/(-(1 - (-b - 2*c*x)**2/b**2
)**(1/3) - sqrt(3) + 1)**2)*(c*(-b*x - c*x**2)/b**2)**(4/3)*(b +
2*c*x)) + 3*2**(1/3)*(1 - (-b - 2*c*x)**2/b**2)**(4/3)*(b + 2*c*x
)*(b*x + c*x**2)**(4/3)/(176*c*(c*(-b*x - c*x**2)/b**2)**(4/3)) +
 3*2**(1/3)*(1 - (-b - 2*c*x)**2/b**2)**(1/3)*(b + 2*c*x)*(b*x +
c*x**2)**(4/3)/(110*c*(c*(-b*x - c*x**2)/b**2)**(4/3))
```

Mathematica [C] time = 0.0840112, size = 94, normalized size = 0.21

$$\frac{3x\left(2b^4\left(\frac{cx}{b}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{cx}{b}\right)-2b^4-b^3cx+16b^2c^2x^2+25bc^3x^3+10c^4x^4\right)}{110c^2(x(b+cx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(4/3),x]
```

```
[Out] (3*x*(-2*b^4 - b^3*c*x + 16*b^2*c^2*x^2 + 25*b*c^3*x^3 + 10*c^4*x
^4 + 2*b^4*(1 + (c*x)/b)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -
((c*x)/b)]))/(110*c^2*(x*(b + c*x))^(2/3))
```

Maple [F] time = 0.057, size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{4}{3}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x)^(4/3),x)
```

```
[Out] int((c*x^2+b*x)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2 + bx\right)^{\frac{4}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(4/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(4/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx + cx^2\right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x)**(4/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(4/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(4/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(4/3), x)
```

3.31 $\int \sqrt[3]{bx+cx^2}\,dx$

Optimal. Leaf size=387

$$\frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\sqrt[3]{bx+cx^2}}{10c\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}}+\frac{3^{3/4}\sqrt{2-\sqrt{3}}b^2\sqrt[3]{bx+cx^2}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-}\right.\right.}{5\,2^{2/3}c(b+2cx)\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3*(-((c*x*(b + c*x))/b^2))^(1/3)*(b + 2*c*x)*(b*x + c*x^2)^(1/3)
)/(10*c*(-((c*(b*x + c*x^2))/b^2))^(1/3)) + (3^(3/4)*Sqrt[2 - Sqr
t[3]]*b^2*(b*x + c*x^2)^(1/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2
))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^
(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((
c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^
(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((
c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(5*2^(2/3)*c*(b +
2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(1/3)*Sqrt[-((1 - 2^(2/3)*(-((c
*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*
x))/b^2))^(1/3))^2)])
```

Rubi [A] time = 0.932088, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$

$$\frac{3\sqrt[3]{-\frac{cx(b+cx)}{b^2}}(b+2cx)\sqrt[3]{bx+cx^2}}{10c\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}} + \frac{3^{3/4}\sqrt{2-\sqrt{3}}b^2\sqrt[3]{bx+cx^2}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-}\right.\right.}{5\,2^{2/3}c(b+2cx)\sqrt[3]{-\frac{c(bx+cx^2)}{b^2}}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)^(1/3),x]
```

```
[Out] (3*(-((c*x*(b + c*x))/b^2))^(1/3)*(b + 2*c*x)*(b*x + c*x^2)^(1/3)
)/(10*c*(-((c*(b*x + c*x^2))/b^2))^(1/3)) + (3^(3/4)*Sqrt[2 - Sqr
t[3]]*b^2*(b*x + c*x^2)^(1/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2
))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^
(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((
c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^
(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((
c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(5*2^(2/3)*c*(b +
2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(1/3)*Sqrt[-((1 - 2^(2/3)*(-((c
*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*
x))/b^2))^(1/3))^2)])
```

Rubi in Sympy [A] time = 32.4011, size = 332, normalized size = 0.86

$$\frac{\sqrt[3]{2}\cdot 3^{\frac{3}{4}} b^2 \sqrt{\frac{\left(1-\frac{(-b-2cx)^2}{b^2}\right)^{\frac{2}{3}}+\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}+1}{\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-\sqrt{3}+1\right)^2}}\sqrt{-\sqrt{3}+2}\sqrt[3]{bx+cx^2}\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}+1\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{1-\frac{(-b-2cx)}{b^2}}}{-\sqrt[3]{1-\frac{(-b-2cx)}{b^2}}}\right.\right.}{10c\sqrt{\frac{\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-1}{\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-\sqrt{3}+1\right)^2}}\sqrt[3]{\frac{c(-bx-cx^2)}{b^2}}(b+2cx)}$$

$$+\frac{3\sqrt[3]{2}\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}(b+2cx)\sqrt[3]{bx+cx^2}}{20c\sqrt[3]{\frac{c(-bx-cx^2)}{b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x)**(1/3),x)
```

```
[Out] 2**(1/3)*3**(3/4)*b**2*sqrt(((1 - (-b - 2*c*x)**2/b**2)**(2/3) +
(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)/(-(1 - (-b - 2*c*x)**2/b**
2)**(1/3) - sqrt(3) + 1)**2)*sqrt(-sqrt(3) + 2)*(b*x + c*x**2)**(
1/3)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)*elliptic_f(asin((-(
1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1 + sqrt(3))/(-(1 - (-b - 2*c*
x)**2/b**2)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/(10*c*sqrt(((
1 - (-b - 2*c*x)**2/b**2)**(1/3) - 1)/(-(1 - (-b - 2*c*x)**2/b**2
)**(1/3) - sqrt(3) + 1)**2)*(c*(-b*x - c*x**2)/b**2)**(1/3)*(b +
2*c*x)) + 3*2**(1/3)*(1 - (-b - 2*c*x)**2/b**2)**(1/3)*(b + 2*c*x
)*(b*x + c*x**2)**(1/3)/(20*c*(c*(-b*x - c*x**2)/b**2)**(1/3))
```

Mathematica [C] time = 0.0570459, size = 70, normalized size = 0.18

$$\frac{3x\left(b^2\left(-\left(\frac{cx}{b}+1\right)^{2/3}\right){}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{cx}{b}\right)+b^2+3bcx+2c^2x^2\right)}{10c(x(b+cx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(1/3),x]
```

```
[Out] (3*x*(b^2 + 3*b*c*x + 2*c^2*x^2 - b^2*(1 + (c*x)/b)^(2/3)*Hyperge
ometric2F1[1/3, 2/3, 4/3, -((c*x)/b)]))/(10*c*(x*(b + c*x))^(2/3)
)
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \sqrt[3]{c x^2 + b x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x)^(1/3),x)
```

```
[Out] int((c*x^2+b*x)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(c x^2 + b x\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(c x^2 + b x\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(1/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[3]{bx + cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x)**(1/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(1/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{1}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(1/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(1/3), x)
```

3.32 $\int \frac{1}{(bx+cx^2)^{2/3}}\,dx$

Optimal. Leaf size=322

$$\frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(\cdots}{}}}{-2^{2/3}\sqrt[3]{-\frac{cx(\cdots}{}}}\cdots\right.\right.}{c(b+2cx)\,(bx+cx^2)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (2^(1/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))
^(2/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(
2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x)
)/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1
/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x)
)/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1
/3))], -7 + 4*Sqrt[3]])/(c*(b + 2*c*x)*(b*x + c*x^2)^(2/3)*Sqrt[-
((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2
/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi [A] time = 0.788418, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$\frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(\cdots}{}}}{-2^{2/3}\sqrt[3]{-\frac{cx(\cdots}{}}}\cdots\right.\right.}{c(b+2cx)\,(bx+cx^2)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)^(-2/3),x]
```

```
[Out] (2^(1/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))
^(2/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(
```

```
2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x)
)/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1
/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x)
)/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1
/3))], -7 + 4*Sqrt[3]]])/(c*(b + 2*c*x)*(b*x + c*x^2)^(2/3)*Sqrt[-
((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2
/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2])
```

Rubi in Sympy [A] time = 23.123, size = 264, normalized size = 0.82

$$\frac{\sqrt[3]{2}\cdot 3^{\frac{3}{4}}b^{2}\sqrt{\frac{\left(1-\frac{(-b-2cx)^{2}}{b^{2}}\right)^{\frac{2}{3}}+\sqrt[3]{1-\frac{(-b-2cx)^{2}}{b^{2}}}+1}{\left(-\sqrt[3]{1-\frac{(-b-2cx)^{2}}{b^{2}}}-\sqrt{3}+1\right)^{2}}}\left(\frac{c\left(-bx-cx^{2}\right)}{b^{2}}\right)^{\frac{2}{3}}\sqrt{-\sqrt{3}+2}\left(-\sqrt[3]{1-\frac{(-b-2cx)^{2}}{b^{2}}}+1\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{1-\frac{(-b-\cdots}{b\cdots}}}{-\sqrt[3]{1-\frac{(-b-\cdots}{b\cdots}}}\right.\right.}{c\sqrt{\frac{\sqrt[3]{1-\frac{(-b-2cx)^{2}}{b^{2}}}-1}{\left(-\sqrt[3]{1-\frac{(-b-2cx)^{2}}{b^{2}}}-\sqrt{3}+1\right)^{2}}}\left(b+2cx\right)\left(bx+cx^{2}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+b*x)**(2/3),x)
```

```
[Out] 2**(1/3)*3**(3/4)*b**2*sqrt(((1 - (-b - 2*c*x)**2/b**2)**(2/3) +
(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)/(-(1 - (-b - 2*c*x)**2/b**
2)**(1/3) - sqrt(3) + 1)**2)*(c*(-b*x - c*x**2)/b**2)**(2/3)*sqrt
(-sqrt(3) + 2)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)*elliptic_
f(asin((-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1 + sqrt(3))/(-(1 -
(-b - 2*c*x)**2/b**2)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/(c*
sqrt(((1 - (-b - 2*c*x)**2/b**2)**(1/3) - 1)/(-(1 - (-b - 2*c*x)*
*2/b**2)**(1/3) - sqrt(3) + 1)**2)*(b + 2*c*x)*(b*x + c*x**2)**(2
/3))
```

Mathematica [C] time = 0.0232084, size = 44, normalized size = 0.14

$$\frac{3x\left(\frac{b+cx}{b}\right)^{2/3}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{cx}{b}\right)}{(x(b+cx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(-2/3),x]
```

```
[Out] (3*x*((b + c*x)/b)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((c*x)
/b)])/(x*(b + c*x))^(2/3)
```

Maple [F] time = 0.055, size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+b*x)^(2/3),x)
```

```
[Out] int(1/(c*x^2+b*x)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cx^2 + bx)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-2/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(-2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx + cx^2)^{\frac{2}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+b*x)**(2/3),x)

[Out] Integral((b*x + c*x**2)**(-2/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx)^{\frac{2}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-2/3),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(-2/3), x)
```

3.33 $\int \frac{1}{(bx+cx^2)^{5/3}}\,dx$

Optimal. Leaf size=384

$$\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}{2c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{5/3}} + \frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx}{}}}{-2^{2/3}\sqrt[3]{-\frac{cx}{}}}\right.\right.}{c(b+2cx)(bx+cx^2)^{5/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(5/3))/(2*c*(-((c*x*(b
+ c*x))/b^2))^(2/3)*(b*x + c*x^2)^(5/3)) + (2^(1/3)*3^(3/4)*Sqrt[
2 - Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(5/3)*(1 - 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x)
)/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sq
rt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcS
in[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sq
rt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]]
)/(c*(b + 2*c*x)*(b*x + c*x^2)^(5/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*
(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))
/b^2))^(1/3))^2])
```

Rubi [A] time = 0.921619, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$

$$\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}}{2c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{5/3}} + \frac{\sqrt[3]{2}\,3^{3/4}\sqrt{2-\sqrt{3}}\,b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\cdots}}{-2^{2/3}\sqrt[3]{-\cdots}}\cdots\right.\right.}{c(b+2cx)(bx+cx^2)^{5/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)^(-5/3),x]
```

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(5/3))/(2*c*(-((c*x*(b
+ c*x))/b^2))^(2/3)*(b*x + c*x^2)^(5/3)) + (2^(1/3)*3^(3/4)*Sqrt[
2 - Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(5/3)*(1 - 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x)
)/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sq
rt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcS
in[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sq
rt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]]
)/(c*(b + 2*c*x)*(b*x + c*x^2)^(5/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*
(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))
/b^2))^(1/3))^2)])
```

Rubi in Sympy [A] time = 32.4619, size = 328, normalized size = 0.85

$$\frac{\sqrt[3]{2}\cdot 3^{\frac{3}{4}}b^2\sqrt{\frac{\left(1-\frac{(-b-2cx)^2}{b^2}\right)^{\frac{2}{3}}+\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}+1}{\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-\sqrt{3}+1\right)^2}}\left(\frac{c(-bx-cx^2)}{b^2}\right)^{\frac{5}{3}}\sqrt{-\sqrt{3}+2}\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}+1\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{1-\frac{(-b-\cdots}{b\cdots}}}{-\sqrt[3]{1-\frac{(-b-\cdots}{b\cdots}}}\right.\right.\cdots}{c\sqrt{\frac{\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-1}{\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-\sqrt{3}+1\right)^2}}(b+2cx)(bx+cx^2)^{\frac{5}{3}}}$$

$$+\frac{3\sqrt[3]{2}\left(\frac{c(-bx-cx^2)}{b^2}\right)^{\frac{5}{3}}(b+2cx)}{c\left(1-\frac{(-b-2cx)^2}{b^2}\right)^{\frac{2}{3}}(bx+cx^2)^{\frac{5}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+b*x)**(5/3),x)
```

```
[Out] 2**(1/3)*3**(3/4)*b**2*sqrt(((1 - (-b - 2*c*x)**2/b**2)**(2/3) +
(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)/(-(1 - (-b - 2*c*x)**2/b**
2)**(1/3) - sqrt(3) + 1)**2)*(c*(-b*x - c*x**2)/b**2)**(5/3)*sqrt
(-sqrt(3) + 2)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)*elliptic_
f(asin((-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1 + sqrt(3))/(-(1 -
(-b - 2*c*x)**2/b**2)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/(c*
sqrt(((1 - (-b - 2*c*x)**2/b**2)**(1/3) - 1)/(-(1 - (-b - 2*c*x)*
*2/b**2)**(1/3) - sqrt(3) + 1)**2)*(b + 2*c*x)*(b*x + c*x**2)**(5
/3)) + 3*2**(1/3)*(c*(-b*x - c*x**2)/b**2)**(5/3)*(b + 2*c*x)/(c*
(1 - (-b - 2*c*x)**2/b**2)**(2/3)*(b*x + c*x**2)**(5/3))
```

Mathematica [C] time = 0.048871, size = 57, normalized size = 0.15

$$-\frac{3\left(2cx\left(\frac{cx}{b}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{cx}{b}\right)+b+2cx\right)}{2b^2(x(b+cx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(-5/3),x]
```

```
[Out] (-3*(b + 2*c*x + 2*c*x*(1 + (c*x)/b)^(2/3)*Hypergeometric2F1[1/3,
 2/3, 4/3, -((c*x)/b)]))/(2*b^2*(x*(b + c*x))^(2/3))
```

Maple [F] time = 0.084, size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{-\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+b*x)^(5/3),x)

[Out] int(1/(c*x^2+b*x)^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-5/3),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(-5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cx^2 + bx)^{\frac{5}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-5/3),x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(-5/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx + cx^2)^{\frac{5}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+b*x)**(5/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(-5/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx)^{\frac{5}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-5/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-5/3), x)
```

3.34 $\int \frac{1}{(bx+cx^2)^{8/3}}\, dx$

Optimal. Leaf size=448

$$\frac{21(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{8/3}}+\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(bx+cx^2)^{8/3}}$$

$$+\frac{14\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\cdots}}{-2^{2/3}\sqrt[3]{-\cdots}}\right.\right.}{5c(b+2cx)(bx+cx^2)^{8/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(8/3))/(5*c*(-((c*x*(b
+ c*x))/b^2))^(5/3)*(b*x + c*x^2)^(8/3)) + (21*(b + 2*c*x)*(-((c*
(b*x + c*x^2))/b^2))^(8/3))/(5*c*(-((c*x*(b + c*x))/b^2))^(2/3)*(
b*x + c*x^2)^(8/3)) + (14*2^(1/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(
-((c*(b*x + c*x^2))/b^2))^(8/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b
^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*
2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(5*c*(b + 2*c*x)
*(b*x + c*x^2)^(8/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))
^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)
])
```

Rubi [A] time = 1.06324, antiderivative size = 448, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$

$$\frac{21(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(bx+cx^2)^{8/3}}+\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}}{5c\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(bx+cx^2)^{8/3}}$$

$$+\frac{14\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\left(-\frac{c(bx+cx^2)}{b^2}\right)^{8/3}F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\cdots}}{-2^{2/3}\sqrt[3]{-\cdots}}\right.\right.}{5c(b+2cx)(bx+cx^2)^{8/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)^(-8/3),x]
```

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(8/3))/(5*c*(-((c*x*(b
+ c*x))/b^2))^(5/3)*(b*x + c*x^2)^(8/3)) + (21*(b + 2*c*x)*(-((c*
(b*x + c*x^2))/b^2))^(8/3))/(5*c*(-((c*x*(b + c*x))/b^2))^(2/3)*(
b*x + c*x^2)^(8/3)) + (14*2^(1/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*b^2*(
-((c*(b*x + c*x^2))/b^2))^(8/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b
^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*
2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] -
2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(5*c*(b + 2*c*x)
*(b*x + c*x^2)^(8/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))
^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)
])
```

Rubi in Sympy [A] time = 41.4834, size = 400, normalized size = 0.89

$$\frac{14\sqrt[3]{2}\cdot 3^{\frac{3}{4}}b^2\sqrt{\frac{\left(1-\frac{(-b-2cx)^2}{b^2}\right)^{\frac{2}{3}}+\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}+1}{\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-\sqrt{3}+1\right)^2}}\left(\frac{c(-bx-cx^2)}{b^2}\right)^{\frac{8}{3}}\sqrt{-\sqrt{3}+2}\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}+1\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{1-\frac{(-b}{}}}{-\sqrt[3]{1-\frac{(-b}{}}}\right.\right.}{5c\sqrt{\frac{\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-1}{\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-\sqrt{3}+1\right)^2}}(b+2cx)\left(bx+cx^2\right)^{\frac{8}{3}}}$$

$$+\frac{42\sqrt[3]{2}\left(\frac{c(-bx-cx^2)}{b^2}\right)^{\frac{8}{3}}(b+2cx)}{5c\left(1-\frac{(-b-2cx)^2}{b^2}\right)^{\frac{2}{3}}\left(bx+cx^2\right)^{\frac{8}{3}}}+\frac{24\sqrt[3]{2}\left(\frac{c(-bx-cx^2)}{b^2}\right)^{\frac{8}{3}}(b+2cx)}{5c\left(1-\frac{(-b-2cx)^2}{b^2}\right)^{\frac{5}{3}}\left(bx+cx^2\right)^{\frac{8}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+b*x)**(8/3),x)
```

```
[Out] 14*2**(1/3)*3**(3/4)*b**2*sqrt(((1 - (-b - 2*c*x)**2/b**2)**(2/3)
 + (1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)/(-(1 - (-b - 2*c*x)**2/
b**2)**(1/3) - sqrt(3) + 1)**2)*(c*(-b*x - c*x**2)/b**2)**(8/3)*s
qrt(-sqrt(3) + 2)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)*ellipt
ic_f(asin((-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1 + sqrt(3))/(-(1
 - (-b - 2*c*x)**2/b**2)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/
(5*c*sqrt(((1 - (-b - 2*c*x)**2/b**2)**(1/3) - 1)/(-(1 - (-b - 2*
c*x)**2/b**2)**(1/3) - sqrt(3) + 1)**2)*(b + 2*c*x)*(b*x + c*x**2
)**(8/3)) + 42*2**(1/3)*(c*(-b*x - c*x**2)/b**2)**(8/3)*(b + 2*c*
x)/(5*c*(1 - (-b - 2*c*x)**2/b**2)**(2/3)*(b*x + c*x**2)**(8/3))
+ 24*2**(1/3)*(c*(-b*x - c*x**2)/b**2)**(8/3)*(b + 2*c*x)/(5*c*(1
 - (-b - 2*c*x)**2/b**2)**(5/3)*(b*x + c*x**2)**(8/3))
```

Mathematica [C] time = 0.0972201, size = 90, normalized size = 0.2

$$\frac{-3b^3+15b^2cx+42c^2x^2(b+cx)\left(\frac{cx}{b}+1\right)^{2/3}{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{4}{3};-\frac{cx}{b}\right)+63bc^2x^2+42c^3x^3}{5b^4(x(b+cx))^{5/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(-8/3),x]
```

```
[Out] (-3*b^3 + 15*b^2*c*x + 63*b*c^2*x^2 + 42*c^3*x^3 + 42*c^2*x^2*(b
+ c*x)*(1 + (c*x)/b)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((c*
x)/b)])/(5*b^4*(x*(b + c*x))^(5/3))
```

Maple [F] time = 0.142, size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{-\frac{8}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+b*x)^(8/3),x)
```

```
[Out] int(1/(c*x^2+b*x)^(8/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx)^{\frac{8}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-8/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-8/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(c^2x^4 + 2\,bcx^3 + b^2x^2)(cx^2 + bx)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-8/3),x, algorithm="fricas")
```

```
[Out] integral(1/((c^2*x^4 + 2*b*c*x^3 + b^2*x^2)*(c*x^2 + b*x)^(2/3)),
 x)
```

<u>**Sympy**</u> **[F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx + cx^2)^{\frac{8}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+b*x)**(8/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(-8/3), x)
```

<u>**GIAC/XCAS**</u> **[F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx)^{\frac{8}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-8/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-8/3), x)
```

3.35 $\int \left(bx + cx^2\right)^{5/3} \, dx$

Optimal. Leaf size=842

$$-\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(cx^2+bx\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+c}{b^2}}}{-2^{2/3}\sqrt[3]{-\frac{cx(b+c}{b^2}}}\right.\right.}{364\sqrt[3]{2}c(b+2cx)\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

$$+\frac{5\,3^{3/4}\left(cx^2+bx\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)\right.}{91\,2^{5/6}c(b+2cx)\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

$$-\frac{15(b+2cx)\left(cx^2+bx\right)^{5/3}}{182\sqrt[3]{2}c\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}$$

$$+\frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(b+2cx)\left(cx^2+bx\right)^{5/3}}{26c\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}}+\frac{15\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(cx^2+bx\right)^{5/3}}{364c\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}}$$

```
[Out] (15*(-((c*x*(b + c*x))/b^2))^(2/3)*(b + 2*c*x)*(b*x + c*x^2)^(5/3
))/(364*c*(-((c*(b*x + c*x^2))/b^2))^(5/3)) + (3*(-((c*x*(b + c*x
))/b^2))^(5/3)*(b + 2*c*x)*(b*x + c*x^2)^(5/3))/(26*c*(-((c*(b*x
+ c*x^2))/b^2))^(5/3)) - (15*(b + 2*c*x)*(b*x + c*x^2)^(5/3))/(18
2*2^(1/3)*c*(-((c*(b*x + c*x^2))/b^2))^(5/3)*(1 - Sqrt[3] - 2^(2/
3)*(-((c*x*(b + c*x))/b^2))^(1/3))) - (15*3^(1/4)*Sqrt[2 + Sqrt[3
]]*b^2*(b*x + c*x^2)^(5/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^
(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/
3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x
*(b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - 2^(2/
3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x
*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(364*2^(1/3)*c*(b + 2
*c*x)*(-((c*(b*x + c*x^2))/b^2))^(5/3)*Sqrt[-((1 - 2^(2/3)*(-((c*
x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x
))/b^2))^(1/3))^2)]) + (5*3^(3/4)*b^2*(b*x + c*x^2)^(5/3)*(1 - 2^
```

```
(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(
b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))
/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*Ellipt
icF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))
/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*
Sqrt[3]])/(91*2^(5/6)*c*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(5
/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt
[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi [A] time = 2.02013, antiderivative size = 842, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538

$$-\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}\left(cx^2+bx\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+c}{b^2}}}{-2^{2/3}\sqrt[3]{-\frac{cx(b+c}{b^2}}}\right.\right.}{364\sqrt[3]{2}c(b+2cx)\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

$$+\frac{5\ 3^{3/4}\left(cx^2+bx\right)^{5/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right.\right.}{91\ 2^{5/6}c(b+2cx)\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

$$-\frac{15(b+2cx)\left(cx^2+bx\right)^{5/3}}{182\sqrt[3]{2}c\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}$$

$$+\frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{5/3}(b+2cx)\left(cx^2+bx\right)^{5/3}}{26c\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}}+\frac{15\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(cx^2+bx\right)^{5/3}}{364c\left(-\frac{c(cx^2+bx)}{b^2}\right)^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(b*x + c*x^2)^(5/3),x]
```

```
[Out] (15*(-((c*x*(b + c*x))/b^2))^(2/3)*(b + 2*c*x)*(b*x + c*x^2)^(5/3
))/(364*c*(-((c*(b*x + c*x^2))/b^2))^(5/3)) + (3*(-((c*x*(b + c*x
))/b^2))^(5/3)*(b + 2*c*x)*(b*x + c*x^2)^(5/3))/(26*c*(-((c*(b*x
+ c*x^2))/b^2))^(5/3)) - (15*(b + 2*c*x)*(b*x + c*x^2)^(5/3))/(18
2*2^(1/3)*c*(-((c*(b*x + c*x^2))/b^2))^(5/3)*(1 - Sqrt[3] - 2^(2/
3)*(-((c*x*(b + c*x))/b^2))^(1/3))) - (15*3^(1/4)*Sqrt[2 + Sqrt[3
]]*b^2*(b*x + c*x^2)^(5/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^
(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/
3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x
*(b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - 2^(2/
3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x
*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(364*2^(1/3)*c*(b + 2
*c*x)*(-((c*(b*x + c*x^2))/b^2))^(5/3)*Sqrt[-((1 - 2^(2/3)*(-((c*
x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x
))/b^2))^(1/3))^2)]) + (5*3^(3/4)*b^2*(b*x + c*x^2)^(5/3)*(1 - 2^
(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(
b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))
/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*Ellipt
icF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))
/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*
Sqrt[3]])/(91*2^(5/6)*c*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(5
/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt
[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi in Sympy [A] time = 82.2921, size = 731, normalized size = 0.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**2+b*x)**(5/3),x)
```

```
[Out] -15*2**(2/3)*3**(1/4)*b**2*sqrt(((1 - (-b - 2*c*x)**2/b**2)**(2/3
) + (1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)/(-(1 - (-b - 2*c*x)**2
/b**2)**(1/3) - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(b*x + c*x**2)
**(5/3)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)*elliptic_e(asin(
(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1 + sqrt(3))/(-(1 - (-b - 2
*c*x)**2/b**2)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/(728*c*sqr
t(((1 - (-b - 2*c*x)**2/b**2)**(1/3) - 1)/(-(1 - (-b - 2*c*x)**2/
b**2)**(1/3) - sqrt(3) + 1)**2)*(c*(-b*x - c*x**2)/b**2)**(5/3)*(
b + 2*c*x)) + 5*2**(1/6)*3**(3/4)*b**2*sqrt(((1 - (-b - 2*c*x)**2
/b**2)**(2/3) + (1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)/(-(1 - (-b
 - 2*c*x)**2/b**2)**(1/3) - sqrt(3) + 1)**2)*(b*x + c*x**2)**(5/3
)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)*elliptic_f(asin((-(1 -
 (-b - 2*c*x)**2/b**2)**(1/3) + 1 + sqrt(3))/(-(1 - (-b - 2*c*x)*
*2/b**2)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/(182*c*sqrt(((1
- (-b - 2*c*x)**2/b**2)**(1/3) - 1)/(-(1 - (-b - 2*c*x)**2/b**2)*
*(1/3) - sqrt(3) + 1)**2)*(c*(-b*x - c*x**2)/b**2)**(5/3)*(b + 2*
c*x)) + 3*2**(2/3)*(1 - (-b - 2*c*x)**2/b**2)**(5/3)*(b + 2*c*x)*
```

```
(b*x + c*x**2)**(5/3)/(416*c*(c*(-b*x - c*x**2)/b**2)**(5/3)) + 1
5*2**(2/3)*(1 - (-b - 2*c*x)**2/b**2)**(2/3)*(b + 2*c*x)*(b*x + c
*x**2)**(5/3)/(1456*c*(c*(-b*x - c*x**2)/b**2)**(5/3)) - 15*2**(2
/3)*(b + 2*c*x)*(b*x + c*x**2)**(5/3)/(364*c*(c*(-b*x - c*x**2)/b
**2)**(5/3)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) - sqrt(3) + 1))
```

Mathematica [C] time = 0.0699761, size = 94, normalized size = 0.11

$$\frac{3x\left(5b^4\sqrt[3]{\frac{cx}{b}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{cx}{b}\right)-5b^4-b^3cx+46b^2c^2x^2+70bc^3x^3+28c^4x^4\right)}{364c^2\sqrt[3]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(5/3),x]
```

```
[Out] (3*x*(-5*b^4 - b^3*c*x + 46*b^2*c^2*x^2 + 70*b*c^3*x^3 + 28*c^4*x
^4 + 5*b^4*(1 + (c*x)/b)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -
((c*x)/b)]))/(364*c^2*(x*(b + c*x))^(1/3))
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x)^(5/3),x)
```

```
[Out] int((c*x^2+b*x)^(5/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{5}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(5/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(5/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2 + bx\right)^{\frac{5}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(5/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(5/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx + cx^2\right)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x)**(5/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(5/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{5}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(5/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(5/3), x)
```

3.36 $\int \left(bx + cx^2\right)^{2/3}\, dx$

Optimal. Leaf size=781

$$\frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}} - \frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{7\sqrt[3]{2}c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}$$

$$+\frac{\sqrt[6]{2}3^{3/4}b^2\left(bx+cx^2\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}\cdots}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}\cdots}\right.\right.\cdots}{7c(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\cdots}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}b^2\left(bx+cx^2\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+\cdots}{b^2}}\cdots}{-2^{2/3}\sqrt[3]{-\frac{cx(b+\cdots}{b^2}}\cdots}\right.\right.\cdots}{14\sqrt[3]{2}c(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\cdots}$$

```
[Out] (3*(-((c*x*(b + c*x))/b^2))^(2/3)*(b + 2*c*x)*(b*x + c*x^2)^(2/3)
)/(14*c*(-((c*(b*x + c*x^2))/b^2))^(2/3)) - (3*(b + 2*c*x)*(b*x +
 c*x^2)^(2/3))/(7*2^(1/3)*c*(-((c*(b*x + c*x^2))/b^2))^(2/3)*(1 -
 Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))) - (3*3^(1/4)*
Sqrt[2 + Sqrt[3]]*b^2*(b*x + c*x^2)^(2/3)*(1 - 2^(2/3)*(-((c*x*(b
 + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^
(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] -
2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1 +
Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] -
2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(14*2^
(1/3)*c*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(2/3)*Sqrt[-((1 -
2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3))^2)]) + (2^(1/6)*3^(3/4)*b^2*(b*x +
c*x^2)^(2/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1
 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b
+ c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^
2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b
+ c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^
```

```
2))^(1/3))], -7 + 4*Sqrt[3]])/(7*c*(b + 2*c*x)*(-((c*(b*x + c*x^2
))/b^2))^(2/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3)
)/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi [A] time = 1.82898, antiderivative size = 781, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538

$$\frac{3\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}(b+2cx)\left(bx+cx^2\right)^{2/3}}{14c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}}-\frac{3(b+2cx)\left(bx+cx^2\right)^{2/3}}{7\sqrt[3]{2}c\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}$$
$$+\frac{\sqrt[6]{2}3^{3/4}b^2\left(bx+cx^2\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}\cdots}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}\cdots}\right.\right.}{7c(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$
$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}b^2\left(bx+cx^2\right)^{2/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+\cdots}{b^2}}}{-2^{2/3}\sqrt[3]{-\frac{cx(b+\cdots}{b^2}}}\right.\right.}{14\sqrt[3]{2}c(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{2/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(b*x + c*x^2)^(2/3),x]
```

```
[Out] (3*(-((c*x*(b + c*x))/b^2))^(2/3)*(b + 2*c*x)*(b*x + c*x^2)^(2/3)
)/(14*c*(-((c*(b*x + c*x^2))/b^2))^(2/3)) - (3*(b + 2*c*x)*(b*x +
 c*x^2)^(2/3))/(7*2^(1/3)*c*(-((c*(b*x + c*x^2))/b^2))^(2/3)*(1 -
 Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))) - (3*3^(1/4)*
Sqrt[2 + Sqrt[3]]*b^2*(b*x + c*x^2)^(2/3)*(1 - 2^(2/3)*(-((c*x*(b
 + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^
(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] -
2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1 +
Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] -
```

```
2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(14*2^
(1/3)*c*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(2/3)*Sqrt[-((1 -
2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3))^2)]) + (2^(1/6)*3^(3/4)*b^2*(b*x +
c*x^2)^(2/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1
 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b
+ c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^
2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b
+ c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^
2))^(1/3))], -7 + 4*Sqrt[3]])/(7*c*(b + 2*c*x)*(-((c*(b*x + c*x^2
))/b^2))^(2/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3)
)/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi in Sympy [A] time = 68.5779, size = 663, normalized size = 0.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x)**(2/3),x)
```

```
[Out] -3*2**(2/3)*3**(1/4)*b**2*sqrt(((1 - (-b - 2*c*x)**2/b**2)**(2/3)
 + (1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)/(-(1 - (-b - 2*c*x)**2/
b**2)**(1/3) - sqrt(3) + 1)**2)*sqrt(sqrt(3) + 2)*(b*x + c*x**2)*
*(2/3)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)*elliptic_e(asin((
-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1 + sqrt(3))/(-(1 - (-b - 2*
c*x)**2/b**2)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/(28*c*sqrt(
((1 - (-b - 2*c*x)**2/b**2)**(1/3) - 1)/(-(1 - (-b - 2*c*x)**2/b*
*2)**(1/3) - sqrt(3) + 1)**2)*(c*(-b*x - c*x**2)/b**2)**(2/3)*(b
+ 2*c*x)) + 2**(1/6)*3**(3/4)*b**2*sqrt(((1 - (-b - 2*c*x)**2/b**
2)**(2/3) + (1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)/(-(1 - (-b - 2
*c*x)**2/b**2)**(1/3) - sqrt(3) + 1)**2)*(b*x + c*x**2)**(2/3)*(-
(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)*elliptic_f(asin((-(1 - (-b
 - 2*c*x)**2/b**2)**(1/3) + 1 + sqrt(3))/(-(1 - (-b - 2*c*x)**2/b
**2)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/(7*c*sqrt(((1 - (-b
- 2*c*x)**2/b**2)**(1/3) - 1)/(-(1 - (-b - 2*c*x)**2/b**2)**(1/3)
 - sqrt(3) + 1)**2)*(c*(-b*x - c*x**2)/b**2)**(2/3)*(b + 2*c*x))
+ 3*2**(2/3)*(1 - (-b - 2*c*x)**2/b**2)**(2/3)*(b + 2*c*x)*(b*x +
 c*x**2)**(2/3)/(56*c*(c*(-b*x - c*x**2)/b**2)**(2/3)) - 3*2**(2/
3)*(b + 2*c*x)*(b*x + c*x**2)**(2/3)/(14*c*(c*(-b*x - c*x**2)/b**
2)**(2/3)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) - sqrt(3) + 1))
```

Mathematica [C] time = 0.0526238, size = 70, normalized size = 0.09

$$\frac{3x\left(b^2\left(-\sqrt[3]{\frac{cx}{b}+1}\right)\ {}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{cx}{b}\right)+b^2+3bcx+2c^2x^2\right)}{14c\sqrt[3]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(2/3),x]
```

```
[Out] (3*x*(b^2 + 3*b*c*x + 2*c^2*x^2 - b^2*(1 + (c*x)/b)^(1/3)*Hyperge
ometric2F1[1/3, 2/3, 5/3, -((c*x)/b)]))/(14*c*(x*(b + c*x))^(1/3)
)
```

Maple [F] time = 0.054, size = 0, normalized size = 0.

$$\int \left(c x^{2}+b x \right)^{\frac{2}{3}}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x)^(2/3),x)
```

```
[Out] int((c*x^2+b*x)^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int {\left(c x^{2} + b x\right)}^{\frac{2}{3}}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(2/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left({\left(c x^{2} + b x\right)}^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(2/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(2/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx + cx^2\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x)**(2/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(2/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{2}{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(2/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(2/3), x)
```

3.37 $\displaystyle\int \frac{1}{\sqrt[3]{bx+cx^2}}\,dx$

Optimal. Leaf size=715

$$-\frac{3(b+2cx)\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}}{\sqrt[3]{2}c\sqrt[3]{bx+cx^2}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}+\frac{\sqrt[6]{2}3^{3/4}b^2\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\cdots}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\cdots}\right.\right.}{c(b+2cx)\sqrt[3]{bx+cx^2}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}b^2\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b\cdots}{b\cdots}}}{-2^{2/3}\sqrt[3]{-\frac{cx(b\cdots}{b\cdots}}}\right.\right.}{2\sqrt[3]{2}c(b+2cx)\sqrt[3]{bx+cx^2}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (-3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(1/3))/(2^(1/3)*c*(b*x
 + c*x^2)^(1/3)*(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(
1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^
2))^(1/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 +
2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c
*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))
^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c
*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))
^(1/3))], -7 + 4*Sqrt[3]])/(2*2^(1/3)*c*(b + 2*c*x)*(b*x + c*x^2)
^(1/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - S
qrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)]) + (2^(1/6)*
3^(3/4)*b^2*(-((c*(b*x + c*x^2))/b^2))^(1/3)*(1 - 2^(2/3)*(-((c*x
*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2
))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3]
 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1
 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3]
 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(c*
```

```
(b + 2*c*x)*(b*x + c*x^2)^(1/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b +
c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2)
)^(1/3))^2)])
```

Rubi [A] time = 1.66507, antiderivative size = 715, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$

$$-\frac{3(b+2cx)\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}}{\sqrt[3]{2}c\sqrt[3]{bx+cx^2}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}$$

$$+\frac{\sqrt[6]{2}3^{3/4}b^2\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\cdots}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\cdots}\right)\cdots\right)}{c(b+2cx)\sqrt[3]{bx+cx^2}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

$$-\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}b^2\sqrt[3]{-\frac{c\left(bx+cx^2\right)}{b^2}}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b\cdots}{b\cdots}}\cdots}{-2^{2/3}\sqrt[3]{-\frac{cx(b\cdots}{b\cdots}}\cdots}\right)\cdots\right)}{2\sqrt[3]{2}c(b+2cx)\sqrt[3]{bx+cx^2}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(b*x + c*x^2)^(-1/3),x]
```

```
[Out] (-3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(1/3))/(2^(1/3)*c*(b*x
 + c*x^2)^(1/3)*(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(
1/3))) - (3*3^(1/4)*Sqrt[2 + Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^
2))^(1/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 +
2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c
*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))
^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c
*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))
```

```
^(1/3))], -7 + 4*Sqrt[3]])/(2*2^(1/3)*c*(b + 2*c*x)*(b*x + c*x^2)
^(1/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - S
qrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)]) + (2^(1/6)*
3^(3/4)*b^2*(-((c*(b*x + c*x^2))/b^2))^(1/3)*(1 - 2^(2/3)*(-((c*x
*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2
))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3]
 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1
 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3]
 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(c*
(b + 2*c*x)*(b*x + c*x^2)^(1/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b +
c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2)
)^(1/3))^2)])
```

Rubi in Sympy [A] time = 55.4489, size = 595, normalized size = 0.83

$$-\frac{3\cdot 2^{\frac{2}{3}}\sqrt[4]{3}b^2\sqrt{\frac{\left(1-\frac{(-b-2cx)^2}{b^2}\right)^{\frac{2}{3}}+\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}+1}{\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-\sqrt{3}+1\right)^2}}\sqrt[3]{\frac{c\left(-bx-cx^2\right)}{b^2}}\sqrt{\sqrt{3}+2}\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}+1\right)E\left(\operatorname{asin}\left(\frac{-\sqrt[3]{1-\frac{(-b}{}}}{-\sqrt[3]{1-\frac{(-b}{}}}\right.\right.}{4c\sqrt{\frac{\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-1}{\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-\sqrt{3}+1\right)^2}}(b+2cx)\sqrt[3]{bx+cx^2}}$$

$$+\frac{\sqrt[6]{2}\cdot 3^{\frac{3}{4}}b^2\sqrt{\frac{\left(1-\frac{(-b-2cx)^2}{b^2}\right)^{\frac{2}{3}}+\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}+1}{\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-\sqrt{3}+1\right)^2}}\sqrt[3]{\frac{c\left(-bx-cx^2\right)}{b^2}}\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}+1\right)F\left(\operatorname{asin}\left(\frac{-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}+1}{-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-}\right.\right.}{c\sqrt{\frac{\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-1}{\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-\sqrt{3}+1\right)^2}}(b+2cx)\sqrt[3]{bx+cx^2}}$$

$$-\frac{3\cdot 2^{\frac{2}{3}}\sqrt[3]{\frac{c\left(-bx-cx^2\right)}{b^2}}(b+2cx)}{2c\sqrt[3]{bx+cx^2}\left(-\sqrt[3]{1-\frac{(-b-2cx)^2}{b^2}}-\sqrt{3}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+b*x)**(1/3),x)
```

```
[Out] -3*2**(2/3)*3**(1/4)*b**2*sqrt(((1 - (-b - 2*c*x)**2/b**2)**(2/3)
 + (1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)/(-(1 - (-b - 2*c*x)**2/
b**2)**(1/3) - sqrt(3) + 1)**2)*(c*(-b*x - c*x**2)/b**2)**(1/3)*s
qrt(sqrt(3) + 2)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)*ellipti
c_e(asin((-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1 + sqrt(3))/(-(1
- (-b - 2*c*x)**2/b**2)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/(
4*c*sqrt(((1 - (-b - 2*c*x)**2/b**2)**(1/3) - 1)/(-(1 - (-b - 2*c
*x)**2/b**2)**(1/3) - sqrt(3) + 1)**2)*(b + 2*c*x)*(b*x + c*x**2)
**(1/3)) + 2**(1/6)*3**(3/4)*b**2*sqrt(((1 - (-b - 2*c*x)**2/b**2
)**(2/3) + (1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)/(-(1 - (-b - 2*
c*x)**2/b**2)**(1/3) - sqrt(3) + 1)**2)*(c*(-b*x - c*x**2)/b**2)*
*(1/3)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)*elliptic_f(asin((
-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1 + sqrt(3))/(-(1 - (-b - 2*
c*x)**2/b**2)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/(c*sqrt(((1
 - (-b - 2*c*x)**2/b**2)**(1/3) - 1)/(-(1 - (-b - 2*c*x)**2/b**2)
**(1/3) - sqrt(3) + 1)**2)*(b + 2*c*x)*(b*x + c*x**2)**(1/3)) - 3
*2**(2/3)*(c*(-b*x - c*x**2)/b**2)**(1/3)*(b + 2*c*x)/(2*c*(b*x +
 c*x**2)**(1/3)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) - sqrt(3) + 1
))
```

Mathematica [C] time = 0.026365, size = 46, normalized size = 0.06

$$\frac{3x\sqrt[3]{\frac{b+cx}{b}}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{cx}{b}\right)}{2\sqrt[3]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(-1/3),x]
```

```
[Out] (3*x*((b + c*x)/b)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((c*x)
/b)])/(2*(x*(b + c*x))^(1/3))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+b*x)^(1/3),x)

[Out] int(1/(c*x^2+b*x)^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx)^{\frac{1}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-1/3),x, algorithm="maxima")

[Out] integrate((c*x^2 + b*x)^(-1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cx^2 + bx)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-1/3),x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(-1/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[3]{bx + cx^2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+b*x)**(1/3),x)

[Out] Integral((b*x + c*x**2)**(-1/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-1/3),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(-1/3), x)
```

3.38 $\int \frac{1}{(bx+cx^2)^{4/3}}\,dx$

Optimal. Leaf size=773

$$\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(bx+cx^2)^{4/3}}+\frac{3\,2^{2/3}(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\,(bx+cx^2)^{4/3}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}$$

$$-\frac{2\sqrt[6]{2}3^{3/4}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\cdots}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\cdots}\right.\right.}{c(b+2cx)\,(bx+cx^2)^{4/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

$$+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b}{}}\cdots}{-2^{2/3}\sqrt[3]{-\frac{cx(b}{}}\cdots}\right.\right.}{\sqrt[3]{2}c(b+2cx)\,(bx+cx^2)^{4/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(4/3))/(c*(-((c*x*(b +
c*x))/b^2))^(1/3)*(b*x + c*x^2)^(4/3)) + (3*2^(2/3)*(b + 2*c*x)*(
-((c*(b*x + c*x^2))/b^2))^(4/3))/(c*(b*x + c*x^2)^(4/3)*(1 - Sqrt
[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))) + (3*3^(1/4)*Sqrt[
2 + Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(4/3)*(1 - 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x)
)/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sq
rt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcS
in[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sq
rt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]]
)/(2^(1/3)*c*(b + 2*c*x)*(b*x + c*x^2)^(4/3)*Sqrt[-((1 - 2^(2/3)*
(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b
 + c*x))/b^2))^(1/3))^2)]) - (2*2^(1/6)*3^(3/4)*b^2*(-((c*(b*x +
c*x^2))/b^2))^(4/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*
Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((
c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c
*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((
c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c
```

```
*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(c*(b + 2*c*x)*(b*x + c*x^2)
^(4/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - S
qrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi [A] time = 1.83522, antiderivative size = 773, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538

$$\frac{3(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\,(bx+cx^2)^{4/3}}+\frac{3\,2^{2/3}(b+2cx)\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}}{c\,(bx+cx^2)^{4/3}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}-\frac{2\sqrt[6]{2}3^{3/4}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\cdots}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\cdots}\right.\right.}{c(b+2cx)\,(bx+cx^2)^{4/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}+\frac{3\sqrt[4]{3}\sqrt{2+\sqrt{3}}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{4/3}\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{c x(b}{b}}\cdots}{-2^{2/3}\sqrt[3]{-\frac{cx(b}{b}}\cdots}\right.\right.}{\sqrt[3]{2}c(b+2cx)\,(bx+cx^2)^{4/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(b*x + c*x^2)^(-4/3),x]
```

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(4/3))/(c*(-((c*x*(b +
c*x))/b^2))^(1/3)*(b*x + c*x^2)^(4/3)) + (3*2^(2/3)*(b + 2*c*x)*(
-((c*(b*x + c*x^2))/b^2))^(4/3))/(c*(b*x + c*x^2)^(4/3)*(1 - Sqrt
[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))) + (3*3^(1/4)*Sqrt[
2 + Sqrt[3]]*b^2*(-((c*(b*x + c*x^2))/b^2))^(4/3)*(1 - 2^(2/3)*(-
((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x)
)/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sq
rt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcS
in[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sq
```

```
rt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]]
)/(2^(1/3)*c*(b + 2*c*x)*(b*x + c*x^2)^(4/3)*Sqrt[-((1 - 2^(2/3)*
(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b
 + c*x))/b^2))^(1/3))^2)]) - (2*2^(1/6)*3^(3/4)*b^2*(-((c*(b*x +
c*x^2))/b^2))^(4/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*
Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((
c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c
*x))/b^2))^(1/3))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((
c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c
*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(c*(b + 2*c*x)*(b*x + c*x^2)
^(4/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - S
qrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi in Sympy [A] time = 68.9371, size = 660, normalized size = 0.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+b*x)**(4/3),x)
```

```
[Out] 3*2**(2/3)*3**(1/4)*b**2*sqrt(((1 - (-b - 2*c*x)**2/b**2)**(2/3)
+ (1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)/(-(1 - (-b - 2*c*x)**2/b
**2)**(1/3) - sqrt(3) + 1)**2)*(c*(-b*x - c*x**2)/b**2)**(4/3)*sq
rt(sqrt(3) + 2)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)*elliptic
_e(asin((-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1 + sqrt(3))/(-(1 -
 (-b - 2*c*x)**2/b**2)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/(2
*c*sqrt(((1 - (-b - 2*c*x)**2/b**2)**(1/3) - 1)/(-(1 - (-b - 2*c*
x)**2/b**2)**(1/3) - sqrt(3) + 1)**2)*(b + 2*c*x)*(b*x + c*x**2)*
*(4/3)) - 2*2**(1/6)*3**(3/4)*b**2*sqrt(((1 - (-b - 2*c*x)**2/b**
2)**(2/3) + (1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)/(-(1 - (-b - 2
*c*x)**2/b**2)**(1/3) - sqrt(3) + 1)**2)*(c*(-b*x - c*x**2)/b**2)
**(4/3)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)*elliptic_f(asin(
(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1 + sqrt(3))/(-(1 - (-b - 2
*c*x)**2/b**2)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/(c*sqrt(((
1 - (-b - 2*c*x)**2/b**2)**(1/3) - 1)/(-(1 - (-b - 2*c*x)**2/b**2
)**(1/3) - sqrt(3) + 1)**2)*(b + 2*c*x)*(b*x + c*x**2)**(4/3)) +
3*2**(2/3)*(c*(-b*x - c*x**2)/b**2)**(4/3)*(b + 2*c*x)/(c*(b*x +
c*x**2)**(4/3)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) - sqrt(3) + 1)
) + 3*2**(2/3)*(c*(-b*x - c*x**2)/b**2)**(4/3)*(b + 2*c*x)/(c*(1
- (-b - 2*c*x)**2/b**2)**(1/3)*(b*x + c*x**2)**(4/3))
```

Mathematica [C] time = 0.0464318, size = 57, normalized size = 0.07

$$\frac{3cx\sqrt[3]{\frac{cx}{b}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{cx}{b}\right)-3(b+2cx)}{b^2\sqrt[3]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(-4/3),x]
```

```
[Out] (-3*(b + 2*c*x) + 3*c*x*(1 + (c*x)/b)^(1/3)*Hypergeometric2F1[1/3
, 2/3, 5/3, -((c*x)/b)])/(b^2*(x*(b + c*x))^(1/3))
```

Maple [F] time = 0.087, size = 0, normalized size = 0.

$$\int \left(c x^{2}+b x\right)^{-\frac{4}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+b*x)^(4/3),x)
```

```
[Out] int(1/(c*x^2+b*x)^(4/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2+bx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-4/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cx^2+bx)^{\frac{4}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-4/3),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(-4/3), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+cx^2)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+b*x)**(4/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(-4/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2+bx)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-4/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-4/3), x)
```

3.39 $\int \frac{1}{(bx+cx^2)^{7/3}}\,dx$

Optimal. Leaf size=838

$$\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}\,b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)\middle|-7\right.}{2\sqrt[3]{2}\,c(b+2cx)\left(cx^2+bx\right)^{7/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

$$-\frac{5\sqrt[6]{2}\,3^{3/4}b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}\,F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)\middle|-7+4\right.}{c(b+2cx)\left(cx^2+bx\right)^{7/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

$$+\frac{15(b+2cx)\left(-\frac{c(cx^2+bx)}{b^2}\right)^{7/3}}{\sqrt[3]{2}c\left(cx^2+bx\right)^{7/3}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}$$

$$+\frac{15(b+2cx)\left(-\frac{c(cx^2+bx)}{b^2}\right)^{7/3}}{2c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\left(cx^2+bx\right)^{7/3}}+\frac{3(b+2cx)\left(-\frac{c(cx^2+bx)}{b^2}\right)^{7/3}}{4c\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}\left(cx^2+bx\right)^{7/3}}$$

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(7/3))/(4*c*(-((c*x*(b
+ c*x))/b^2))^(4/3)*(b*x + c*x^2)^(7/3)) + (15*(b + 2*c*x)*(-((c*
(b*x + c*x^2))/b^2))^(7/3))/(2*c*(-((c*x*(b + c*x))/b^2))^(1/3)*(
b*x + c*x^2)^(7/3)) + (15*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^
(7/3))/(2^(1/3)*c*(b*x + c*x^2)^(7/3)*(1 - Sqrt[3] - 2^(2/3)*(-((
c*x*(b + c*x))/b^2))^(1/3))) + (15*3^(1/4)*Sqrt[2 + Sqrt[3]]*b^2*
(-((c*(b*x + c*x^2))/b^2))^(7/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/
b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2
*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(
-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(
-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(2*2^(1/3)*c*(b
 + 2*c*x)*(b*x + c*x^2)^(7/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*
```

```
x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^
(1/3))^2)]) - (5*2^(1/6)*3^(3/4)*b^2*(-((c*(b*x + c*x^2))/b^2))^(
7/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/
3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/
b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3
))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/
b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3
))], -7 + 4*Sqrt[3]])/(c*(b + 2*c*x)*(b*x + c*x^2)^(7/3)*Sqrt[-((
1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3
)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi [A] time = 1.99409, antiderivative size = 838, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$

$$\frac{15\sqrt[4]{3}\sqrt{2+\sqrt{3}}b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}E\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)|-7}{2\sqrt[3]{2}c(b+2cx)\left(cx^2+bx\right)^{7/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

$$-\frac{5\sqrt[6]{2}3^{3/4}b^2\left(1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\right)\sqrt{\frac{2\sqrt[3]{2}\left(-\frac{cx(b+cx)}{b^2}\right)^{2/3}+2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+1}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}F\left(\sin^{-1}\left(\frac{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}+\sqrt{3}+1}{-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1}\right)|-7+4}{c(b+2cx)\left(cx^2+bx\right)^{7/3}\sqrt{-\frac{1-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}}{\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)^2}}}$$

$$+\frac{15(b+2cx)\left(-\frac{c\left(cx^2+bx\right)}{b^2}\right)^{7/3}}{\sqrt[3]{2}c\left(cx^2+bx\right)^{7/3}\left(-2^{2/3}\sqrt[3]{-\frac{cx(b+cx)}{b^2}}-\sqrt{3}+1\right)}$$

$$+\frac{15(b+2cx)\left(-\frac{c\left(cx^2+bx\right)}{b^2}\right)^{7/3}}{2c\sqrt[3]{-\frac{cx(b+cx)}{b^2}}\left(cx^2+bx\right)^{7/3}}+\frac{3(b+2cx)\left(-\frac{c\left(cx^2+bx\right)}{b^2}\right)^{7/3}}{4c\left(-\frac{cx(b+cx)}{b^2}\right)^{4/3}\left(cx^2+bx\right)^{7/3}}$$

Warning: Unable to verify antiderivative.

```
[In]  Int[(b*x + c*x^2)^(-7/3),x]
```

```
[Out] (3*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^(7/3))/(4*c*(-((c*x*(b
+ c*x))/b^2))^(4/3)*(b*x + c*x^2)^(7/3)) + (15*(b + 2*c*x)*(-((c*
(b*x + c*x^2))/b^2))^(7/3))/(2*c*(-((c*x*(b + c*x))/b^2))^(1/3)*(
b*x + c*x^2)^(7/3)) + (15*(b + 2*c*x)*(-((c*(b*x + c*x^2))/b^2))^
(7/3))/(2^(1/3)*c*(b*x + c*x^2)^(7/3)*(1 - Sqrt[3] - 2^(2/3)*(-((
c*x*(b + c*x))/b^2))^(1/3))) + (15*3^(1/4)*Sqrt[2 + Sqrt[3]]*b^2*
(-((c*(b*x + c*x^2))/b^2))^(7/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/
b^2))^(1/3))*Sqrt[(1 + 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2
*2^(1/3)*(-((c*x*(b + c*x))/b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(
-((c*x*(b + c*x))/b^2))^(1/3))^2]*EllipticE[ArcSin[(1 + Sqrt[3] -
 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(
-((c*x*(b + c*x))/b^2))^(1/3))], -7 + 4*Sqrt[3]])/(2*2^(1/3)*c*(b
 + 2*c*x)*(b*x + c*x^2)^(7/3)*Sqrt[-((1 - 2^(2/3)*(-((c*x*(b + c*
x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^
(1/3))^2)]) - (5*2^(1/6)*3^(3/4)*b^2*(-((c*(b*x + c*x^2))/b^2))^(
7/3)*(1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))*Sqrt[(1 + 2^(2/
3)*(-((c*x*(b + c*x))/b^2))^(1/3) + 2*2^(1/3)*(-((c*x*(b + c*x))/
b^2))^(2/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3
))^2]*EllipticF[ArcSin[(1 + Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/
b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3
))], -7 + 4*Sqrt[3]])/(c*(b + 2*c*x)*(b*x + c*x^2)^(7/3)*Sqrt[-((
1 - 2^(2/3)*(-((c*x*(b + c*x))/b^2))^(1/3))/(1 - Sqrt[3] - 2^(2/3
)*(-((c*x*(b + c*x))/b^2))^(1/3))^2)])
```

Rubi in Sympy [A] time = 82.6812, size = 728, normalized size = 0.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+b*x)**(7/3),x)
```

```
[Out] 15*2**(2/3)*3**(1/4)*b**2*sqrt(((1 - (-b - 2*c*x)**2/b**2)**(2/3)
 + (1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)/(-(1 - (-b - 2*c*x)**2/
b**2)**(1/3) - sqrt(3) + 1)**2)*(c*(-b*x - c*x**2)/b**2)**(7/3)*s
qrt(sqrt(3) + 2)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)*ellipti
c_e(asin((-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1 + sqrt(3))/(-(1
- (-b - 2*c*x)**2/b**2)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/(
4*c*sqrt(((1 - (-b - 2*c*x)**2/b**2)**(1/3) - 1)/(-(1 - (-b - 2*c
*x)**2/b**2)**(1/3) - sqrt(3) + 1)**2)*(b + 2*c*x)*(b*x + c*x**2)
**(7/3)) - 5*2**(1/6)*3**(3/4)*b**2*sqrt(((1 - (-b - 2*c*x)**2/b*
*2)**(2/3) + (1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)/(-(1 - (-b -
2*c*x)**2/b**2)**(1/3) - sqrt(3) + 1)**2)*(c*(-b*x - c*x**2)/b**2
)**(7/3)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1)*elliptic_f(asin
((-(1 - (-b - 2*c*x)**2/b**2)**(1/3) + 1 + sqrt(3))/(-(1 - (-b -
2*c*x)**2/b**2)**(1/3) - sqrt(3) + 1)), -7 + 4*sqrt(3))/(c*sqrt((
(1 - (-b - 2*c*x)**2/b**2)**(1/3) - 1)/(-(1 - (-b - 2*c*x)**2/b**
```

```
2)**(1/3) - sqrt(3) + 1)**2)*(b + 2*c*x)*(b*x + c*x**2)**(7/3)) +
 15*2**(2/3)*(c*(-b*x - c*x**2)/b**2)**(7/3)*(b + 2*c*x)/(2*c*(b*
x + c*x**2)**(7/3)*(-(1 - (-b - 2*c*x)**2/b**2)**(1/3) - sqrt(3)
+ 1)) + 15*2**(2/3)*(c*(-b*x - c*x**2)/b**2)**(7/3)*(b + 2*c*x)/(
2*c*(1 - (-b - 2*c*x)**2/b**2)**(1/3)*(b*x + c*x**2)**(7/3)) + 3*
2**(2/3)*(c*(-b*x - c*x**2)/b**2)**(7/3)*(b + 2*c*x)/(c*(1 - (-b
- 2*c*x)**2/b**2)**(4/3)*(b*x + c*x**2)**(7/3))
```

Mathematica [C] time = 0.0986271, size = 90, normalized size = 0.11

$$\frac{-3b^3+24b^2cx-30c^2x^2(b+cx)\sqrt[3]{\frac{cx}{b}+1}\,{}_2F_1\left(\frac{1}{3},\frac{2}{3};\frac{5}{3};-\frac{cx}{b}\right)+90bc^2x^2+60c^3x^3}{4b^4(x(b+cx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(-7/3),x]
```

```
[Out] (-3*b^3 + 24*b^2*c*x + 90*b*c^2*x^2 + 60*c^3*x^3 - 30*c^2*x^2*(b
+ c*x)*(1 + (c*x)/b)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((c*
x)/b)])/(4*b^4*(x*(b + c*x))^(4/3))
```

Maple [F] time = 0.144, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{-\frac{7}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+b*x)^(7/3),x)
```

```
[Out] int(1/(c*x^2+b*x)^(7/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2+bx)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-7/3),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-7/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(c^2x^4 + 2\,bcx^3 + b^2x^2)(cx^2 + bx)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-7/3),x, algorithm="fricas")
```

```
[Out] integral(1/((c^2*x^4 + 2*b*c*x^3 + b^2*x^2)*(c*x^2 + b*x)^(1/3)),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx + cx^2)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+b*x)**(7/3),x)
```

```
[Out] Integral((b*x + c*x**2)**(-7/3), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx)^{\frac{7}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-7/3),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-7/3), x)
```

3.40 $\int \left(bx+cx^2\right)^{5/4}\,dx$

Optimal. Leaf size=119

$$-\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2}+\frac{5b^5\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{84\sqrt{2}c^3\left(bx+cx^2\right)^{3/4}}+\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}$$

```
[Out] (-5*b^2*(b + 2*c*x)*(b*x + c*x^2)^(1/4))/(84*c^2) + ((b + 2*c*x)*
(b*x + c*x^2)^(5/4))/(7*c) + (5*b^5*(-((c*(b*x + c*x^2))/b^2))^(3
/4)*EllipticF[ArcSin[1 + (2*c*x)/b]/2, 2])/(84*Sqrt[2]*c^3*(b*x +
 c*x^2)^(3/4))
```

Rubi [A] time = 0.110834, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$-\frac{5b^2(b+2cx)\sqrt[4]{bx+cx^2}}{84c^2}+\frac{5b^5\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{84\sqrt{2}c^3\left(bx+cx^2\right)^{3/4}}+\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)^(5/4),x]
```

```
[Out] (-5*b^2*(b + 2*c*x)*(b*x + c*x^2)^(1/4))/(84*c^2) + ((b + 2*c*x)*
(b*x + c*x^2)^(5/4))/(7*c) + (5*b^5*(-((c*(b*x + c*x^2))/b^2))^(3
/4)*EllipticF[ArcSin[1 + (2*c*x)/b]/2, 2])/(84*Sqrt[2]*c^3*(b*x +
 c*x^2)^(3/4))
```

Rubi in Sympy [A] time = 17.2335, size = 109, normalized size = 0.92

$$\frac{5\sqrt{2}b^5\left(\frac{c(-bx-cx^2)}{b^2}\right)^{\frac{3}{4}}F\left(\frac{\operatorname{asin}\left(1+\frac{2cx}{b}\right)}{2}\middle|2\right)}{168c^3\left(bx+cx^2\right)^{\frac{3}{4}}}-\frac{5b^2\left(b+2cx\right)\sqrt[4]{bx+cx^2}}{84c^2}+\frac{\left(b+2cx\right)\left(bx+cx^2\right)^{\frac{5}{4}}}{7c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x)**(5/4),x)
```

```
[Out] 5*sqrt(2)*b**5*(c*(-b*x - c*x**2)/b**2)**(3/4)*elliptic_f(asin(1
+ 2*c*x/b)/2, 2)/(168*c**3*(b*x + c*x**2)**(3/4)) - 5*b**2*(b + 2
```

```
*c*x)*(b*x + c*x**2)**(1/4)/(84*c**2) + (b + 2*c*x)*(b*x + c*x**2
)**(5/4)/(7*c)
```

Mathematica [C] time = 0.070116, size = 94, normalized size = 0.79

$$\frac{x\left(5b^4\left(\frac{cx}{b}+1\right)^{3/4}{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{cx}{b}\right)-5b^4-3b^3cx+38b^2c^2x^2+60bc^3x^3+24c^4x^4\right)}{84c^2(x(b+cx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(5/4),x]
```

```
[Out] (x*(-5*b^4 - 3*b^3*c*x + 38*b^2*c^2*x^2 + 60*b*c^3*x^3 + 24*c^4*x
^4 + 5*b^4*(1 + (c*x)/b)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -
((c*x)/b)]))/(84*c^2*(x*(b + c*x))^(3/4))
```

Maple [F] time = 0.063, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x)^(5/4),x)
```

```
[Out] int((c*x^2+b*x)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2 + bx\right)^{\frac{5}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(5/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(5/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx + cx^2\right)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x)**(5/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(5/4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{5}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(5/4), x)
```

3.41 $\int \left(bx+cx^2\right)^{3/4}\,dx$

Optimal. Leaf size=90

$$\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}-\frac{3b^3\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{10\sqrt{2}c^2\sqrt[4]{bx+cx^2}}$$

```
[Out] ((b + 2*c*x)*(b*x + c*x^2)^(3/4))/(5*c) - (3*b^3*(-((c*(b*x + c*x
^2))/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]/2, 2])/(10*Sqrt[
2]*c^2*(b*x + c*x^2)^(1/4))
```

Rubi [A] time = 0.0779072, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}-\frac{3b^3\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{10\sqrt{2}c^2\sqrt[4]{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)^(3/4),x]
```

```
[Out] ((b + 2*c*x)*(b*x + c*x^2)^(3/4))/(5*c) - (3*b^3*(-((c*(b*x + c*x
^2))/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]/2, 2])/(10*Sqrt[
2]*c^2*(b*x + c*x^2)^(1/4))
```

Rubi in Sympy [A] time = 14.028, size = 80, normalized size = 0.89

$$-\frac{3\sqrt{2}b^3\sqrt[4]{\frac{c\left(-bx-cx^2\right)}{b^2}}E\left(\frac{\operatorname{asin}\left(1+\frac{2cx}{b}\right)}{2}\middle|2\right)}{20c^2\sqrt[4]{bx+cx^2}}+\frac{(b+2cx)\left(bx+cx^2\right)^{\frac{3}{4}}}{5c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x)**(3/4),x)
```

```
[Out] -3*sqrt(2)*b**3*(c*(-b*x - c*x**2)/b**2)**(1/4)*elliptic_e(asin(1
 + 2*c*x/b)/2, 2)/(20*c**2*(b*x + c*x**2)**(1/4)) + (b + 2*c*x)*(
```

```
b*x + c*x**2)**(3/4)/(5*c)
```

Mathematica [C] time = 0.0639374, size = 70, normalized size = 0.78

$$\frac{x\left(b^2\left(-\sqrt[4]{\frac{cx}{b}+1}\right)\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{cx}{b}\right)+b^2+3bcx+2c^2x^2\right)}{5c\sqrt[4]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(3/4),x]
```

```
[Out] (x*(b^2 + 3*b*c*x + 2*c^2*x^2 - b^2*(1 + (c*x)/b)^(1/4)*Hypergeom
etric2F1[1/4, 3/4, 7/4, -((c*x)/b)]))/(5*c*(x*(b + c*x))^(1/4))
```

Maple [F] time = 0.056, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x)^(3/4),x)
```

```
[Out] int((c*x^2+b*x)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2 + bx\right)^{\frac{3}{4}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(3/4),x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(3/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(bx + cx^2\right)^{\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x)**(3/4),x)

[Out] Integral((b*x + c*x**2)**(3/4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{3}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(3/4),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(3/4), x)
```

3.42 $\int \sqrt[4]{bx+cx^2}\,dx$

Optimal. Leaf size=90

$$\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}-\frac{b^3\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{3\sqrt{2}c^2\left(bx+cx^2\right)^{3/4}}$$

```
[Out] ((b + 2*c*x)*(b*x + c*x^2)^(1/4))/(3*c) - (b^3*(-((c*(b*x + c*x^2
))/b^2))^(3/4)*EllipticF[ArcSin[1 + (2*c*x)/b]/2, 2])/(3*Sqrt[2]*
c^2*(b*x + c*x^2)^(3/4))
```

Rubi [A] time = 0.0751646, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}-\frac{b^3\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{3\sqrt{2}c^2\left(bx+cx^2\right)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)^(1/4),x]
```

```
[Out] ((b + 2*c*x)*(b*x + c*x^2)^(1/4))/(3*c) - (b^3*(-((c*(b*x + c*x^2
))/b^2))^(3/4)*EllipticF[ArcSin[1 + (2*c*x)/b]/2, 2])/(3*Sqrt[2]*
c^2*(b*x + c*x^2)^(3/4))
```

Rubi in Sympy [A] time = 13.8965, size = 78, normalized size = 0.87

$$-\frac{\sqrt{2}b^3\left(\frac{c(-bx-cx^2)}{b^2}\right)^{\frac{3}{4}}F\left(\frac{\operatorname{asin}\left(1+\frac{2cx}{b}\right)}{2}\middle|2\right)}{6c^2\left(bx+cx^2\right)^{\frac{3}{4}}}+\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x)**(1/4),x)
```

```
[Out] -sqrt(2)*b**3*(c*(-b*x - c*x**2)/b**2)**(3/4)*elliptic_f(asin(1 +
 2*c*x/b)/2, 2)/(6*c**2*(b*x + c*x**2)**(3/4)) + (b + 2*c*x)*(b*x
 + c*x**2)**(1/4)/(3*c)
```

Mathematica [C] time = 0.0519025, size = 70, normalized size = 0.78

$$\frac{x\left(b^2\left(-\left(\frac{cx}{b}+1\right)^{3/4}\right)\ {}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{cx}{b}\right)+b^2+3bcx+2c^2x^2\right)}{3c(x(b+cx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(1/4),x]
```

```
[Out] (x*(b^2 + 3*b*c*x + 2*c^2*x^2 - b^2*(1 + (c*x)/b)^(3/4)*Hypergeom
etric2F1[1/4, 3/4, 5/4, -((c*x)/b)]))/(3*c*(x*(b + c*x))^(3/4))
```

Maple [F] time = 0.049, size = 0, normalized size = 0.

$$\int \sqrt[4]{cx^2+bx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x)^(1/4),x)
```

```
[Out] int((c*x^2+b*x)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{\frac{1}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx\right)^{\frac{1}{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(1/4),x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^(1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt[4]{bx + cx^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x)**(1/4),x)

[Out] Integral((b*x + c*x**2)**(1/4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{\frac{1}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(1/4),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(1/4), x)
```

3.43 $\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{2}b\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{c\sqrt[4]{bx+cx^2}}$$

```
[Out] (Sqrt[2]*b*(-((c*(b*x + c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[1 +
(2*c*x)/b]/2, 2])/(c*(b*x + c*x^2)^(1/4))
```

Rubi [A] time = 0.0507608, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{\sqrt{2}b\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{c\sqrt[4]{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)^(-1/4),x]
```

```
[Out] (Sqrt[2]*b*(-((c*(b*x + c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[1 +
(2*c*x)/b]/2, 2])/(c*(b*x + c*x^2)^(1/4))
```

Rubi in Sympy [A] time = 11.8577, size = 51, normalized size = 0.88

$$\frac{\sqrt{2}b\sqrt[4]{\frac{c(-bx-cx^2)}{b^2}}E\left(\frac{\operatorname{asin}\left(1+\frac{2cx}{b}\right)}{2}\middle|2\right)}{c\sqrt[4]{bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+b*x)**(1/4),x)
```

```
[Out] sqrt(2)*b*(c*(-b*x - c*x**2)/b**2)**(1/4)*elliptic_e(asin(1 + 2*c
*x/b)/2, 2)/(c*(b*x + c*x**2)**(1/4))
```

Mathematica [C] time = 0.0287287, size = 46, normalized size = 0.79

$$\frac{4x\sqrt[4]{\frac{b+cx}{b}}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{cx}{b}\right)}{3\sqrt[4]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(-1/4),x]
```

```
[Out] (4*x*((b + c*x)/b)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((c*x)
/b)])/(3*(x*(b + c*x))^(1/4))
```

Maple [F] time = 0.047, size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{cx^2+bx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+b*x)^(1/4),x)
```

```
[Out] int(1/(c*x^2+b*x)^(1/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2+bx)^{\frac{1}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-1/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-1/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cx^2+bx)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-1/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(-1/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt[4]{bx + cx^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+b*x)**(1/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(-1/4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx)^{\frac{1}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-1/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-1/4), x)
```

3.44 $\int \frac{1}{(bx+cx^2)^{3/4}}\,dx$

Optimal. Leaf size=59

$$\frac{2\sqrt{2}b\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{c\,(bx+cx^2)^{3/4}}$$

```
[Out] (2*Sqrt[2]*b*(-((c*(b*x + c*x^2))/b^2))^(3/4)*EllipticF[ArcSin[1
+ (2*c*x)/b]/2, 2])/(c*(b*x + c*x^2)^(3/4))
```

Rubi [A] time = 0.0503874, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{2\sqrt{2}b\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{c\,(bx+cx^2)^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)^(-3/4),x]
```

```
[Out] (2*Sqrt[2]*b*(-((c*(b*x + c*x^2))/b^2))^(3/4)*EllipticF[ArcSin[1
+ (2*c*x)/b]/2, 2])/(c*(b*x + c*x^2)^(3/4))
```

Rubi in Sympy [A] time = 11.9314, size = 53, normalized size = 0.9

$$\frac{2\sqrt{2}b\left(\frac{c(-bx-cx^2)}{b^2}\right)^{\frac{3}{4}}F\left(\frac{\operatorname{asin}\left(1+\frac{2cx}{b}\right)}{2}\middle|2\right)}{c\,(bx+cx^2)^{\frac{3}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+b*x)**(3/4),x)
```

```
[Out] 2*sqrt(2)*b*(c*(-b*x - c*x**2)/b**2)**(3/4)*elliptic_f(asin(1 + 2
*c*x/b)/2, 2)/(c*(b*x + c*x**2)**(3/4))
```

Mathematica [C] time = 0.0221383, size = 44, normalized size = 0.75

$$\frac{4x\left(\frac{b+cx}{b}\right)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{5}{4};-\frac{cx}{b}\right)}{(x(b+cx))^{3/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(-3/4),x]
```

```
[Out] (4*x*((b + c*x)/b)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((c*x)
/b)])/(x*(b + c*x))^(3/4)
```

Maple [F] time = 0.058, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{-\frac{3}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+b*x)^(3/4),x)
```

```
[Out] int(1/(c*x^2+b*x)^(3/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2+bx)^{\frac{3}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-3/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-3/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cx^2+bx)^{\frac{3}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-3/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(-3/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx + cx^2)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+b*x)**(3/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(-3/4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx)^{\frac{3}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-3/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-3/4), x)
```

3.45 $\int \frac{1}{(bx+cx^2)^{5/4}}\,dx$

Optimal. Leaf size=83

$$\frac{4\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{b\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}$$

```
[Out] (-4*(b + 2*c*x))/(b^2*(b*x + c*x^2)^(1/4)) + (4*Sqrt[2]*(-((c*(b*
x + c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]/2, 2])/(b
*(b*x + c*x^2)^(1/4))
```

Rubi [A] time = 0.0732425, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$\frac{4\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{b\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)^(-5/4),x]
```

```
[Out] (-4*(b + 2*c*x))/(b^2*(b*x + c*x^2)^(1/4)) + (4*Sqrt[2]*(-((c*(b*
x + c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]/2, 2])/(b
*(b*x + c*x^2)^(1/4))
```

Rubi in Sympy [A] time = 13.5915, size = 75, normalized size = 0.9

$$\frac{4\sqrt{2}\sqrt[4]{\frac{c(-bx-cx^2)}{b^2}}E\left(\frac{\operatorname{asin}\left(1+\frac{2cx}{b}\right)}{2}\middle|2\right)}{b\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+b*x)**(5/4),x)
```

```
[Out] 4*sqrt(2)*(c*(-b*x - c*x**2)/b**2)**(1/4)*elliptic_e(asin(1 + 2*c
*x/b)/2, 2)/(b*(b*x + c*x**2)**(1/4)) - 4*(b + 2*c*x)/(b**2*(b*x
```

```
+ c*x**2)**(1/4))
```

Mathematica [C] time = 0.053344, size = 59, normalized size = 0.71

$$-\frac{4\left(-4cx\sqrt[4]{\frac{cx}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{cx}{b}\right)+3b+6cx\right)}{3b^2\sqrt[4]{x(b+cx)}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(-5/4),x]
```

```
[Out] (-4*(3*b + 6*c*x - 4*c*x*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[1/
4, 3/4, 7/4, -((c*x)/b)]))/(3*b^2*(x*(b + c*x))^(1/4))
```

Maple [F] time = 0.09, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{-\frac{5}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+b*x)^(5/4),x)
```

```
[Out] int(1/(c*x^2+b*x)^(5/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(cx^2+bx\right)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-5/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-5/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(cx^2+bx)^{\frac{5}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-5/4),x, algorithm="fricas")
```

```
[Out] integral((c*x^2 + b*x)^(-5/4), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx+cx^2)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+b*x)**(5/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(-5/4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2+bx)^{\frac{5}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-5/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-5/4), x)
```

3.46 $\int \frac{1}{(bx+cx^2)^{9/4}}\,dx$

Optimal. Leaf size=115

$$\frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/4}}-\frac{48\sqrt{2}c\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{5b^3\sqrt[4]{bx+cx^2}}$$

```
[Out] (-4*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/4)) + (48*c*(b + 2*c*x))
/(5*b^4*(b*x + c*x^2)^(1/4)) - (48*Sqrt[2]*c*(-((c*(b*x + c*x^2))
/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]/2, 2])/(5*b^3*(b*x +
 c*x^2)^(1/4))
```

Rubi [A] time = 0.101361, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$\frac{48c(b+2cx)}{5b^4\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/4}}-\frac{48\sqrt{2}c\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{5b^3\sqrt[4]{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)^(-9/4),x]
```

```
[Out] (-4*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/4)) + (48*c*(b + 2*c*x))
/(5*b^4*(b*x + c*x^2)^(1/4)) - (48*Sqrt[2]*c*(-((c*(b*x + c*x^2))
/b^2))^(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]/2, 2])/(5*b^3*(b*x +
 c*x^2)^(1/4))
```

Rubi in Sympy [A] time = 16.2416, size = 109, normalized size = 0.95

$$-\frac{4\left(b+2cx\right)}{5b^2\left(bx+cx^2\right)^{\frac{5}{4}}}-\frac{48\sqrt{2}c\sqrt[4]{\frac{c\left(-bx-cx^2\right)}{b^2}}E\left(\frac{\operatorname{asin}\left(1+\frac{2cx}{b}\right)}{2}\middle|2\right)}{5b^3\sqrt[4]{bx+cx^2}}+\frac{48c\left(b+2cx\right)}{5b^4\sqrt[4]{bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+b*x)**(9/4),x)
```

```
[Out] -4*(b + 2*c*x)/(5*b**2*(b*x + c*x**2)**(5/4)) - 48*sqrt(2)*c*(c*(
-b*x - c*x**2)/b**2)**(1/4)*elliptic_e(asin(1 + 2*c*x/b)/2, 2)/(5
*b**3*(b*x + c*x**2)**(1/4)) + 48*c*(b + 2*c*x)/(5*b**4*(b*x + c*
x**2)**(1/4))
```

Mathematica [C] time = 0.105024, size = 90, normalized size = 0.78

$$\frac{-4b^3 + 40b^2cx - 64c^2x^2(b + cx)\sqrt[4]{\frac{cx}{b} + 1}\,{}_2F_1\left(\frac{1}{4}, \frac{3}{4}; \frac{7}{4}; -\frac{cx}{b}\right) + 144bc^2x^2 + 96c^3x^3}{5b^4(x(b + cx))^{5/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(-9/4),x]
```

```
[Out] (-4*b^3 + 40*b^2*c*x + 144*b*c^2*x^2 + 96*c^3*x^3 - 64*c^2*x^2*(b
 + c*x)*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((c
*x)/b)])/(5*b^4*(x*(b + c*x))^(5/4))
```

Maple [F] time = 0.153, size = 0, normalized size = 0.

$$\int \left(cx^2 + bx\right)^{-\frac{9}{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+b*x)^(9/4),x)
```

```
[Out] int(1/(c*x^2+b*x)^(9/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx)^{\frac{9}{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-9/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-9/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(c^2x^4 + 2\,bcx^3 + b^2x^2)(cx^2 + bx)^{\frac{1}{4}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-9/4),x, algorithm="fricas")

[Out] integral(1/((c^2*x^4 + 2*b*c*x^3 + b^2*x^2)*(c*x^2 + b*x)^(1/4)),
 x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(bx + cx^2)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+b*x)**(9/4),x)

[Out] Integral((b*x + c*x**2)**(-9/4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx)^{\frac{9}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-9/4),x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^(-9/4), x)
```

3.47 $\int \frac{1}{(bx+cx^2)^{13/4}}\,dx$

Optimal. Leaf size=146

$$-\frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}}+\frac{112c(b+2cx)}{45b^4(bx+cx^2)^{5/4}}-\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}}+\frac{448\sqrt{2}c^2\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{15b^5\sqrt[4]{bx+cx^2}}$$

```
[Out] (-4*(b + 2*c*x))/(9*b^2*(b*x + c*x^2)^(9/4)) + (112*c*(b + 2*c*x)
)/(45*b^4*(b*x + c*x^2)^(5/4)) - (448*c^2*(b + 2*c*x))/(15*b^6*(b
*x + c*x^2)^(1/4)) + (448*Sqrt[2]*c^2*(-((c*(b*x + c*x^2))/b^2))^
(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]/2, 2])/(15*b^5*(b*x + c*x^2
)^(1/4))
```

Rubi [A] time = 0.129558, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308

$$-\frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}}+\frac{112c(b+2cx)}{45b^4(bx+cx^2)^{5/4}}-\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}}+\frac{448\sqrt{2}c^2\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\sin^{-1}\left(\frac{2cx}{b}+1\right)\middle|2\right)}{15b^5\sqrt[4]{bx+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)^(-13/4),x]
```

```
[Out] (-4*(b + 2*c*x))/(9*b^2*(b*x + c*x^2)^(9/4)) + (112*c*(b + 2*c*x)
)/(45*b^4*(b*x + c*x^2)^(5/4)) - (448*c^2*(b + 2*c*x))/(15*b^6*(b
*x + c*x^2)^(1/4)) + (448*Sqrt[2]*c^2*(-((c*(b*x + c*x^2))/b^2))^
(1/4)*EllipticE[ArcSin[1 + (2*c*x)/b]/2, 2])/(15*b^5*(b*x + c*x^2
)^(1/4))
```

Rubi in Sympy [A] time = 19.6136, size = 139, normalized size = 0.95

$$-\frac{4(b+2cx)}{9b^2(bx+cx^2)^{\frac{9}{4}}}+\frac{112c(b+2cx)}{45b^4(bx+cx^2)^{\frac{5}{4}}}+\frac{448\sqrt{2}c^2\sqrt[4]{\frac{c(-bx-cx^2)}{b^2}}E\left(\frac{\operatorname{asin}\left(1+\frac{2cx}{b}\right)}{2}\middle|2\right)}{15b^5\sqrt[4]{bx+cx^2}}-\frac{448c^2(b+2cx)}{15b^6\sqrt[4]{bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+b*x)**(13/4),x)
```

```
[Out] -4*(b + 2*c*x)/(9*b**2*(b*x + c*x**2)**(9/4)) + 112*c*(b + 2*c*x)
/(45*b**4*(b*x + c*x**2)**(5/4)) + 448*sqrt(2)*c**2*(c*(-b*x - c*
x**2)/b**2)**(1/4)*elliptic_e(asin(1 + 2*c*x/b)/2, 2)/(15*b**5*(b
*x + c*x**2)**(1/4)) - 448*c**2*(b + 2*c*x)/(15*b**6*(b*x + c*x**
2)**(1/4))
```

Mathematica [C] time = 0.144171, size = 114, normalized size = 0.78

$$-\frac{4\left(5b^5-18b^4cx+252b^3c^2x^2+1288b^2c^3x^3+1680bc^4x^4-448c^3x^3(b+cx)^2\sqrt[4]{\frac{cx}{b}+1}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\frac{7}{4};-\frac{cx}{b}\right)+672c^5x^5\right)}{45b^6(x(b+cx))^{9/4}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^(-13/4),x]
```

```
[Out] (-4*(5*b^5 - 18*b^4*c*x + 252*b^3*c^2*x^2 + 1288*b^2*c^3*x^3 + 16
80*b*c^4*x^4 + 672*c^5*x^5 - 448*c^3*x^3*(b + c*x)^2*(1 + (c*x)/b
)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((c*x)/b)]))/(45*b^6*(x
*(b + c*x))^(9/4))
```

Maple [F] time = 0.1, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^{-\frac{13}{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+b*x)^(13/4),x)
```

```
[Out] int(1/(c*x^2+b*x)^(13/4),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2+bx)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-13/4),x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^(-13/4), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\left(c^3x^6+3\,bc^2x^5+3\,b^2cx^4+b^3x^3\right)\left(cx^2+bx\right)^{\frac{1}{4}}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-13/4),x, algorithm="fricas")
```

```
[Out] integral(1/((c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^3)*(c*x^
2 + b*x)^(1/4)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(bx+cx^2\right)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+b*x)**(13/4),x)
```

```
[Out] Integral((b*x + c*x**2)**(-13/4), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\left(cx^2+bx\right)^{\frac{13}{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^(-13/4),x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + b*x)^(-13/4), x)
```

3.48 $\int \left(bx+cx^2\right)^p \, dx$

Optimal. Leaf size=55

$$-\frac{\left(-\frac{cx}{b}\right)^{-p-1}\left(bx+cx^2\right)^{p+1} {}_2F_1\left(-p,p+1;p+2;\frac{b+cx}{b}\right)}{b(p+1)}$$

```
[Out] -(((-((c*x)/b))^(-1 - p)*(b*x + c*x^2)^(1 + p)*Hypergeometric2F1[
-p, 1 + p, 2 + p, (b + c*x)/b])/(b*(1 + p)))
```

Rubi [A] time = 0.0321692, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$-\frac{\left(-\frac{cx}{b}\right)^{-p-1}\left(bx+cx^2\right)^{p+1} {}_2F_1\left(-p,p+1;p+2;\frac{b+cx}{b}\right)}{b(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(b*x + c*x^2)^p,x]
```

```
[Out] -(((-((c*x)/b))^(-1 - p)*(b*x + c*x^2)^(1 + p)*Hypergeometric2F1[
-p, 1 + p, 2 + p, (b + c*x)/b])/(b*(1 + p)))
```

Rubi in Sympy [A] time = 2.75342, size = 42, normalized size = 0.76

$$-\frac{\left(-\frac{cx}{b}\right)^{-p-1}\left(bx+cx^2\right)^{p+1} {}_2F_1\left(\left.\begin{matrix}-p,p+1\\p+2\end{matrix}\right|\frac{b+cx}{b}\right)}{b\left(p+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x)**p,x)
```

```
[Out] -(-c*x/b)**(-p - 1)*(b*x + c*x**2)**(p + 1)*hyper((-p, p + 1), (p
 + 2,), (b + c*x)/b)/(b*(p + 1))
```

Mathematica [A] time = 0.0407748, size = 45, normalized size = 0.82

$$\frac{x(x(b+cx))^p \left(\frac{cx}{b}+1\right)^{-p} {}_2F_1\left(-p,p+1;p+2;-\frac{cx}{b}\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b*x + c*x^2)^p,x]
```

```
[Out] (x*(x*(b + c*x))^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x)/b)
])/((1 + p)*(1 + (c*x)/b)^p)
```

Maple [F] time = 0.044, size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x)^p,x)
```

```
[Out] int((c*x^2+b*x)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^p,x, algorithm="fricas")

[Out] integral((c*x^2 + b*x)^p, x)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \left(bx+cx^{2}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x)**p,x)

[Out] Integral((b*x + c*x**2)**p, x)
```

GIAC/XCAS [F]   time = 0., size = 0, normalized size = 0.

$$\int (cx^{2}+bx)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x)^p,x, algorithm="giac")

[Out] integrate((c*x^2 + b*x)^p, x)
```

3.49 $\int \left(a+cx^2\right)^4 \, dx$

Optimal. Leaf size=51

$$a^4x+\frac{4}{3}a^3cx^3+\frac{6}{5}a^2c^2x^5+\frac{4}{7}ac^3x^7+\frac{c^4x^9}{9}$$

```
[Out] a^4*x + (4*a^3*c*x^3)/3 + (6*a^2*c^2*x^5)/5 + (4*a*c^3*x^7)/7 + (
c^4*x^9)/9
```

Rubi [A] time = 0.0438834, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$a^4x+\frac{4}{3}a^3cx^3+\frac{6}{5}a^2c^2x^5+\frac{4}{7}ac^3x^7+\frac{c^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)^4,x]
```

```
[Out] a^4*x + (4*a^3*c*x^3)/3 + (6*a^2*c^2*x^5)/5 + (4*a*c^3*x^7)/7 + (
c^4*x^9)/9
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{4a^3cx^3}{3}+\frac{6a^2c^2x^5}{5}+\frac{4ac^3x^7}{7}+\frac{c^4x^9}{9}+\int a^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)**4,x)
```

```
[Out] 4*a**3*c*x**3/3 + 6*a**2*c**2*x**5/5 + 4*a*c**3*x**7/7 + c**4*x**
9/9 + Integral(a**4, x)
```

Mathematica [A] time = 0.00265362, size = 51, normalized size = 1.

$$a^4x+\frac{4}{3}a^3cx^3+\frac{6}{5}a^2c^2x^5+\frac{4}{7}ac^3x^7+\frac{c^4x^9}{9}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)^4,x]
```

```
[Out] a^4*x + (4*a^3*c*x^3)/3 + (6*a^2*c^2*x^5)/5 + (4*a*c^3*x^7)/7 + (
c^4*x^9)/9
```

Maple [A] time = 0.001, size = 44, normalized size = 0.9

$$a^{4}x+\frac{4\,a^{3}cx^{3}}{3}+\frac{6\,a^{2}c^{2}x^{5}}{5}+\frac{4\,ac^{3}x^{7}}{7}+\frac{c^{4}x^{9}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)^4,x)
```

```
[Out] a^4*x+4/3*a^3*c*x^3+6/5*a^2*c^2*x^5+4/7*a*c^3*x^7+1/9*c^4*x^9
```

Maxima [A] time = 0.701094, size = 58, normalized size = 1.14

$$\frac{1}{9}\,c^{4}x^{9}+\frac{4}{7}\,ac^{3}x^{7}+\frac{6}{5}\,a^{2}c^{2}x^{5}+\frac{4}{3}\,a^{3}cx^{3}+a^{4}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^4,x, algorithm="maxima")
```

```
[Out] 1/9*c^4*x^9 + 4/7*a*c^3*x^7 + 6/5*a^2*c^2*x^5 + 4/3*a^3*c*x^3 + a
^4*x
```

Fricas [A] time = 0.185464, size = 1, normalized size = 0.02

$$\frac{1}{9}x^{9}c^{4}+\frac{4}{7}x^{7}c^{3}a+\frac{6}{5}x^{5}c^{2}a^{2}+\frac{4}{3}x^{3}ca^{3}+xa^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^4,x, algorithm="fricas")
```

```
[Out] 1/9*x^9*c^4 + 4/7*x^7*c^3*a + 6/5*x^5*c^2*a^2 + 4/3*x^3*c*a^3 + x
*a^4
```

Sympy [A] time = 0.106809, size = 49, normalized size = 0.96

$$a^4x + \frac{4a^3cx^3}{3} + \frac{6a^2c^2x^5}{5} + \frac{4ac^3x^7}{7} + \frac{c^4x^9}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)**4,x)
```

```
[Out] a**4*x + 4*a**3*c*x**3/3 + 6*a**2*c**2*x**5/5 + 4*a*c**3*x**7/7 +
 c**4*x**9/9
```

GIAC/XCAS [A] time = 0.208722, size = 58, normalized size = 1.14

$$\frac{1}{9}\,c^4x^9 + \frac{4}{7}\,ac^3x^7 + \frac{6}{5}\,a^2c^2x^5 + \frac{4}{3}\,a^3cx^3 + a^4x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^4,x, algorithm="giac")
```

```
[Out] 1/9*c^4*x^9 + 4/7*a*c^3*x^7 + 6/5*a^2*c^2*x^5 + 4/3*a^3*c*x^3 + a
^4*x
```

3.50 $\int \left(a+cx^2\right)^3 \, dx$

Optimal. Leaf size=35

$$a^3x+a^2cx^3+\frac{3}{5}ac^2x^5+\frac{c^3x^7}{7}$$

```
[Out] a^3*x + a^2*c*x^3 + (3*a*c^2*x^5)/5 + (c^3*x^7)/7
```

Rubi [A] time = 0.0290113, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$a^3x+a^2cx^3+\frac{3}{5}ac^2x^5+\frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)^3,x]

[Out] a^3*x + a^2*c*x^3 + (3*a*c^2*x^5)/5 + (c^3*x^7)/7
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2cx^3+\frac{3ac^2x^5}{5}+\frac{c^3x^7}{7}+\int a^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)**3,x)

[Out] a**2*c*x**3 + 3*a*c**2*x**5/5 + c**3*x**7/7 + Integral(a**3, x)
```

Mathematica [A] time = 0.00196918, size = 35, normalized size = 1.

$$a^3x+a^2cx^3+\frac{3}{5}ac^2x^5+\frac{c^3x^7}{7}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)^3,x]
```

```
[Out] a^3*x + a^2*c*x^3 + (3*a*c^2*x^5)/5 + (c^3*x^7)/7
```

Maple [A] time = 0.002, size = 32, normalized size = 0.9

$$a^3x + a^2cx^3 + \frac{3\,ac^2x^5}{5} + \frac{c^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)^3,x)
```

```
[Out] a^3*x+a^2*c*x^3+3/5*a*c^2*x^5+1/7*c^3*x^7
```

Maxima [A] time = 0.694496, size = 42, normalized size = 1.2

$$\frac{1}{7}\,c^3x^7 + \frac{3}{5}\,ac^2x^5 + a^2cx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^3,x, algorithm="maxima")
```

```
[Out] 1/7*c^3*x^7 + 3/5*a*c^2*x^5 + a^2*c*x^3 + a^3*x
```

Fricas [A] time = 0.187323, size = 1, normalized size = 0.03

$$\frac{1}{7}x^7c^3 + \frac{3}{5}x^5c^2a + x^3ca^2 + xa^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^3,x, algorithm="fricas")
```

```
[Out] 1/7*x^7*c^3 + 3/5*x^5*c^2*a + x^3*c*a^2 + x*a^3
```

Sympy [A] time = 0.098392, size = 32, normalized size = 0.91

$$a^3x + a^2cx^3 + \frac{3ac^2x^5}{5} + \frac{c^3x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)**3,x)
```

```
[Out] a**3*x + a**2*c*x**3 + 3*a*c**2*x**5/5 + c**3*x**7/7
```

GIAC/XCAS [A] time = 0.207707, size = 42, normalized size = 1.2

$$\frac{1}{7}c^3x^7 + \frac{3}{5}ac^2x^5 + a^2cx^3 + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^3,x, algorithm="giac")
```

```
[Out] 1/7*c^3*x^7 + 3/5*a*c^2*x^5 + a^2*c*x^3 + a^3*x
```

3.51 $\int \left(a + cx^2\right)^2 \, dx$

Optimal. Leaf size=25

$$a^2x + \frac{2}{3}acx^3 + \frac{c^2x^5}{5}$$

```
[Out] a^2*x + (2*a*c*x^3)/3 + (c^2*x^5)/5
```

Rubi [A] time = 0.0193471, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$a^2x + \frac{2}{3}acx^3 + \frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)^2,x]

[Out] a^2*x + (2*a*c*x^3)/3 + (c^2*x^5)/5
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{2acx^3}{3} + \frac{c^2x^5}{5} + \int a^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)**2,x)

[Out] 2*a*c*x**3/3 + c**2*x**5/5 + Integral(a**2, x)
```

Mathematica [A] time = 0.00170871, size = 25, normalized size = 1.

$$a^2x + \frac{2}{3}acx^3 + \frac{c^2x^5}{5}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)^2,x]
```

```
[Out] a^2*x + (2*a*c*x^3)/3 + (c^2*x^5)/5
```

Maple [A] time = 0.002, size = 22, normalized size = 0.9

$$a^2x + \frac{2\,ax^3c}{3} + \frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)^2,x)
```

```
[Out] a^2*x+2/3*a*x^3*c+1/5*c^2*x^5
```

Maxima [A] time = 0.703682, size = 28, normalized size = 1.12

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,acx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^2,x, algorithm="maxima")
```

```
[Out] 1/5*c^2*x^5 + 2/3*a*c*x^3 + a^2*x
```

Fricas [A] time = 0.191099, size = 1, normalized size = 0.04

$$\frac{1}{5}x^5c^2 + \frac{2}{3}x^3ca + xa^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^2,x, algorithm="fricas")
```

```
[Out] 1/5*x^5*c^2 + 2/3*x^3*c*a + x*a^2
```

Sympy [A] time = 0.087772, size = 22, normalized size = 0.88

$$a^2x + \frac{2acx^3}{3} + \frac{c^2x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)**2,x)

[Out] a**2*x + 2*a*c*x**3/3 + c**2*x**5/5
```

GIAC/XCAS [A] time = 0.209627, size = 28, normalized size = 1.12

$$\frac{1}{5}\,c^2x^5 + \frac{2}{3}\,acx^3 + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^2,x, algorithm="giac")

[Out] 1/5*c^2*x^5 + 2/3*a*c*x^3 + a^2*x
```

3.52 $\int (a + cx^2) \, dx$

Optimal. Leaf size=12

$$ax + \frac{cx^3}{3}$$

```
[Out] a*x + (c*x^3)/3
```

Rubi [A] time = 0.00791862, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 0, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.$

$$ax + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Int[a + c*x^2,x]

[Out] a*x + (c*x^3)/3
```

Rubi in Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{cx^3}{3} + \int a \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(c*x**2+a,x)

[Out] c*x**3/3 + Integral(a, x)
```

Mathematica [A] time = 0.000075836, size = 12, normalized size = 1.

$$ax + \frac{cx^3}{3}$$

Antiderivative was successfully verified.

```
[In]  Integrate[a + c*x^2,x]
```

```
[Out] a*x + (c*x^3)/3
```

Maple [A] time = 0.001, size = 11, normalized size = 0.9

$$ax + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(c*x^2+a,x)
```

```
[Out] a*x+1/3*c*x^3
```

Maxima [A] time = 0.697821, size = 14, normalized size = 1.17

$$\frac{1}{3}\,cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x^2 + a,x, algorithm="maxima")
```

```
[Out] 1/3*c*x^3 + a*x
```

Fricas [A] time = 0.189583, size = 1, normalized size = 0.08

$$\frac{1}{3}x^3c + xa$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x^2 + a,x, algorithm="fricas")
```

```
[Out] 1/3*x^3*c + x*a
```

Sympy [A] time = 0.06707, size = 8, normalized size = 0.67

$$ax + \frac{cx^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x**2+a,x)
```

```
[Out] a*x + c*x**3/3
```

GIAC/XCAS [A] time = 0.208659, size = 14, normalized size = 1.17

$$\frac{1}{3}\,cx^3 + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(c*x^2 + a,x, algorithm="giac")
```

```
[Out] 1/3*c*x^3 + a*x
```

3.53 $\int \frac{1}{a+cx^2}\,dx$

Optimal. Leaf size=24

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[c])
```

Rubi [A] time = 0.0187577, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)^(-1),x]

[Out] ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[c])
```

Rubi in Sympy [A] time = 2.44701, size = 22, normalized size = 0.92

$$\frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+a),x)

[Out] atan(sqrt(c)*x/sqrt(a))/(sqrt(a)*sqrt(c))
```

Mathematica [A] time = 0.00783446, size = 24, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(Sqrt[a]*Sqrt[c])
```

Maple [A] time = 0.005, size = 16, normalized size = 0.7

$$1 \arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+a),x)
```

```
[Out] 1/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.221177, size = 1, normalized size = 0.04

$$\left[\frac{\log\left(\frac{2\,acx+(cx^2-a)\sqrt{-ac}}{cx^2+a}\right)}{2\sqrt{-ac}}, \frac{\arctan\left(\frac{\sqrt{ac}x}{a}\right)}{\sqrt{ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/2*log((2*a*c*x + (c*x^2 - a)*sqrt(-a*c))/(c*x^2 + a))/sqrt(-a*
c), arctan(sqrt(a*c)*x/a)/sqrt(a*c)]
```

Sympy [A] time = 0.289894, size = 53, normalized size = 2.21

$$-\frac{\sqrt{-\frac{1}{ac}}\log\left(-a\sqrt{-\frac{1}{ac}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{ac}}\log\left(a\sqrt{-\frac{1}{ac}}+x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+a),x)
```

```
[Out] -sqrt(-1/(a*c))*log(-a*sqrt(-1/(a*c)) + x)/2 + sqrt(-1/(a*c))*log
(a*sqrt(-1/(a*c)) + x)/2
```

GIAC/XCAS [A] time = 0.208717, size = 20, normalized size = 0.83

$$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x^2 + a),x, algorithm="giac")
```

```
[Out] arctan(c*x/sqrt(a*c))/sqrt(a*c)
```

3.54 $\int \frac{1}{(a+cx^2)^2}\,dx$

Optimal. Leaf size=45

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a\left(a+cx^2\right)}$$

```
[Out] x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt
[c])
```

Rubi [A] time = 0.028627, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)^(-2),x]
```

```
[Out] x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt
[c])
```

Rubi in Sympy [A] time = 3.77339, size = 36, normalized size = 0.8

$$\frac{x}{2a\left(a+cx^2\right)}+\frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{\frac{3}{2}}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+a)**2,x)
```

```
[Out] x/(2*a*(a + c*x**2)) + atan(sqrt(c)*x/sqrt(a))/(2*a**(3/2)*sqrt(c
))
```

Mathematica [A] time = 0.045163, size = 45, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{c}}+\frac{x}{2a\left(a+cx^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)^(-2),x]
```

```
[Out] x/(2*a*(a + c*x^2)) + ArcTan[(Sqrt[c]*x)/Sqrt[a]]/(2*a^(3/2)*Sqrt
[c])
```

Maple [A] time = 0.002, size = 36, normalized size = 0.8

$$\frac{x}{2\,a\left(cx^2+a\right)}+\frac{1}{2\,a}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+a)^2,x)
```

```
[Out] 1/2*x/a/(c*x^2+a)+1/2/a/(a*c)^(1/2)*arctan(c*x/(a*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.229139, size = 1, normalized size = 0.02

$$\left[\frac{\left(cx^2+a\right)\log\left(\frac{2\,acx+\left(cx^2-a\right)\sqrt{-ac}}{cx^2+a}\right)+2\,\sqrt{-ac}x}{4\left(acx^2+a^2\right)\sqrt{-ac}},\frac{\left(cx^2+a\right)\arctan\left(\frac{\sqrt{ac}x}{a}\right)+\sqrt{ac}x}{2\left(acx^2+a^2\right)\sqrt{ac}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-2),x, algorithm="fricas")
```

```
[Out] [1/4*((c*x^2 + a)*log((2*a*c*x + (c*x^2 - a)*sqrt(-a*c))/(c*x^2 +
 a)) + 2*sqrt(-a*c)*x)/((a*c*x^2 + a^2)*sqrt(-a*c)), 1/2*((c*x^2
+ a)*arctan(sqrt(a*c)*x/a) + sqrt(a*c)*x)/((a*c*x^2 + a^2)*sqrt(a
*c))]
```

Sympy [A] time = 1.40999, size = 78, normalized size = 1.73

$$\frac{x}{2a^2 + 2acx^2} - \frac{\sqrt{-\frac{1}{a^3c}}\log\left(-a^2\sqrt{-\frac{1}{a^3c}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{a^3c}}\log\left(a^2\sqrt{-\frac{1}{a^3c}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+a)**2,x)
```

```
[Out] x/(2*a**2 + 2*a*c*x**2) - sqrt(-1/(a**3*c))*log(-a**2*sqrt(-1/(a*
*3*c)) + x)/4 + sqrt(-1/(a**3*c))*log(a**2*sqrt(-1/(a**3*c)) + x)
/4
```

GIAC/XCAS [A] time = 0.210347, size = 47, normalized size = 1.04

$$\frac{\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{2\sqrt{ac}a} + \frac{x}{2(cx^2 + a)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a) + 1/2*x/((c*x^2 + a)*a)
```

3.55 $\int \frac{1}{(a+cx^2)^3} \, dx$

Optimal. Leaf size=62

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}+\frac{3x}{8a^2\left(a+cx^2\right)}+\frac{x}{4a\left(a+cx^2\right)^2}$$

```
[Out] x/(4*a*(a + c*x^2)^2) + (3*x)/(8*a^2*(a + c*x^2)) + (3*ArcTan[(Sq
rt[c]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[c])
```

Rubi [A] time = 0.0411095, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}+\frac{3x}{8a^2\left(a+cx^2\right)}+\frac{x}{4a\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)^(-3),x]
```

```
[Out] x/(4*a*(a + c*x^2)^2) + (3*x)/(8*a^2*(a + c*x^2)) + (3*ArcTan[(Sq
rt[c]*x)/Sqrt[a]])/(8*a^(5/2)*Sqrt[c])
```

Rubi in Sympy [A] time = 5.70768, size = 54, normalized size = 0.87

$$\frac{x}{4a\left(a+cx^2\right)^2}+\frac{3x}{8a^2\left(a+cx^2\right)}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{\frac{5}{2}}\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+a)**3,x)
```

```
[Out] x/(4*a*(a + c*x**2)**2) + 3*x/(8*a**2*(a + c*x**2)) + 3*atan(sqrt
(c)*x/sqrt(a))/(8*a**(5/2)*sqrt(c))
```

Mathematica [A] time = 0.0875115, size = 55, normalized size = 0.89

$$\frac{3\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{c}}+\frac{5ax+3cx^3}{8a^2\left(a+cx^2\right)^2}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)^(-3),x]
```

```
[Out] (5*a*x + 3*c*x^3)/(8*a^2*(a + c*x^2)^2) + (3*ArcTan[(Sqrt[c]*x)/S
qrt[a]])/(8*a^(5/2)*Sqrt[c])
```

Maple [A] time = 0.002, size = 51, normalized size = 0.8

$$\frac{x}{4\,a\left(cx^2+a\right)^2}+\frac{3\,x}{8\,a^2\left(cx^2+a\right)}+\frac{3}{8\,a^2}\arctan\left(cx\frac{1}{\sqrt{ac}}\right)\frac{1}{\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+a)^3,x)
```

```
[Out] 1/4*x/a/(c*x^2+a)^2+3/8*x/a^2/(c*x^2+a)+3/8/a^2/(a*c)^(1/2)*arcta
n(c*x/(a*c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.230261, size = 1, normalized size = 0.02

$$\left[\frac{3\left(c^2x^4+2\,acx^2+a^2\right)\log\left(\frac{2\,acx+\left(cx^2-a\right)\sqrt{-ac}}{cx^2+a}\right)+2\left(3\,cx^3+5\,ax\right)\sqrt{-ac}}{16\left(a^2c^2x^4+2\,a^3cx^2+a^4\right)\sqrt{-ac}},\frac{3\left(c^2x^4+2\,acx^2+a^2\right)\arctan\left(\frac{\sqrt{ac}x}{a}\right)+\left(3\,cx^3+5\,a\right.}{8\left(a^2c^2x^4+2\,a^3cx^2+a^4\right)\sqrt{ac}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-3),x, algorithm="fricas")
```

```
[Out] [1/16*(3*(c^2*x^4 + 2*a*c*x^2 + a^2)*log((2*a*c*x + (c*x^2 - a)*s
qrt(-a*c))/(c*x^2 + a)) + 2*(3*c*x^3 + 5*a*x)*sqrt(-a*c))/((a^2*c
^2*x^4 + 2*a^3*c*x^2 + a^4)*sqrt(-a*c)), 1/8*(3*(c^2*x^4 + 2*a*c*
x^2 + a^2)*arctan(sqrt(a*c)*x/a) + (3*c*x^3 + 5*a*x)*sqrt(a*c))/(
(a^2*c^2*x^4 + 2*a^3*c*x^2 + a^4)*sqrt(a*c))]
```

Sympy [A] time = 1.83422, size = 105, normalized size = 1.69

$$-\frac{3\sqrt{-\frac{1}{a^5c}}\log\left(-a^3\sqrt{-\frac{1}{a^5c}}+x\right)}{16}+\frac{3\sqrt{-\frac{1}{a^5c}}\log\left(a^3\sqrt{-\frac{1}{a^5c}}+x\right)}{16}+\frac{5ax+3cx^3}{8a^4+16a^3cx^2+8a^2c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+a)**3,x)
```

```
[Out] -3*sqrt(-1/(a**5*c))*log(-a**3*sqrt(-1/(a**5*c)) + x)/16 + 3*sqrt
(-1/(a**5*c))*log(a**3*sqrt(-1/(a**5*c)) + x)/16 + (5*a*x + 3*c*x
**3)/(8*a**4 + 16*a**3*c*x**2 + 8*a**2*c**2*x**4)
```

GIAC/XCAS [A] time = 0.209193, size = 61, normalized size = 0.98

$$\frac{3\,\arctan\left(\frac{cx}{\sqrt{ac}}\right)}{8\,\sqrt{ac}a^2}+\frac{3\,cx^3+5\,ax}{8\,(cx^2+a)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-3),x, algorithm="giac")
```

```
[Out] 3/8*arctan(c*x/sqrt(a*c))/(sqrt(a*c)*a^2) + 1/8*(3*c*x^3 + 5*a*x)
/((c*x^2 + a)^2*a^2)
```

3.56 $\int \left(a+cx^2\right)^{5/2} \, dx$

Optimal. Leaf size=84

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}}+\frac{5}{16}a^2x\sqrt{a+cx^2}+\frac{5}{24}ax\left(a+cx^2\right)^{3/2}+\frac{1}{6}x\left(a+cx^2\right)^{5/2}$$

```
[Out] (5*a^2*x*Sqrt[a + c*x^2])/16 + (5*a*x*(a + c*x^2)^(3/2))/24 + (x*
(a + c*x^2)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]
])/(16*Sqrt[c])
```

Rubi [A] time = 0.0536519, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{5a^3\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}}+\frac{5}{16}a^2x\sqrt{a+cx^2}+\frac{5}{24}ax\left(a+cx^2\right)^{3/2}+\frac{1}{6}x\left(a+cx^2\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)^(5/2),x]
```

```
[Out] (5*a^2*x*Sqrt[a + c*x^2])/16 + (5*a*x*(a + c*x^2)^(3/2))/24 + (x*
(a + c*x^2)^(5/2))/6 + (5*a^3*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]
])/(16*Sqrt[c])
```

Rubi in Sympy [A] time = 5.83583, size = 78, normalized size = 0.93

$$\frac{5a^3\operatorname{atanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{16\sqrt{c}}+\frac{5a^2x\sqrt{a+cx^2}}{16}+\frac{5ax\left(a+cx^2\right)^{\frac{3}{2}}}{24}+\frac{x\left(a+cx^2\right)^{\frac{5}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)**(5/2),x)
```

```
[Out] 5*a**3*atanh(sqrt(c)*x/sqrt(a + c*x**2))/(16*sqrt(c)) + 5*a**2*x*
sqrt(a + c*x**2)/16 + 5*a*x*(a + c*x**2)**(3/2)/24 + x*(a + c*x**
2)**(5/2)/6
```

Mathematica [A] time = 0.0811618, size = 71, normalized size = 0.85

$$\frac{1}{48}\left(\frac{15a^3\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{\sqrt{c}}+x\sqrt{a+cx^2}\left(33a^2+26acx^2+8c^2x^4\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)^(5/2),x]
```

```
[Out] (x*Sqrt[a + c*x^2]*(33*a^2 + 26*a*c*x^2 + 8*c^2*x^4) + (15*a^3*Lo
g[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/Sqrt[c])/48
```

Maple [A] time = 0.006, size = 66, normalized size = 0.8

$$\frac{x}{6}\left(cx^2+a\right)^{\frac{5}{2}}+\frac{5\,ax}{24}\left(cx^2+a\right)^{\frac{3}{2}}+\frac{5\,a^2x}{16}\sqrt{cx^2+a}+\frac{5\,a^3}{16}\ln\left(\sqrt{c}x+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)^(5/2),x)
```

```
[Out] 1/6*x*(c*x^2+a)^(5/2)+5/24*a*x*(c*x^2+a)^(3/2)+5/16*a^2*x*(c*x^2+
a)^(1/2)+5/16*a^3/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.248609, size = 1, normalized size = 0.01

$$\left[\frac{15\,a^3\log\left(-2\,\sqrt{cx^2+a}cx-\left(2\,cx^2+a\right)\sqrt{c}\right)+2\left(8\,c^2x^5+26\,acx^3+33\,a^2x\right)\sqrt{cx^2+a}\sqrt{c}}{96\,\sqrt{c}},\frac{15\,a^3\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)+\left(8\,c^2x^5+\right.}{48\,\sqrt{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/96*(15*a^3*log(-2*sqrt(c*x^2 + a)*c*x - (2*c*x^2 + a)*sqrt(c))
 + 2*(8*c^2*x^5 + 26*a*c*x^3 + 33*a^2*x)*sqrt(c*x^2 + a)*sqrt(c))
/sqrt(c), 1/48*(15*a^3*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (8*c^
2*x^5 + 26*a*c*x^3 + 33*a^2*x)*sqrt(c*x^2 + a)*sqrt(-c))/sqrt(-c)
]
```

Sympy [A] time = 14.3432, size = 97, normalized size = 1.15

$$\frac{11a^{\frac{5}{2}}x\sqrt{1+\frac{cx^2}{a}}}{16}+\frac{13a^{\frac{3}{2}}cx^3\sqrt{1+\frac{cx^2}{a}}}{24}+\frac{\sqrt{a}c^2x^5\sqrt{1+\frac{cx^2}{a}}}{6}+\frac{5a^3\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{16\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)**(5/2),x)
```

```
[Out] 11*a**(5/2)*x*sqrt(1 + c*x**2/a)/16 + 13*a**(3/2)*c*x**3*sqrt(1 +
 c*x**2/a)/24 + sqrt(a)*c**2*x**5*sqrt(1 + c*x**2/a)/6 + 5*a**3*a
sinh(sqrt(c)*x/sqrt(a))/(16*sqrt(c))
```

GIAC/XCAS [A] time = 0.215024, size = 85, normalized size = 1.01

$$-\frac{5\,a^3\ln\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{16\,\sqrt{c}}+\frac{1}{48}\left(2\left(4\,c^2x^2+13\,ac\right)x^2+33\,a^2\right)\sqrt{cx^2+a}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(5/2),x, algorithm="giac")
```

```
[Out] -5/16*a^3*ln(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c) + 1/48*(2
*(4*c^2*x^2 + 13*a*c)*x^2 + 33*a^2)*sqrt(c*x^2 + a)*x
```

3.57 $\int \left(a+cx^2\right)^{3/2} dx$

Optimal. Leaf size=65

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}}+\frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}$$

```
[Out] (3*a*x*Sqrt[a + c*x^2])/8 + (x*(a + c*x^2)^(3/2))/4 + (3*a^2*ArcT
anh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqrt[c])
```

Rubi [A] time = 0.0374962, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}}+\frac{3}{8}ax\sqrt{a+cx^2}+\frac{1}{4}x\left(a+cx^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)^(3/2),x]
```

```
[Out] (3*a*x*Sqrt[a + c*x^2])/8 + (x*(a + c*x^2)^(3/2))/4 + (3*a^2*ArcT
anh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(8*Sqrt[c])
```

Rubi in Sympy [A] time = 4.41791, size = 60, normalized size = 0.92

$$\frac{3a^2\operatorname{atanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}}+\frac{3ax\sqrt{a+cx^2}}{8}+\frac{x\left(a+cx^2\right)^{\frac{3}{2}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)**(3/2),x)
```

```
[Out] 3*a**2*atanh(sqrt(c)*x/sqrt(a + c*x**2))/(8*sqrt(c)) + 3*a*x*sqrt
(a + c*x**2)/8 + x*(a + c*x**2)**(3/2)/4
```

Mathematica [A] time = 0.0582948, size = 62, normalized size = 0.95

$$\frac{3a^2\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{8\sqrt{c}}+\sqrt{a+cx^2}\left(\frac{5ax}{8}+\frac{cx^3}{4}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)^(3/2),x]
```

```
[Out] Sqrt[a + c*x^2]*((5*a*x)/8 + (c*x^3)/4) + (3*a^2*Log[c*x + Sqrt[c
]*Sqrt[a + c*x^2]])/(8*Sqrt[c])
```

Maple [A] time = 0.004, size = 51, normalized size = 0.8

$$\frac{x}{4}\left(cx^2+a\right)^{\frac{3}{2}}+\frac{3\,ax}{8}\sqrt{cx^2+a}+\frac{3\,a^2}{8}\ln\left(\sqrt{c}x+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)^(3/2),x)
```

```
[Out] 1/4*x*(c*x^2+a)^(3/2)+3/8*a*x*(c*x^2+a)^(1/2)+3/8*a^2*ln(c^(1/2)*
x+(c*x^2+a)^(1/2))/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.240402, size = 1, normalized size = 0.02

$$\left[\frac{3\,a^2\log\left(-2\,\sqrt{cx^2+a}cx-\left(2\,cx^2+a\right)\sqrt{c}\right)+2\,\left(2\,cx^3+5\,ax\right)\sqrt{cx^2+a}\sqrt{c}}{16\,\sqrt{c}},\frac{3\,a^2\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)+\left(2\,cx^3+5\,ax\right)\sqrt{cx^2+a}\sqrt{}}{8\,\sqrt{-c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*a^2*log(-2*sqrt(c*x^2 + a)*c*x - (2*c*x^2 + a)*sqrt(c))
+ 2*(2*c*x^3 + 5*a*x)*sqrt(c*x^2 + a)*sqrt(c))/sqrt(c), 1/8*(3*a^
2*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (2*c*x^3 + 5*a*x)*sqrt(c*x
^2 + a)*sqrt(-c))/sqrt(-c)]
```

Sympy [A] time = 9.94338, size = 70, normalized size = 1.08

$$\frac{5a^{\frac{3}{2}}x\sqrt{1+\frac{cx^2}{a}}}{8}+\frac{\sqrt{a}cx^3\sqrt{1+\frac{cx^2}{a}}}{4}+\frac{3a^2\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{8\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)**(3/2),x)
```

```
[Out] 5*a**(3/2)*x*sqrt(1 + c*x**2/a)/8 + sqrt(a)*c*x**3*sqrt(1 + c*x**
2/a)/4 + 3*a**2*asinh(sqrt(c)*x/sqrt(a))/(8*sqrt(c))
```

GIAC/XCAS [A] time = 0.215111, size = 66, normalized size = 1.02

$$\frac{1}{8}\left(2\,cx^2+5\,a\right)\sqrt{cx^2+a}x-\frac{3\,a^2\ln\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{8\,\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*c*x^2 + 5*a)*sqrt(c*x^2 + a)*x - 3/8*a^2*ln(abs(-sqrt(c)*x
 + sqrt(c*x^2 + a)))/sqrt(c)
```

3.58 $\int \sqrt{a+cx^2}\,dx$

Optimal. Leaf size=46

$$\frac{1}{2}x\sqrt{a+cx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}$$

```
[Out] (x*Sqrt[a + c*x^2])/2 + (a*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/
(2*Sqrt[c])
```

Rubi [A] time = 0.0242806, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{1}{2}x\sqrt{a+cx^2}+\frac{a\tanh^{-1}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[a + c*x^2],x]
```

```
[Out] (x*Sqrt[a + c*x^2])/2 + (a*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/
(2*Sqrt[c])
```

Rubi in Sympy [A] time = 3.17443, size = 39, normalized size = 0.85

$$\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{c}\,x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}}+\frac{x\sqrt{a+cx^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+a)**(1/2),x)
```

```
[Out] a*atanh(sqrt(c)*x/sqrt(a + c*x**2))/(2*sqrt(c)) + x*sqrt(a + c*x*
*2)/2
```

Mathematica [A] time = 0.023936, size = 49, normalized size = 1.07

$$\frac{1}{2}x\sqrt{a+cx^2}+\frac{a\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[a + c*x^2],x]
```

```
[Out] (x*Sqrt[a + c*x^2])/2 + (a*Log[c*x + Sqrt[c]*Sqrt[a + c*x^2]])/(2
*Sqrt[c])
```

Maple [A] time = 0.003, size = 36, normalized size = 0.8

$$\frac{x}{2}\sqrt{cx^2+a}+\frac{a}{2}\ln\left(\sqrt{c}x+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+a)^(1/2),x)
```

```
[Out] 1/2*x*(c*x^2+a)^(1/2)+1/2*a/c^(1/2)*ln(c^(1/2)*x+(c*x^2+a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.236027, size = 1, normalized size = 0.02

$$\left[\frac{2\sqrt{cx^2+a}\sqrt{c}x+a\log\left(-2\sqrt{cx^2+a}cx-\left(2cx^2+a\right)\sqrt{c}\right)}{4\sqrt{c}},\frac{\sqrt{cx^2+a}\sqrt{-c}x+a\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)}{2\sqrt{-c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*sqrt(c*x^2 + a)*sqrt(c)*x + a*log(-2*sqrt(c*x^2 + a)*c*x
- (2*c*x^2 + a)*sqrt(c)))/sqrt(c), 1/2*(sqrt(c*x^2 + a)*sqrt(-c)*
x + a*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)))/sqrt(-c)]
```

Sympy [A] time = 6.22564, size = 41, normalized size = 0.89

$$\frac{\sqrt{a}x\sqrt{1+\frac{cx^2}{a}}}{2}+\frac{a\,\mathrm{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+a)**(1/2),x)
```

```
[Out] sqrt(a)*x*sqrt(1 + c*x**2/a)/2 + a*asinh(sqrt(c)*x/sqrt(a))/(2*sq
rt(c))
```

GIAC/XCAS [A] time = 0.210636, size = 50, normalized size = 1.09

$$\frac{1}{2}\sqrt{cx^2+a}x-\frac{a\ln\left(\left|-\sqrt{c}x+\sqrt{cx^2+a}\right|\right)}{2\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(c*x^2 + a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^2 + a)*x - 1/2*a*ln(abs(-sqrt(c)*x + sqrt(c*x^2 + a)
))/sqrt(c)
```

3.59 $\int \frac{1}{\sqrt{a+cx^2}}\,dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/Sqrt[c]
```

Rubi [A] time = 0.014828, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[a + c*x^2],x]

[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/Sqrt[c]
```

Rubi in Sympy [A] time = 2.43196, size = 22, normalized size = 0.88

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+a)**(1/2),x)

[Out] atanh(sqrt(c)*x/sqrt(a + c*x**2))/sqrt(c)
```

Mathematica [A] time = 0.010187, size = 25, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[a + c*x^2],x]
```

```
[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/Sqrt[c]
```

Maple [A] time = 0.003, size = 21, normalized size = 0.8

$$1\ln\left(\sqrt{c}x+\sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+a)^(1/2),x)
```

```
[Out] ln(c^(1/2)*x+(c*x^2+a)^(1/2))/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^2 + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.226916, size = 1, normalized size = 0.04

$$\left[\frac{\log\left(-2\sqrt{cx^2+a}cx-\left(2cx^2+a\right)\sqrt{c}\right)}{2\sqrt{c}},\frac{\arctan\left(\frac{\sqrt{-c}x}{\sqrt{cx^2+a}}\right)}{\sqrt{-c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^2 + a),x, algorithm="fricas")
```

```
[Out] [1/2*log(-2*sqrt(c*x^2 + a)*c*x - (2*c*x^2 + a)*sqrt(c))/sqrt(c),
 arctan(sqrt(-c)*x/sqrt(c*x^2 + a))/sqrt(-c)]
```

Sympy [A] time = 3.57301, size = 17, normalized size = 0.68

$$\frac{\operatorname{asinh}\left(\frac{\sqrt{c}x}{\sqrt{a}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+a)**(1/2),x)

[Out] asinh(sqrt(c)*x/sqrt(a))/sqrt(c)
```

GIAC/XCAS [A] time = 0.215241, size = 31, normalized size = 1.24

$$-\frac{\ln\left(\left|-\sqrt{c}x + \sqrt{cx^2 + a}\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(c*x^2 + a),x, algorithm="giac")

[Out] -ln(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/sqrt(c)
```

3.60 $\int \frac{1}{(a+cx^2)^{3/2}} dx$

Optimal. Leaf size=16

$$\frac{x}{a\sqrt{a+cx^2}}$$

```
[Out] x/(a*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.00809269, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091

$$\frac{x}{a\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)^(-3/2),x]

[Out] x/(a*Sqrt[a + c*x^2])
```

Rubi in Sympy [A] time = 1.27416, size = 12, normalized size = 0.75

$$\frac{x}{a\sqrt{a+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+a)**(3/2),x)

[Out] x/(a*sqrt(a + c*x**2))
```

Mathematica [A] time = 0.012431, size = 16, normalized size = 1.

$$\frac{x}{a\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)^(-3/2),x]
```

```
[Out] x/(a*Sqrt[a + c*x^2])
```

Maple [A] time = 0.004, size = 15, normalized size = 0.9

$$\frac{x}{a}\frac{1}{\sqrt{cx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+a)^(3/2),x)
```

```
[Out] x/a/(c*x^2+a)^(1/2)
```

Maxima [A] time = 0.705913, size = 19, normalized size = 1.19

$$\frac{x}{\sqrt{cx^2+a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-3/2),x, algorithm="maxima")
```

```
[Out] x/(sqrt(c*x^2 + a)*a)
```

Fricas [A] time = 0.215021, size = 31, normalized size = 1.94

$$\frac{\sqrt{cx^2+a}x}{acx^2+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-3/2),x, algorithm="fricas")
```

```
[Out] sqrt(c*x^2 + a)*x/(a*c*x^2 + a^2)
```

Sympy [A] time = 1.80337, size = 17, normalized size = 1.06

$$\frac{x}{a^{\frac{3}{2}}\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+a)**(3/2),x)
```

```
[Out] x/(a**(3/2)*sqrt(1 + c*x**2/a))
```

GIAC/XCAS [A] time = 0.2149, size = 19, normalized size = 1.19

$$\frac{x}{\sqrt{cx^2 + a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-3/2),x, algorithm="giac")
```

```
[Out] x/(sqrt(c*x^2 + a)*a)
```

3.61 $\int \frac{1}{(a+cx^2)^{5/2}}\,dx$

Optimal. Leaf size=39

$$\frac{2x}{3a^2\sqrt{a+cx^2}}+\frac{x}{3a\,(a+cx^2)^{3/2}}$$

```
[Out] x/(3*a*(a + c*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.020612, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{2x}{3a^2\sqrt{a+cx^2}}+\frac{x}{3a\,(a+cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)^(-5/2),x]

[Out] x/(3*a*(a + c*x^2)^(3/2)) + (2*x)/(3*a^2*Sqrt[a + c*x^2])
```

Rubi in Sympy [A] time = 2.01375, size = 32, normalized size = 0.82

$$\frac{x}{3a\,(a+cx^2)^{\frac{3}{2}}}+\frac{2x}{3a^2\sqrt{a+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+a)**(5/2),x)

[Out] x/(3*a*(a + c*x**2)**(3/2)) + 2*x/(3*a**2*sqrt(a + c*x**2))
```

Mathematica [A] time = 0.0210235, size = 29, normalized size = 0.74

$$\frac{x\left(3a+2cx^2\right)}{3a^2\,(a+cx^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)^(-5/2),x]
```

```
[Out] (x*(3*a + 2*c*x^2))/(3*a^2*(a + c*x^2)^(3/2))
```

Maple [A] time = 0.004, size = 26, normalized size = 0.7

$$\frac{x\left(2\,cx^2+3\,a\right)}{3\,a^2}\left(cx^2+a\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+a)^(5/2),x)
```

```
[Out] 1/3*x*(2*c*x^2+3*a)/(c*x^2+a)^(3/2)/a^2
```

Maxima [A] time = 0.702105, size = 42, normalized size = 1.08

$$\frac{2\,x}{3\,\sqrt{cx^2+a}a^2}+\frac{x}{3\,(cx^2+a)^{\frac{3}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-5/2),x, algorithm="maxima")
```

```
[Out] 2/3*x/(sqrt(c*x^2 + a)*a^2) + 1/3*x/((c*x^2 + a)^(3/2)*a)
```

Fricas [A] time = 0.225107, size = 63, normalized size = 1.62

$$\frac{\left(2\,cx^3+3\,ax\right)\sqrt{cx^2+a}}{3\,(a^2c^2x^4+2\,a^3cx^2+a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*c*x^3 + 3*a*x)*sqrt(c*x^2 + a)/(a^2*c^2*x^4 + 2*a^3*c*x^2
+ a^4)
```

Sympy [A] time = 2.66581, size = 95, normalized size = 2.44

$$\frac{3ax}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^2}{a}}+3a^{\frac{5}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}+\frac{2cx^3}{3a^{\frac{7}{2}}\sqrt{1+\frac{cx^2}{a}}+3a^{\frac{5}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+a)**(5/2),x)
```

```
[Out] 3*a*x/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2)*c*x**2*sqrt(1 +
 c*x**2/a)) + 2*c*x**3/(3*a**(7/2)*sqrt(1 + c*x**2/a) + 3*a**(5/2
)*c*x**2*sqrt(1 + c*x**2/a))
```

GIAC/XCAS [A] time = 0.21492, size = 36, normalized size = 0.92

$$\frac{x\left(\frac{2\,cx^2}{a^2}+\frac{3}{a}\right)}{3\,(cx^2+a)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-5/2),x, algorithm="giac")
```

```
[Out] 1/3*x*(2*c*x^2/a^2 + 3/a)/(c*x^2 + a)^(3/2)
```

3.62 $\int \frac{1}{(a+cx^2)^{7/2}} \, dx$

Optimal. Leaf size=58

$$\frac{8x}{15a^3\sqrt{a+cx^2}} + \frac{4x}{15a^2(a+cx^2)^{3/2}} + \frac{x}{5a(a+cx^2)^{5/2}}$$

```
[Out] x/(5*a*(a + c*x^2)^(5/2)) + (4*x)/(15*a^2*(a + c*x^2)^(3/2)) + (8
*x)/(15*a^3*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.0297162, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{8x}{15a^3\sqrt{a+cx^2}} + \frac{4x}{15a^2(a+cx^2)^{3/2}} + \frac{x}{5a(a+cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)^(-7/2),x]
```

```
[Out] x/(5*a*(a + c*x^2)^(5/2)) + (4*x)/(15*a^2*(a + c*x^2)^(3/2)) + (8
*x)/(15*a^3*Sqrt[a + c*x^2])
```

Rubi in Sympy [A] time = 3.32707, size = 51, normalized size = 0.88

$$\frac{x}{5a(a+cx^2)^{\frac{5}{2}}} + \frac{4x}{15a^2(a+cx^2)^{\frac{3}{2}}} + \frac{8x}{15a^3\sqrt{a+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+a)**(7/2),x)
```

```
[Out] x/(5*a*(a + c*x**2)**(5/2)) + 4*x/(15*a**2*(a + c*x**2)**(3/2)) +
 8*x/(15*a**3*sqrt(a + c*x**2))
```

Mathematica [A] time = 0.0270443, size = 40, normalized size = 0.69

$$\frac{x\left(15a^2 + 20acx^2 + 8c^2x^4\right)}{15a^3(a+cx^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)^(-7/2),x]
```

```
[Out] (x*(15*a^2 + 20*a*c*x^2 + 8*c^2*x^4))/(15*a^3*(a + c*x^2)^(5/2))
```

Maple [A] time = 0.004, size = 37, normalized size = 0.6

$$\frac{x\left(8\,c^2x^4+20\,ax^2c+15\,a^2\right)}{15\,a^3}\left(cx^2+a\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+a)^(7/2),x)
```

```
[Out] 1/15*x*(8*c^2*x^4+20*a*c*x^2+15*a^2)/(c*x^2+a)^(5/2)/a^3
```

Maxima [A] time = 0.708095, size = 62, normalized size = 1.07

$$\frac{8\,x}{15\,\sqrt{cx^2+a}a^3}+\frac{4\,x}{15\,(cx^2+a)^{\frac{3}{2}}a^2}+\frac{x}{5\,(cx^2+a)^{\frac{5}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-7/2),x, algorithm="maxima")
```

```
[Out] 8/15*x/(sqrt(c*x^2 + a)*a^3) + 4/15*x/((c*x^2 + a)^(3/2)*a^2) + 1
/5*x/((c*x^2 + a)^(5/2)*a)
```

Fricas [A] time = 0.235426, size = 93, normalized size = 1.6

$$\frac{\left(8\,c^2x^5+20\,acx^3+15\,a^2x\right)\sqrt{cx^2+a}}{15\,(a^3c^3x^6+3\,a^4c^2x^4+3\,a^5cx^2+a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*c^2*x^5 + 20*a*c*x^3 + 15*a^2*x)*sqrt(c*x^2 + a)/(a^3*c^3
*x^6 + 3*a^4*c^2*x^4 + 3*a^5*c*x^2 + a^6)
```

Sympy [A] time = 5.25936, size = 413, normalized size = 7.12

$$\frac{15a^5x}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}} + \frac{35a^4cx^3}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}} + \frac{28a^3c^2x^5}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}} + \frac{8a^2c^3x^7}{15a^{\frac{17}{2}}\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{15}{2}}cx^2\sqrt{1+\frac{cx^2}{a}}+45a^{\frac{13}{2}}c^2x^4\sqrt{1+\frac{cx^2}{a}}+15a^{\frac{11}{2}}c^3x^6\sqrt{1+\frac{cx^2}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(c*x**2+a)**(7/2),x)
```

```
[Out] 15*a**5*x/(15*a**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*
sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) +
15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a)) + 35*a**4*c*x**3/(15*a
**(17/2)*sqrt(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2
/a) + 45*a**(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c*
*3*x**6*sqrt(1 + c*x**2/a)) + 28*a**3*c**2*x**5/(15*a**(17/2)*sqr
t(1 + c*x**2/a) + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**
(13/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt
(1 + c*x**2/a)) + 8*a**2*c**3*x**7/(15*a**(17/2)*sqrt(1 + c*x**2/
a) + 45*a**(15/2)*c*x**2*sqrt(1 + c*x**2/a) + 45*a**(13/2)*c**2*x
**4*sqrt(1 + c*x**2/a) + 15*a**(11/2)*c**3*x**6*sqrt(1 + c*x**2/a
))
```

GIAC/XCAS [A] time = 0.216998, size = 55, normalized size = 0.95

$$\frac{\left(4\,x^2\left(\frac{2\,c^2x^2}{a^3}+\frac{5\,c}{a^2}\right)+\frac{15}{a}\right)x}{15\,(cx^2+a)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-7/2),x, algorithm="giac")
```

```
[Out] 1/15*(4*x^2*(2*c^2*x^2/a^3 + 5*c/a^2) + 15/a)*x/(c*x^2 + a)^(5/2)
```

3.63 $\int \frac{1}{(a+cx^2)^{9/2}}\,dx$

Optimal. Leaf size=77

$$\frac{16x}{35a^4\sqrt{a+cx^2}}+\frac{8x}{35a^3\left(a+cx^2\right)^{3/2}}+\frac{6x}{35a^2\left(a+cx^2\right)^{5/2}}+\frac{x}{7a\left(a+cx^2\right)^{7/2}}$$

```
[Out] x/(7*a*(a + c*x^2)^(7/2)) + (6*x)/(35*a^2*(a + c*x^2)^(5/2)) + (8
*x)/(35*a^3*(a + c*x^2)^(3/2)) + (16*x)/(35*a^4*Sqrt[a + c*x^2])
```

Rubi [A] time = 0.0425395, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182

$$\frac{16x}{35a^4\sqrt{a+cx^2}}+\frac{8x}{35a^3\left(a+cx^2\right)^{3/2}}+\frac{6x}{35a^2\left(a+cx^2\right)^{5/2}}+\frac{x}{7a\left(a+cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x^2)^(-9/2),x]
```

```
[Out] x/(7*a*(a + c*x^2)^(7/2)) + (6*x)/(35*a^2*(a + c*x^2)^(5/2)) + (8
*x)/(35*a^3*(a + c*x^2)^(3/2)) + (16*x)/(35*a^4*Sqrt[a + c*x^2])
```

Rubi in Sympy [A] time = 5.01404, size = 70, normalized size = 0.91

$$\frac{x}{7a\left(a+cx^2\right)^{\frac{7}{2}}}+\frac{6x}{35a^2\left(a+cx^2\right)^{\frac{5}{2}}}+\frac{8x}{35a^3\left(a+cx^2\right)^{\frac{3}{2}}}+\frac{16x}{35a^4\sqrt{a+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(c*x**2+a)**(9/2),x)
```

```
[Out] x/(7*a*(a + c*x**2)**(7/2)) + 6*x/(35*a**2*(a + c*x**2)**(5/2)) +
 8*x/(35*a**3*(a + c*x**2)**(3/2)) + 16*x/(35*a**4*sqrt(a + c*x**
2))
```

Mathematica [A] time = 0.0316585, size = 51, normalized size = 0.66

$$\frac{x\left(35a^3+70a^2cx^2+56ac^2x^4+16c^3x^6\right)}{35a^4\left(a+cx^2\right)^{7/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x^2)^(-9/2),x]
```

```
[Out] (x*(35*a^3 + 70*a^2*c*x^2 + 56*a*c^2*x^4 + 16*c^3*x^6))/(35*a^4*(
a + c*x^2)^(7/2))
```

Maple [A] time = 0.006, size = 48, normalized size = 0.6

$$\frac{x\left(16\,c^{3}x^{6}+56\,ac^{2}x^{4}+70\,a^{2}cx^{2}+35\,a^{3}\right)}{35\,a^{4}}\left(cx^{2}+a\right)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(c*x^2+a)^(9/2),x)
```

```
[Out] 1/35*x*(16*c^3*x^6+56*a*c^2*x^4+70*a^2*c*x^2+35*a^3)/(c*x^2+a)^(7
/2)/a^4
```

Maxima [A] time = 0.698505, size = 82, normalized size = 1.06

$$\frac{16\,x}{35\,\sqrt{cx^{2}+a}a^{4}}+\frac{8\,x}{35\,(cx^{2}+a)^{\frac{3}{2}}a^{3}}+\frac{6\,x}{35\,(cx^{2}+a)^{\frac{5}{2}}a^{2}}+\frac{x}{7\,(cx^{2}+a)^{\frac{7}{2}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-9/2),x, algorithm="maxima")
```

```
[Out] 16/35*x/(sqrt(c*x^2 + a)*a^4) + 8/35*x/((c*x^2 + a)^(3/2)*a^3) +
6/35*x/((c*x^2 + a)^(5/2)*a^2) + 1/7*x/((c*x^2 + a)^(7/2)*a)
```

Fricas [A] time = 0.251975, size = 123, normalized size = 1.6

$$\frac{\left(16\,c^{3}x^{7}+56\,ac^{2}x^{5}+70\,a^{2}cx^{3}+35\,a^{3}x\right)\sqrt{cx^{2}+a}}{35\,(a^{4}c^{4}x^{8}+4\,a^{5}c^{3}x^{6}+6\,a^{6}c^{2}x^{4}+4\,a^{7}cx^{2}+a^{8})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-9/2),x, algorithm="fricas")
```

```
[Out] 1/35*(16*c^3*x^7 + 56*a*c^2*x^5 + 70*a^2*c*x^3 + 35*a^3*x)*sqrt(c
*x^2 + a)/(a^4*c^4*x^8 + 4*a^5*c^3*x^6 + 6*a^6*c^2*x^4 + 4*a^7*c*
x^2 + a^8)
```

Sympy [A] time = 9.96288, size = 1265, normalized size = 16.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**2+a)**(9/2),x)
```

```
[Out] 35*a**14*x/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**
2*sqrt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a)
 + 700*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**
4*x**8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x
**2/a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 175*a**13*
c*x**3/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sq
rt(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 7
00*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x*
*8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/
a) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 371*a**12*c**2
*x**5/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqr
t(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 70
0*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**
8*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a
) + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 429*a**11*c**3*
x**7/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt
(1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700
*a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8
*sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a)
 + 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 286*a**10*c**4*x
**9/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(
1 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*
a**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*
sqrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a)
+ 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 104*a**9*c**5*x**
11/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1
 + c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a
**(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*s
qrt(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) +
 35*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a)) + 16*a**8*c**6*x**13
/(35*a**(37/2)*sqrt(1 + c*x**2/a) + 210*a**(35/2)*c*x**2*sqrt(1 +
 c*x**2/a) + 525*a**(33/2)*c**2*x**4*sqrt(1 + c*x**2/a) + 700*a**
(31/2)*c**3*x**6*sqrt(1 + c*x**2/a) + 525*a**(29/2)*c**4*x**8*sqr
t(1 + c*x**2/a) + 210*a**(27/2)*c**5*x**10*sqrt(1 + c*x**2/a) + 3
5*a**(25/2)*c**6*x**12*sqrt(1 + c*x**2/a))
```

GIAC/XCAS [A] time = 0.216599, size = 74, normalized size = 0.96

$$\frac{\left(2\left(4x^2\left(\frac{2c^3x^2}{a^4}+\frac{7c^2}{a^3}\right)+\frac{35c}{a^2}\right)x^2+\frac{35}{a}\right)x}{35\,(cx^2+a)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + a)^(-9/2),x, algorithm="giac")
```

```
[Out] 1/35*(2*(4*x^2*(2*c^3*x^2/a^4 + 7*c^2/a^3) + 35*c/a^2)*x^2 + 35/a
)*x/(c*x^2 + a)^(7/2)
```

3.64 $\int \left(4+12x+9x^2\right)^{3/2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{12}(3x+2)\left(9x^2+12x+4\right)^{3/2}$$

```
[Out] ((2 + 3*x)*(4 + 12*x + 9*x^2)^(3/2))/12
```

Rubi [A] time = 0.00968493, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$\frac{1}{12}(3x+2)\left(9x^2+12x+4\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(4 + 12*x + 9*x^2)^(3/2),x]

[Out] ((2 + 3*x)*(4 + 12*x + 9*x^2)^(3/2))/12
```

Rubi in Sympy [A] time = 1.2197, size = 19, normalized size = 0.83

$$\frac{(18x+12)\left(9x^2+12x+4\right)^{\frac{3}{2}}}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((9*x**2+12*x+4)**(3/2),x)

[Out] (18*x + 12)*(9*x**2 + 12*x + 4)**(3/2)/72
```

Mathematica [A] time = 0.0157694, size = 20, normalized size = 0.87

$$\frac{1}{12}(3x+2)\left((3x+2)^2\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(4 + 12*x + 9*x^2)^(3/2),x]
```

```
[Out] ((2 + 3*x)*((2 + 3*x)^2)^(3/2))/12
```

Maple [A] time = 0.004, size = 35, normalized size = 1.5

$$\frac{x\left(27\,x^{3}+72\,x^{2}+72\,x+32\right)}{4\,\left(2+3\,x\right)^{3}}\left(\left(2+3\,x\right)^{2}\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((9*x^2+12*x+4)^(3/2),x)
```

```
[Out] 1/4*x*(27*x^3+72*x^2+72*x+32)*((2+3*x)^2)^(3/2)/(2+3*x)^3
```

Maxima [A] time = 0.803982, size = 41, normalized size = 1.78

$$\frac{1}{4}\left(9\,x^{2}+12\,x+4\right)^{\frac{3}{2}}x+\frac{1}{6}\left(9\,x^{2}+12\,x+4\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((9*x^2 + 12*x + 4)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(9*x^2 + 12*x + 4)^(3/2)*x + 1/6*(9*x^2 + 12*x + 4)^(3/2)
```

Fricas [A] time = 0.209458, size = 26, normalized size = 1.13

$$\frac{27}{4}\,x^{4}+18\,x^{3}+18\,x^{2}+8\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((9*x^2 + 12*x + 4)^(3/2),x, algorithm="fricas")
```

```
[Out] 27/4*x^4 + 18*x^3 + 18*x^2 + 8*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(9x^{2}+12x+4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((9*x**2+12*x+4)**(3/2),x)
```

```
[Out] Integral((9*x**2 + 12*x + 4)**(3/2), x)
```

GIAC/XCAS [A] time = 0.208803, size = 69, normalized size = 3.

$$\frac{27}{4}\,x^{4}\mathrm{sign}\left(3\,x+2\right)+18\,x^{3}\mathrm{sign}\left(3\,x+2\right)+18\,x^{2}\mathrm{sign}\left(3\,x+2\right)+8\,x\mathrm{sign}\left(3\,x+2\right)+\frac{4}{3}\,\mathrm{sign}\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((9*x^2 + 12*x + 4)^(3/2),x, algorithm="giac")
```

```
[Out] 27/4*x^4*sign(3*x + 2) + 18*x^3*sign(3*x + 2) + 18*x^2*sign(3*x +
 2) + 8*x*sign(3*x + 2) + 4/3*sign(3*x + 2)
```

3.65 $\int \sqrt{4+12x+9x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{6}(3x+2)\sqrt{9x^2+12x+4}$$

```
[Out] ((2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])/6
```

Rubi [A] time = 0.00965837, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{1}{6}(3x+2)\sqrt{9x^2+12x+4}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[4 + 12*x + 9*x^2],x]

[Out] ((2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])/6
```

Rubi in Sympy [A] time = 1.24647, size = 19, normalized size = 0.83

$$\frac{(18x+12)\sqrt{9x^2+12x+4}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((9*x**2+12*x+4)**(1/2),x)

[Out] (18*x + 12)*sqrt(9*x**2 + 12*x + 4)/36
```

Mathematica [A] time = 0.00949838, size = 25, normalized size = 1.09

$$\frac{x\sqrt{(3x+2)^2}(3x+4)}{6x+4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] (x*Sqrt[(2 + 3*x)^2]*(4 + 3*x))/(4 + 6*x)
```

Maple [A] time = 0.004, size = 25, normalized size = 1.1

$$\frac{x(3x+4)}{4+6x}\sqrt{(2+3x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((9*x^2+12*x+4)^(1/2),x)
```

```
[Out] 1/2*x*(3*x+4)*((2+3*x)^2)^(1/2)/(2+3*x)
```

Maxima [A] time = 0.769286, size = 41, normalized size = 1.78

$$\frac{1}{2}\sqrt{9x^2+12x+4}x+\frac{1}{3}\sqrt{9x^2+12x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(9*x^2 + 12*x + 4),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(9*x^2 + 12*x + 4)*x + 1/3*sqrt(9*x^2 + 12*x + 4)
```

Fricas [A] time = 0.219046, size = 12, normalized size = 0.52

$$\frac{3}{2}x^2+2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(9*x^2 + 12*x + 4),x, algorithm="fricas")
```

```
[Out] 3/2*x^2 + 2*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{9x^2+12x+4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((9*x**2+12*x+4)**(1/2),x)
```

```
[Out] Integral(sqrt(9*x**2 + 12*x + 4), x)
```

GIAC/XCAS [A] time = 0.210094, size = 35, normalized size = 1.52

$$\frac{1}{2}\left(3\,x^2+4\,x\right)\mathrm{sign}\left(3\,x+2\right)+\frac{2}{3}\,\mathrm{sign}\left(3\,x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(9*x^2 + 12*x + 4),x, algorithm="giac")
```

```
[Out] 1/2*(3*x^2 + 4*x)*sign(3*x + 2) + 2/3*sign(3*x + 2)
```

3.66 $\int \frac{1}{\sqrt{4+12x+9x^2}} \, dx$

Optimal. Leaf size=29

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{9x^2+12x+4}}$$

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[4 + 12*x + 9*x^2])
```

Rubi [A] time = 0.0140105, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{9x^2+12x+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[4 + 12*x + 9*x^2],x]

[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[4 + 12*x + 9*x^2])
```

Rubi in Sympy [A] time = 1.54644, size = 26, normalized size = 0.9

$$\frac{(9x+6)\log(3x+2)}{9\sqrt{9x^2+12x+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(9*x**2+12*x+4)**(1/2),x)

[Out] (9*x + 6)*log(3*x + 2)/(9*sqrt(9*x**2 + 12*x + 4))
```

Mathematica [A] time = 0.0102155, size = 26, normalized size = 0.9

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{(3x+2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[4 + 12*x + 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[(2 + 3*x)^2])
```

Maple [A] time = 0.007, size = 23, normalized size = 0.8

$$\frac{(2+3x)\ln(2+3x)}{3}\frac{1}{\sqrt{(2+3x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(9*x^2+12*x+4)^(1/2),x)
```

```
[Out] 1/3*(2+3*x)*ln(2+3*x)/((2+3*x)^2)^(1/2)
```

Maxima [A] time = 0.791097, size = 8, normalized size = 0.28

$$\frac{1}{3}\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(9*x^2 + 12*x + 4),x, algorithm="maxima")
```

```
[Out] 1/3*log(x + 2/3)
```

Fricas [A] time = 0.211927, size = 11, normalized size = 0.38

$$\frac{1}{3}\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(9*x^2 + 12*x + 4),x, algorithm="fricas")
```

```
[Out] 1/3*log(3*x + 2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{9x^2 + 12x + 4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(9*x**2+12*x+4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(9*x**2 + 12*x + 4), x)
```

GIAC/XCAS [A] time = 0.21183, size = 34, normalized size = 1.17

$$\frac{\ln\left(|3\,x + 2||\mathrm{sign}\left(3\,x + 2\right)|\right)}{3\,\mathrm{sign}\left(3\,x + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(9*x^2 + 12*x + 4),x, algorithm="giac")
```

```
[Out] 1/3*ln(abs(3*x + 2)*abs(sign(3*x + 2)))/sign(3*x + 2)
```

3.67 $\int \frac{1}{(4+12x+9x^2)^{3/2}}\,dx$

Optimal. Leaf size=25

$$-\frac{1}{6(3x+2)\sqrt{9x^2+12x+4}}$$

```
[Out] -1/(6*(2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])
```

Rubi [A] time = 0.0101844, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$-\frac{1}{6(3x+2)\sqrt{9x^2+12x+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[(4 + 12*x + 9*x^2)^(-3/2),x]

[Out] -1/(6*(2 + 3*x)*Sqrt[4 + 12*x + 9*x^2])
```

Rubi in Sympy [A] time = 1.22399, size = 20, normalized size = 0.8

$$-\frac{18x+12}{36\left(9x^2+12x+4\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(9*x**2+12*x+4)**(3/2),x)

[Out] -(18*x + 12)/(36*(9*x**2 + 12*x + 4)**(3/2))
```

Mathematica [A] time = 0.00990187, size = 20, normalized size = 0.8

$$-\frac{3x+2}{6\left((3x+2)^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 + 12*x + 9*x^2)^(-3/2),x]

[Out] -(2 + 3*x)/(6*((2 + 3*x)^2)^(3/2))
```

Maple [A] time = 0.005, size = 17, normalized size = 0.7

$$-\frac{2+3x}{6}\left((2+3x)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(9*x^2+12*x+4)^(3/2),x)

[Out] -1/6*(2+3*x)/((2+3*x)^2)^(3/2)
```

Maxima [A] time = 0.78654, size = 12, normalized size = 0.48

$$-\frac{1}{6(3x+2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((9*x^2 + 12*x + 4)^(-3/2),x, algorithm="maxima")

[Out] -1/6/(3*x + 2)^2
```

Fricas [A] time = 0.220998, size = 19, normalized size = 0.76

$$-\frac{1}{6(9x^2+12x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((9*x^2 + 12*x + 4)^(-3/2),x, algorithm="fricas")

[Out] -1/6/(9*x^2 + 12*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(9x^2 + 12x + 4)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(9*x**2+12*x+4)**(3/2),x)
```

```
[Out] Integral((9*x**2 + 12*x + 4)**(-3/2), x)
```

GIAC/XCAS [A] time = 0.561854, size = 4, normalized size = 0.16

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((9*x^2 + 12*x + 4)^(-3/2),x, algorithm="giac")
```

```
[Out] sage0*x
```

3.68 $\int \sqrt{4-12x+9x^2}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{6}(2-3x)\sqrt{9x^2-12x+4}$$

```
[Out] -((2 - 3*x)*Sqrt[4 - 12*x + 9*x^2])/6
```

Rubi [A] time = 0.00992491, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$-\frac{1}{6}(2-3x)\sqrt{9x^2-12x+4}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[4 - 12*x + 9*x^2],x]

[Out] -((2 - 3*x)*Sqrt[4 - 12*x + 9*x^2])/6
```

Rubi in Sympy [A] time = 1.40134, size = 20, normalized size = 0.87

$$-\frac{(-18x+12)\sqrt{9x^2-12x+4}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(((-2+3*x)**2)**(1/2),x)

[Out] -(-18*x + 12)*sqrt(9*x**2 - 12*x + 4)/36
```

Mathematica [A] time = 0.0195161, size = 25, normalized size = 1.09

$$\frac{\sqrt{(2-3x)^2}x(3x-4)}{6x-4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[4 - 12*x + 9*x^2],x]
```

```
[Out] (Sqrt[(2 - 3*x)^2]*x*(-4 + 3*x))/(-4 + 6*x)
```

Maple [A] time = 0.005, size = 25, normalized size = 1.1

$$\frac{x\,(3\,x-4)}{-4+6\,x}\sqrt{(-2+3\,x)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(((-2+3*x)^2)^(1/2),x)
```

```
[Out] 1/2*x*(3*x-4)*((-2+3*x)^2)^(1/2)/(-2+3*x)
```

Maxima [A] time = 0.798965, size = 41, normalized size = 1.78

$$\frac{1}{2}\,\sqrt{9\,x^{2}-12\,x+4}x-\frac{1}{3}\,\sqrt{9\,x^{2}-12\,x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((3*x - 2)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(9*x^2 - 12*x + 4)*x - 1/3*sqrt(9*x^2 - 12*x + 4)
```

Fricas [A] time = 0.215914, size = 12, normalized size = 0.52

$$\frac{3}{2}\,x^{2}-2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((3*x - 2)^2),x, algorithm="fricas")
```

```
[Out] 3/2*x^2 - 2*x
```

Sympy [A] time = 0.105407, size = 8, normalized size = 0.35

$$\frac{3x^{2}}{2}-2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(((-2+3*x)**2)**(1/2),x)
```

```
[Out] 3*x**2/2 - 2*x
```

GIAC/XCAS [A] time = 0.211031, size = 35, normalized size = 1.52

$$\frac{1}{2}\left(3\,x^{2}-4\,x\right)\operatorname{sign}\left(3\,x-2\right)+\frac{2}{3}\operatorname{sign}\left(3\,x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt((3*x - 2)^2),x, algorithm="giac")
```

```
[Out] 1/2*(3*x^2 - 4*x)*sign(3*x - 2) + 2/3*sign(3*x - 2)
```

3.69 $\int \frac{1}{\sqrt{4-12x+9x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{9x^2-12x+4}}$$

```
[Out] -((2 - 3*x)*Log[2 - 3*x])/(3*Sqrt[4 - 12*x + 9*x^2])
```

Rubi [A] time = 0.014126, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{9x^2-12x+4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[4 - 12*x + 9*x^2],x]

[Out] -((2 - 3*x)*Log[2 - 3*x])/(3*Sqrt[4 - 12*x + 9*x^2])
```

Rubi in Sympy [A] time = 1.75393, size = 27, normalized size = 0.93

$$-\frac{(-9x+6)\log(-3x+2)}{9\sqrt{9x^2-12x+4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/((-2+3*x)**2)**(1/2),x)

[Out] -(-9*x + 6)*log(-3*x + 2)/(9*sqrt(9*x**2 - 12*x + 4))
```

Mathematica [A] time = 0.0180115, size = 26, normalized size = 0.9

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{(2-3x)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[4 - 12*x + 9*x^2],x]
```

```
[Out] -((2 - 3*x)*Log[2 - 3*x])/(3*Sqrt[(2 - 3*x)^2])
```

Maple [A] time = 0.007, size = 23, normalized size = 0.8

$$\frac{(-2+3x)\ln(-2+3x)}{3}\frac{1}{\sqrt{(-2+3x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/((-2+3*x)^2)^(1/2),x)
```

```
[Out] 1/3/((-2+3*x)^2)^(1/2)*(-2+3*x)*ln(-2+3*x)
```

Maxima [A] time = 0.804821, size = 8, normalized size = 0.28

$$\frac{1}{3}\log\left(x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((3*x - 2)^2),x, algorithm="maxima")
```

```
[Out] 1/3*log(x - 2/3)
```

Fricas [A] time = 0.233132, size = 11, normalized size = 0.38

$$\frac{1}{3}\log(3x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((3*x - 2)^2),x, algorithm="fricas")
```

```
[Out] 1/3*log(3*x - 2)
```

Sympy [A] time = 0.116029, size = 7, normalized size = 0.24

$$\frac{\log(3x-2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/((-2+3*x)**2)**(1/2),x)
```

```
[Out] log(3*x - 2)/3
```

GIAC/XCAS [A] time = 0.210258, size = 20, normalized size = 0.69

$$\frac{1}{3}\ln\left(|3x-2|\right)\mathrm{sign}\left(3x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt((3*x - 2)^2),x, algorithm="giac")
```

```
[Out] 1/3*ln(abs(3*x - 2))*sign(3*x - 2)
```

3.70 $\int \sqrt{-4+12x-9x^2}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{6}(2-3x)\sqrt{-9x^2+12x-4}$$

```
[Out] -((2 - 3*x)*Sqrt[-4 + 12*x - 9*x^2])/6
```

Rubi [A] time = 0.0089736, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{1}{6}(2-3x)\sqrt{-9x^2+12x-4}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-4 + 12*x - 9*x^2],x]

[Out] -((2 - 3*x)*Sqrt[-4 + 12*x - 9*x^2])/6
```

Rubi in Sympy [A] time = 1.45825, size = 20, normalized size = 0.87

$$-\frac{(-18x+12)\sqrt{-9x^2+12x-4}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-(-2+3*x)**2)**(1/2),x)

[Out] -(-18*x + 12)*sqrt(-9*x**2 + 12*x - 4)/36
```

Mathematica [A] time = 0.0132252, size = 27, normalized size = 1.17

$$\frac{\sqrt{-(2-3x)^2}x(3x-4)}{6x-4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-4 + 12*x - 9*x^2],x]
```

```
[Out] (Sqrt[-(2 - 3*x)^2]*x*(-4 + 3*x))/(-4 + 6*x)
```

Maple [A] time = 0.003, size = 27, normalized size = 1.2

$$\frac{x(3x-4)}{-4+6x}\sqrt{-(-2+3x)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(-2+3*x)^2)^(1/2),x)
```

```
[Out] 1/2*x*(3*x-4)*(-(-2+3*x)^2)^(1/2)/(-2+3*x)
```

Maxima [A] time = 0.775862, size = 41, normalized size = 1.78

$$\frac{1}{2}\sqrt{-9x^{2}+12x-4}x-\frac{1}{3}\sqrt{-9x^{2}+12x-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-(3*x - 2)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-9*x^2 + 12*x - 4)*x - 1/3*sqrt(-9*x^2 + 12*x - 4)
```

Fricas [A] time = 0.209313, size = 12, normalized size = 0.52

$$\frac{3}{2}ix^{2}-2ix$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-(3*x - 2)^2),x, algorithm="fricas")
```

```
[Out] 3/2*I*x^2 - 2*I*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-(3x-2)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(-2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(3*x - 2)**2), x)
```

GIAC/XCAS [A] time = 0.210061, size = 39, normalized size = 1.7

$$-\frac{1}{6}\left(3\left(3x^2 - 4x\right)\operatorname{sign}\left(-3x + 2\right) + 4\operatorname{sign}\left(-3x + 2\right)\right)i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(3*x - 2)^2),x, algorithm="giac")
```

```
[Out] -1/6*(3*(3*x^2 - 4*x)*sign(-3*x + 2) + 4*sign(-3*x + 2))*i
```

3.71 $\int \frac{1}{\sqrt{-4+12x-9x^2}}\,dx$

Optimal. Leaf size=29

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-9x^2+12x-4}}$$

```
[Out] -((2 - 3*x)*Log[2 - 3*x])/(3*Sqrt[-4 + 12*x - 9*x^2])
```

Rubi [A] time = 0.0142402, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-9x^2+12x-4}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-4 + 12*x - 9*x^2],x]

[Out] -((2 - 3*x)*Log[2 - 3*x])/(3*Sqrt[-4 + 12*x - 9*x^2])
```

Rubi in Sympy [A] time = 1.7981, size = 27, normalized size = 0.93

$$-\frac{(-9x+6)\log(-3x+2)}{9\sqrt{-9x^2+12x-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-(-2+3*x)**2)**(1/2),x)

[Out] -(-9*x + 6)*log(-3*x + 2)/(9*sqrt(-9*x**2 + 12*x - 4))
```

Mathematica [A] time = 0.0115597, size = 28, normalized size = 0.97

$$-\frac{(2-3x)\log(2-3x)}{3\sqrt{-(2-3x)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-4 + 12*x - 9*x^2],x]

[Out] -((2 - 3*x)*Log[2 - 3*x])/(3*Sqrt[-(2 - 3*x)^2])
```

Maple [A] time = 0.005, size = 25, normalized size = 0.9

$$\frac{(-2+3x)\ln(-2+3x)}{3}\frac{1}{\sqrt{-(-2+3x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-(-2+3*x)^2)^(1/2),x)

[Out] 1/3/(-(-2+3*x)^2)^(1/2)*(-2+3*x)*ln(-2+3*x)
```

Maxima [A] time = 0.80606, size = 8, normalized size = 0.28

$$\frac{1}{3}i\log\left(x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(3*x - 2)^2),x, algorithm="maxima")

[Out] 1/3*I*log(x - 2/3)
```

Fricas [A] time = 0.221142, size = 8, normalized size = 0.28

$$-\frac{1}{3}i\log\left(x-\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(3*x - 2)^2),x, algorithm="fricas")

[Out] -1/3*I*log(x - 2/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(3x-2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-(-2+3*x)**2)**(1/2),x)

[Out] Integral(1/sqrt(-(3*x - 2)**2), x)
```

GIAC/XCAS [A] time = 0.211316, size = 35, normalized size = 1.21

$$\frac{i \ln\left(-i(3\,x-2)\mathrm{sign}\left(-3\,x+2\right)\right)}{3\,\mathrm{sign}\left(-3\,x+2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(3*x - 2)^2),x, algorithm="giac")

[Out] 1/3*i*ln(-i*(3*x - 2)*sign(-3*x + 2))/sign(-3*x + 2)
```

3.72 $\int \sqrt{-4-12x-9x^2}\,dx$

Optimal. Leaf size=23

$$\frac{1}{6}(3x+2)\sqrt{-9x^2-12x-4}$$

```
[Out] ((2 + 3*x)*Sqrt[-4 - 12*x - 9*x^2])/6
```

Rubi [A] time = 0.00982988, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{1}{6}(3x+2)\sqrt{-9x^2-12x-4}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-4 - 12*x - 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[-4 - 12*x - 9*x^2])/6
```

Rubi in Sympy [A] time = 1.39454, size = 20, normalized size = 0.87

$$\frac{(18x+12)\sqrt{-9x^2-12x-4}}{36}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-(2+3*x)**2)**(1/2),x)
```

```
[Out] (18*x + 12)*sqrt(-9*x**2 - 12*x - 4)/36
```

Mathematica [A] time = 0.0108711, size = 27, normalized size = 1.17

$$\frac{x\sqrt{-(3x+2)^2}(3x+4)}{6x+4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-4 - 12*x - 9*x^2],x]
```

```
[Out] (x*Sqrt[-(2 + 3*x)^2]*(4 + 3*x))/(4 + 6*x)
```

Maple [A] time = 0.003, size = 27, normalized size = 1.2

$$\frac{x(3x+4)}{4+6x}\sqrt{-(2+3x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-(2+3*x)^2)^(1/2),x)
```

```
[Out] 1/2*x*(3*x+4)*(-(2+3*x)^2)^(1/2)/(2+3*x)
```

Maxima [A] time = 0.792125, size = 41, normalized size = 1.78

$$\frac{1}{2}\sqrt{-9x^2-12x-4}x+\frac{1}{3}\sqrt{-9x^2-12x-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-(3*x + 2)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-9*x^2 - 12*x - 4)*x + 1/3*sqrt(-9*x^2 - 12*x - 4)
```

Fricas [A] time = 0.215199, size = 12, normalized size = 0.52

$$\frac{3}{2}ix^2+2ix$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-(3*x + 2)^2),x, algorithm="fricas")
```

```
[Out] 3/2*I*x^2 + 2*I*x
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-(3x+2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-(2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-(3*x + 2)**2), x)
```

GIAC/XCAS [A] time = 0.209124, size = 39, normalized size = 1.7

$$-\frac{1}{6}\left(3\left(3\,x^2+4\,x\right)\operatorname{sign}\left(-3\,x-2\right)+4\operatorname{sign}\left(-3\,x-2\right)\right)i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-(3*x + 2)^2),x, algorithm="giac")
```

```
[Out] -1/6*(3*(3*x^2 + 4*x)*sign(-3*x - 2) + 4*sign(-3*x - 2))*i
```

3.73 $\int \frac{1}{\sqrt{-4-12x-9x^2}}\,dx$

Optimal. Leaf size=29

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-9x^2-12x-4}}$$

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-4 - 12*x - 9*x^2])
```

Rubi [A] time = 0.0174394, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-9x^2-12x-4}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-4 - 12*x - 9*x^2],x]

[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-4 - 12*x - 9*x^2])
```

Rubi in Sympy [A] time = 1.88976, size = 27, normalized size = 0.93

$$\frac{(9x+6)\log(3x+2)}{9\sqrt{-9x^2-12x-4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/(-(2+3*x)**2)**(1/2),x)

[Out] (9*x + 6)*log(3*x + 2)/(9*sqrt(-9*x**2 - 12*x - 4))
```

Mathematica [A] time = 0.00728601, size = 28, normalized size = 0.97

$$\frac{(3x+2)\log(3x+2)}{3\sqrt{-(3x+2)^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-4 - 12*x - 9*x^2],x]
```

```
[Out] ((2 + 3*x)*Log[2 + 3*x])/(3*Sqrt[-(2 + 3*x)^2])
```

Maple [A] time = 0.003, size = 25, normalized size = 0.9

$$\frac{(2+3x)\ln(2+3x)}{3}\frac{1}{\sqrt{-(2+3x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-(2+3*x)^2)^(1/2),x)
```

```
[Out] 1/3*(2+3*x)*ln(2+3*x)/(-(2+3*x)^2)^(1/2)
```

Maxima [A] time = 0.795971, size = 8, normalized size = 0.28

$$\frac{1}{3}i\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(3*x + 2)^2),x, algorithm="maxima")
```

```
[Out] 1/3*I*log(x + 2/3)
```

Fricas [A] time = 0.21563, size = 8, normalized size = 0.28

$$-\frac{1}{3}i\log\left(x+\frac{2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(3*x + 2)^2),x, algorithm="fricas")
```

```
[Out] -1/3*I*log(x + 2/3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\left(3x+2\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-(2+3*x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-(3*x + 2)**2), x)
```

GIAC/XCAS [A] time = 0.21228, size = 35, normalized size = 1.21

$$\frac{i\ln\left(-i(3\,x+2)\mathrm{sign}\left(-3\,x-2\right)\right)}{3\,\mathrm{sign}\left(-3\,x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-(3*x + 2)^2),x, algorithm="giac")
```

```
[Out] 1/3*i*ln(-i*(3*x + 2)*sign(-3*x - 2))/sign(-3*x - 2)
```

3.74 $\int \left(\frac{-1+b^2}{4c}+bx+cx^2\right)^5 \, dx$

Optimal. Leaf size=109

$$-\frac{(-b-2cx+1)^{11}}{22528c^6}+\frac{(-b-2cx+1)^{10}}{2048c^6}-\frac{5(-b-2cx+1)^9}{2304c^6}+\frac{5(-b-2cx+1)^8}{1024c^6}-\frac{5(-b-2cx+1)^7}{896c^6}+\frac{(-b-2cx+1)^6}{384c^6}$$

```
[Out] (1 - b - 2*c*x)^6/(384*c^6) - (5*(1 - b - 2*c*x)^7)/(896*c^6) + (
5*(1 - b - 2*c*x)^8)/(1024*c^6) - (5*(1 - b - 2*c*x)^9)/(2304*c^6
) + (1 - b - 2*c*x)^10/(2048*c^6) - (1 - b - 2*c*x)^11/(22528*c^6
)
```

Rubi [A] time = 0.301559, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$-\frac{(-b-2cx+1)^{11}}{22528c^6}+\frac{(-b-2cx+1)^{10}}{2048c^6}-\frac{5(-b-2cx+1)^9}{2304c^6}+\frac{5(-b-2cx+1)^8}{1024c^6}-\frac{5(-b-2cx+1)^7}{896c^6}+\frac{(-b-2cx+1)^6}{384c^6}$$

Antiderivative was successfully verified.

```
[In]  Int[((-1 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] (1 - b - 2*c*x)^6/(384*c^6) - (5*(1 - b - 2*c*x)^7)/(896*c^6) + (
5*(1 - b - 2*c*x)^8)/(1024*c^6) - (5*(1 - b - 2*c*x)^9)/(2304*c^6
) + (1 - b - 2*c*x)^10/(2048*c^6) - (1 - b - 2*c*x)^11/(22528*c^6
)
```

Rubi in Sympy [A] time = 48.3885, size = 95, normalized size = 0.87

$$\frac{(b+2cx+1)^{11}}{22528c^6}-\frac{(b+2cx+1)^{10}}{2048c^6}+\frac{5\,(b+2cx+1)^9}{2304c^6}-\frac{5\,(b+2cx+1)^8}{1024c^6}+\frac{5\,(b+2cx+1)^7}{896c^6}-\frac{(b+2cx+1)^6}{384c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1/4*(b**2-1)/c+b*x+c*x**2)**5,x)
```

```
[Out] (b + 2*c*x + 1)**11/(22528*c**6) - (b + 2*c*x + 1)**10/(2048*c**6
) + 5*(b + 2*c*x + 1)**9/(2304*c**6) - 5*(b + 2*c*x + 1)**8/(1024
```

```
*c**6) + 5*(b + 2*c*x + 1)**7/(896*c**6) - (b + 2*c*x + 1)**6/(38
4*c**6)
```

Mathematica [A] time = 0.0552646, size = 207, normalized size = 1.9

$$\frac{5}{8}\left(3b^3-b\right)c^2x^8+\frac{\left(b^2-1\right)^5x}{1024c^5}+\frac{5b\left(b^2-1\right)^4x^2}{512c^4}+\frac{5}{36}\left(9b^2-1\right)c^3x^9+\frac{5\left(b^2-1\right)^3\left(9b^2-1\right)x^3}{768c^3}+\frac{5b\left(b^2-1\right)^2\left(3b^2-1\right)x^4}{64c^2}+\frac{5}{56}\left(21b^4-14b^2+1\right)cx^7+\frac{\left(b^2-1\right)\left(21b^4-14b^2+1\right)x^5}{32c}+\frac{1}{48}b\left(63b^4-70b^2+15\right)x^6+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-1 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] ((-1 + b^2)^5*x)/(1024*c^5) + (5*b*(-1 + b^2)^4*x^2)/(512*c^4) +
(5*(-1 + b^2)^3*(-1 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-1 + b^2)^2*(
-1 + 3*b^2)*x^4)/(64*c^2) + ((-1 + b^2)*(1 - 14*b^2 + 21*b^4)*x^5
)/(32*c) + (b*(15 - 70*b^2 + 63*b^4)*x^6)/48 + (5*(1 - 14*b^2 + 2
1*b^4)*c*x^7)/56 + (5*(-b + 3*b^3)*c^2*x^8)/8 + (5*(-1 + 9*b^2)*c
^3*x^9)/36 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

Maple [B] time = 0.007, size = 636, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1/4*(b^2-1)/c+b*x+c*x^2)^5,x)
```

```
[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-1)*c^3+4*b^2*c^3+c*(2*
(3/2*b^2-1/2)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-1)*c^2*b+b*(2*(3/2*b^
2-1/2)*c^2+4*b^2*c^2)+c*((b^2-1)*c*b+4*(3/2*b^2-1/2)*b*c))*x^8+1/
7*(1/4*(b^2-1)/c*(2*(3/2*b^2-1/2)*c^2+4*b^2*c^2)+b*((b^2-1)*c*b+4
*(3/2*b^2-1/2)*b*c)+c*(1/8*(b^2-1)^2+2*(b^2-1)*b^2+(3/2*b^2-1/2)^
2))*x^7+1/6*(1/4*(b^2-1)/c*((b^2-1)*c*b+4*(3/2*b^2-1/2)*b*c)+b*(1
/8*(b^2-1)^2+2*(b^2-1)*b^2+(3/2*b^2-1/2)^2)+c*(1/4*(b^2-1)^2/c*b+
(b^2-1)/c*b*(3/2*b^2-1/2)))*x^6+1/5*(1/4*(b^2-1)/c*(1/8*(b^2-1)^2
+2*(b^2-1)*b^2+(3/2*b^2-1/2)^2)+b*(1/4*(b^2-1)^2/c*b+(b^2-1)/c*b*
(3/2*b^2-1/2))+c*(1/8*(b^2-1)^2/c^2*(3/2*b^2-1/2)+1/4*(b^2-1)^2/c
^2*b^2))*x^5+1/4*(1/4*(b^2-1)/c*(1/4*(b^2-1)^2/c*b+(b^2-1)/c*b*(3
/2*b^2-1/2))+b*(1/8*(b^2-1)^2/c^2*(3/2*b^2-1/2)+1/4*(b^2-1)^2/c^2
*b^2)+1/16/c^2*(b^2-1)^3*b)*x^4+1/3*(1/4*(b^2-1)/c*(1/8*(b^2-1)^2
```

```
/c^2*(3/2*b^2-1/2)+1/4*(b^2-1)^2/c^2*b^2)+1/16*b^2*(b^2-1)^3/c^3+
1/256/c^3*(b^2-1)^4)*x^3+5/512*(b^2-1)^4/c^4*b*x^2+1/1024*(b^2-1)
^5/c^5*x
```

Maxima [A] time = 0.721383, size = 316, normalized size = 2.9

$$\frac{1}{11}\,c^5x^{11} + \frac{1}{2}\,bc^4x^{10} + \frac{10}{9}\,b^2c^3x^9 + \frac{5}{4}\,b^3c^2x^8 + \frac{5}{7}\,b^4cx^7 + \frac{1}{6}\,b^5x^6 + \frac{5\left(2\,cx^3 + 3\,bx^2\right)\left(b^2-1\right)^4}{1536\,c^4} + \frac{\left(6\,c^2x^5 + 15\,bcx^4 + 10\,b^2x^3\right)\left(b^2-1\right)^3}{192\,c^3} + \frac{\left(20\,c^3x^7 + 70\,bc^2x^6 + 84\,b^2cx^5 + 35\,b^3x^4\right)\left(b^2-1\right)^2}{224\,c^2} + \frac{\left(70\,c^4x^9 + 315\,bc^3x^8 + 540\,b^2c^2x^7 + 420\,b^3cx^6 + 126\,b^4x^5\right)\left(b^2-1\right)}{504\,c} + \frac{\left(b^2-1\right)^5x}{1024\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/1024*(4*c*x^2 + 4*b*x + (b^2 - 1)/c)^5,x, algorithm="maxima")
```

```
[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x
^8 + 5/7*b^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^
2 - 1)^4/c^4 + 1/192*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 -
 1)^3/c^3 + 1/224*(20*c^3*x^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*
b^3*x^4)*(b^2 - 1)^2/c^2 + 1/504*(70*c^4*x^9 + 315*b*c^3*x^8 + 54
0*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b^2 - 1)/c + 1/1024
*(b^2 - 1)^5*x/c^5
```

Fricas [A] time = 0.219919, size = 315, normalized size = 2.89

$$64512\,c^{10}x^{11} + 354816\,bc^9x^{10} + 98560\left(9\,b^2-1\right)c^8x^9 + 443520\left(3\,b^3-b\right)c^7x^8 + 63360\left(21\,b^4-14\,b^2+1\right)c^6x^7 + 14784\,(63\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/1024*(4*c*x^2 + 4*b*x + (b^2 - 1)/c)^5,x, algorithm="fricas")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 1)
*c^8*x^9 + 443520*(3*b^3 - b)*c^7*x^8 + 63360*(21*b^4 - 14*b^2 +
1)*c^6*x^7 + 14784*(63*b^5 - 70*b^3 + 15*b)*c^5*x^6 + 22176*(21*b
^6 - 35*b^4 + 15*b^2 - 1)*c^4*x^5 + 55440*(3*b^7 - 7*b^5 + 5*b^3
- b)*c^3*x^4 + 4620*(9*b^8 - 28*b^6 + 30*b^4 - 12*b^2 + 1)*c^2*x^
3 + 6930*(b^9 - 4*b^7 + 6*b^5 - 4*b^3 + b)*c*x^2 + 693*(b^10 - 5*
b^8 + 10*b^6 - 10*b^4 + 5*b^2 - 1)*x)/c^5
```

Sympy [A] time = 0.432987, size = 253, normalized size = 2.32

$$\frac{bc^4x^{10}}{2} + \frac{c^5x^{11}}{11} + x^9\left(\frac{5b^2c^3}{4} - \frac{5c^3}{36}\right) + x^8\left(\frac{15b^3c^2}{8} - \frac{5bc^2}{8}\right) + x^7\left(\frac{15b^4c}{8} - \frac{5b^2c}{4} + \frac{5c}{56}\right) + x^6\left(\frac{21b^5}{16} - \frac{35b^3}{24} + \frac{5b}{16}\right) + \frac{x^5\left(21b^6 - 35b^4 + 15b^2 - 1\right)}{32c} + \frac{x^4\left(15b^7 - 35b^5 + 25b^3 - 5b\right)}{64c^2} + \frac{x^3\left(45b^8 - 140b^6 + 150b^4 - 60b^2 + 5\right)}{768c^3} + \frac{x^2\left(5b^9 - 20b^7 + 30b^5 - 20b^3 + 5b\right)}{512c^4} + \frac{x\left(b^{10} - 5b^8 + 10b^6 - 10b^4 + 5b^2 - 1\right)}{1024c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/4*(b**2-1)/c+b*x+c*x**2)**5,x)
```

```
[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/36)
 + x**8*(15*b**3*c**2/8 - 5*b*c**2/8) + x**7*(15*b**4*c/8 - 5*b**
2*c/4 + 5*c/56) + x**6*(21*b**5/16 - 35*b**3/24 + 5*b/16) + x**5*
(21*b**6 - 35*b**4 + 15*b**2 - 1)/(32*c) + x**4*(15*b**7 - 35*b**
5 + 25*b**3 - 5*b)/(64*c**2) + x**3*(45*b**8 - 140*b**6 + 150*b**
4 - 60*b**2 + 5)/(768*c**3) + x**2*(5*b**9 - 20*b**7 + 30*b**5 -
20*b**3 + 5*b)/(512*c**4) + x*(b**10 - 5*b**8 + 10*b**6 - 10*b**4
 + 5*b**2 - 1)/(1024*c**5)
```

GIAC/XCAS [A] time = 0.209394, size = 489, normalized size = 4.49

$$64512\,c^{60}x^{11} + 354816\,bc^{59}x^{10} + 887040\,b^2c^{58}x^9 + 1330560\,b^3c^{57}x^8 + 1330560\,b^4c^{56}x^7 - 98560\,c^{58}x^9 + 931392\,b^5c^{55}x^6 - 443$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/1024*(4*c*x^2 + 4*b*x + (b^2 - 1)/c)^5,x, algorithm="giac")
```

```
[Out] 1/709632*(64512*c^60*x^11 + 354816*b*c^59*x^10 + 887040*b^2*c^58*
x^9 + 1330560*b^3*c^57*x^8 + 1330560*b^4*c^56*x^7 - 98560*c^58*x^
9 + 931392*b^5*c^55*x^6 - 443520*b*c^57*x^8 + 465696*b^6*c^54*x^5
 - 887040*b^2*c^56*x^7 + 166320*b^7*c^53*x^4 - 1034880*b^3*c^55*x
^6 + 41580*b^8*c^52*x^3 - 776160*b^4*c^54*x^5 + 63360*c^56*x^7 +
6930*b^9*c^51*x^2 - 388080*b^5*c^53*x^4 + 221760*b*c^55*x^6 + 693
*b^10*c^50*x - 129360*b^6*c^52*x^3 + 332640*b^2*c^54*x^5 - 27720*
b^7*c^51*x^2 + 277200*b^3*c^53*x^4 - 3465*b^8*c^50*x + 138600*b^4
*c^52*x^3 - 22176*c^54*x^5 + 41580*b^5*c^51*x^2 - 55440*b*c^53*x^
4 + 6930*b^6*c^50*x - 55440*b^2*c^52*x^3 - 27720*b^3*c^51*x^2 - 6
930*b^4*c^50*x + 4620*c^52*x^3 + 6930*b*c^51*x^2 + 3465*b^2*c^50*
x - 693*c^50*x)/c^55
```

3.75 $\int \left(\frac{-4+b^2}{4c}+bx+cx^2\right)^5 dx$

Optimal. Leaf size=109

$$-\frac{(-b-2cx+2)^{11}}{22528c^6}+\frac{(-b-2cx+2)^{10}}{1024c^6}-\frac{5(-b-2cx+2)^9}{576c^6}+\frac{5(-b-2cx+2)^8}{128c^6}-\frac{5(-b-2cx+2)^7}{56c^6}+\frac{(-b-2cx+2)^6}{12c^6}$$

```
[Out] (2 - b - 2*c*x)^6/(12*c^6) - (5*(2 - b - 2*c*x)^7)/(56*c^6) + (5*
(2 - b - 2*c*x)^8)/(128*c^6) - (5*(2 - b - 2*c*x)^9)/(576*c^6) +
(2 - b - 2*c*x)^10/(1024*c^6) - (2 - b - 2*c*x)^11/(22528*c^6)
```

Rubi [A] time = 0.295234, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087

$$-\frac{(-b-2cx+2)^{11}}{22528c^6}+\frac{(-b-2cx+2)^{10}}{1024c^6}-\frac{5(-b-2cx+2)^9}{576c^6}+\frac{5(-b-2cx+2)^8}{128c^6}-\frac{5(-b-2cx+2)^7}{56c^6}+\frac{(-b-2cx+2)^6}{12c^6}$$

Antiderivative was successfully verified.

```
[In]  Int[((-4 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] (2 - b - 2*c*x)^6/(12*c^6) - (5*(2 - b - 2*c*x)^7)/(56*c^6) + (5*
(2 - b - 2*c*x)^8)/(128*c^6) - (5*(2 - b - 2*c*x)^9)/(576*c^6) +
(2 - b - 2*c*x)^10/(1024*c^6) - (2 - b - 2*c*x)^11/(22528*c^6)
```

Rubi in Sympy [A] time = 47.3509, size = 95, normalized size = 0.87

$$\frac{(b+2cx+2)^{11}}{22528c^6}-\frac{(b+2cx+2)^{10}}{1024c^6}+\frac{5\,(b+2cx+2)^9}{576c^6}-\frac{5\,(b+2cx+2)^8}{128c^6}+\frac{5\,(b+2cx+2)^7}{56c^6}-\frac{(b+2cx+2)^6}{12c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1/4*(b**2-4)/c+b*x+c*x**2)**5,x)
```

```
[Out] (b + 2*c*x + 2)**11/(22528*c**6) - (b + 2*c*x + 2)**10/(1024*c**6
) + 5*(b + 2*c*x + 2)**9/(576*c**6) - 5*(b + 2*c*x + 2)**8/(128*c
**6) + 5*(b + 2*c*x + 2)**7/(56*c**6) - (b + 2*c*x + 2)**6/(12*c*
```

```
*6)
```

Mathematica [A] time = 0.0764407, size = 207, normalized size = 1.9

$$\frac{5}{8}\left(3b^3-4b\right)c^2x^8+\frac{\left(b^2-4\right)^5x}{1024c^5}+\frac{5b\left(b^2-4\right)^4x^2}{512c^4}+\frac{5}{36}\left(9b^2-4\right)c^3x^9+\frac{5\left(b^2-4\right)^3\left(9b^2-4\right)x^3}{768c^3}+\frac{5b\left(b^2-4\right)^2\left(3b^2-4\right)x^4}{64c^2}+\frac{5}{56}\left(21b^4-56b^2+16\right)cx^7+\frac{\left(b^2-4\right)\left(21b^4-56b^2+16\right)x^5}{32c}+\frac{1}{48}b\left(63b^4-280b^2+240\right)x^6+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-4 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] ((-4 + b^2)^5*x)/(1024*c^5) + (5*b*(-4 + b^2)^4*x^2)/(512*c^4) +
(5*(-4 + b^2)^3*(-4 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-4 + b^2)^2*(
-4 + 3*b^2)*x^4)/(64*c^2) + ((-4 + b^2)*(16 - 56*b^2 + 21*b^4)*x^
5)/(32*c) + (b*(240 - 280*b^2 + 63*b^4)*x^6)/48 + (5*(16 - 56*b^2
 + 21*b^4)*c*x^7)/56 + (5*(-4*b + 3*b^3)*c^2*x^8)/8 + (5*(-4 + 9*
b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

Maple [B] time = 0.006, size = 636, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1/4*(b^2-4)/c+b*x+c*x^2)^5,x)
```

```
[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-4)*c^3+4*b^2*c^3+c*(2*
(3/2*b^2-2)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-4)*c^2*b+b*(2*(3/2*b^2-
2)*c^2+4*b^2*c^2)+c*((b^2-4)*c*b+4*(3/2*b^2-2)*b*c))*x^8+1/7*(1/4
*(b^2-4)/c*(2*(3/2*b^2-2)*c^2+4*b^2*c^2)+b*((b^2-4)*c*b+4*(3/2*b^
2-2)*b*c)+c*(1/8*(b^2-4)^2+2*(b^2-4)*b^2+(3/2*b^2-2)^2))*x^7+1/6*
(1/4*(b^2-4)/c*((b^2-4)*c*b+4*(3/2*b^2-2)*b*c)+b*(1/8*(b^2-4)^2+2
*(b^2-4)*b^2+(3/2*b^2-2)^2)+c*(1/4*(b^2-4)^2/c*b+(b^2-4)/c*b*(3/2
*b^2-2)))*x^6+1/5*(1/4*(b^2-4)/c*(1/8*(b^2-4)^2+2*(b^2-4)*b^2+(3/
2*b^2-2)^2)+b*(1/4*(b^2-4)^2/c*b+(b^2-4)/c*b*(3/2*b^2-2))+c*(1/8*
(b^2-4)^2/c^2*(3/2*b^2-2)+1/4*(b^2-4)^2/c^2*b^2))*x^5+1/4*(1/4*(b
^2-4)/c*(1/4*(b^2-4)^2/c*b+(b^2-4)/c*b*(3/2*b^2-2))+b*(1/8*(b^2-4
)^2/c^2*(3/2*b^2-2)+1/4*(b^2-4)^2/c^2*b^2)+1/16/c^2*(b^2-4)^3*b)*
x^4+1/3*(1/4*(b^2-4)/c*(1/8*(b^2-4)^2/c^2*(3/2*b^2-2)+1/4*(b^2-4)
^2/c^2*b^2)+1/16*b^2*(b^2-4)^3/c^3+1/256/c^3*(b^2-4)^4)*x^3+5/512
```

```
*(b^2-4)^4/c^4*b*x^2+1/1024*(b^2-4)^5/c^5*x
```

Maxima [A] time = 0.718161, size = 316, normalized size = 2.9

$$\frac{1}{11}\,c^5x^{11}+\frac{1}{2}\,bc^4x^{10}+\frac{10}{9}\,b^2c^3x^9+\frac{5}{4}\,b^3c^2x^8+\frac{5}{7}\,b^4cx^7+\frac{1}{6}\,b^5x^6+\frac{5\left(2\,cx^3+3\,bx^2\right)\left(b^2-4\right)^4}{1536\,c^4}+\frac{\left(6\,c^2x^5+15\,bcx^4+10\,b^2x^3\right)\left(b^2-4\right)^3}{192\,c^3}+\frac{\left(20\,c^3x^7+70\,bc^2x^6+84\,b^2cx^5+35\,b^3x^4\right)\left(b^2-4\right)^2}{224\,c^2}+\frac{\left(70\,c^4x^9+315\,bc^3x^8+540\,b^2c^2x^7+420\,b^3cx^6+126\,b^4x^5\right)\left(b^2-4\right)}{504\,c}+\frac{\left(b^2-4\right)^5x}{1024\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/1024*(4*c*x^2 + 4*b*x + (b^2 - 4)/c)^5,x, algorithm="maxima")
```

```
[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x
^8 + 5/7*b^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^
2 - 4)^4/c^4 + 1/192*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 -
 4)^3/c^3 + 1/224*(20*c^3*x^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*
b^3*x^4)*(b^2 - 4)^2/c^2 + 1/504*(70*c^4*x^9 + 315*b*c^3*x^8 + 54
0*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b^2 - 4)/c + 1/1024
*(b^2 - 4)^5*x/c^5
```

Fricas [A] time = 0.213218, size = 317, normalized size = 2.91

$$\frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+98560\left(9\,b^2-4\right)c^8x^9+443520\left(3\,b^3-4\,b\right)c^7x^8+63360\left(21\,b^4-56\,b^2+16\right)c^6x^7+14784\,(6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/1024*(4*c*x^2 + 4*b*x + (b^2 - 4)/c)^5,x, algorithm="fricas")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 4)
*c^8*x^9 + 443520*(3*b^3 - 4*b)*c^7*x^8 + 63360*(21*b^4 - 56*b^2
+ 16)*c^6*x^7 + 14784*(63*b^5 - 280*b^3 + 240*b)*c^5*x^6 + 22176*
(21*b^6 - 140*b^4 + 240*b^2 - 64)*c^4*x^5 + 55440*(3*b^7 - 28*b^5
 + 80*b^3 - 64*b)*c^3*x^4 + 4620*(9*b^8 - 112*b^6 + 480*b^4 - 768
*b^2 + 256)*c^2*x^3 + 6930*(b^9 - 16*b^7 + 96*b^5 - 256*b^3 + 256
*b)*c*x^2 + 693*(b^10 - 20*b^8 + 160*b^6 - 640*b^4 + 1280*b^2 - 1
024)*x)/c^5
```

Sympy [A] time = 0.418648, size = 250, normalized size = 2.29

$$\frac{bc^4x^{10}}{2}+\frac{c^5x^{11}}{11}+x^9\left(\frac{5b^2c^3}{4}-\frac{5c^3}{9}\right)+x^8\left(\frac{15b^3c^2}{8}-\frac{5bc^2}{2}\right)+x^7\left(\frac{15b^4c}{8}-5b^2c+\frac{10c}{7}\right)+x^6\left(\frac{21b^5}{16}-\frac{35b^3}{6}+5b\right)+\frac{x^5\left(21b^6-140b^4+240b^2-64\right)}{32c}+\frac{x^4\left(15b^7-140b^5+400b^3-320b\right)}{64c^2}+\frac{x^3\left(45b^8-560b^6+2400b^4-3840b^2+1280\right)}{768c^3}+\frac{x^2\left(5b^9-80b^7+480b^5-1280b^3+1280b\right)}{512c^4}+\frac{x\left(b^{10}-20b^8+160b^6-640b^4+1280b^2-1024\right)}{1024c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/4*(b**2-4)/c+b*x+c*x**2)**5,x)
```

```
[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/9)
+ x**8*(15*b**3*c**2/8 - 5*b*c**2/2) + x**7*(15*b**4*c/8 - 5*b**2
*c + 10*c/7) + x**6*(21*b**5/16 - 35*b**3/6 + 5*b) + x**5*(21*b**
6 - 140*b**4 + 240*b**2 - 64)/(32*c) + x**4*(15*b**7 - 140*b**5 +
 400*b**3 - 320*b)/(64*c**2) + x**3*(45*b**8 - 560*b**6 + 2400*b*
*4 - 3840*b**2 + 1280)/(768*c**3) + x**2*(5*b**9 - 80*b**7 + 480*
b**5 - 1280*b**3 + 1280*b)/(512*c**4) + x*(b**10 - 20*b**8 + 160*
b**6 - 640*b**4 + 1280*b**2 - 1024)/(1024*c**5)
```

GIAC/XCAS [A] time = 0.209137, size = 489, normalized size = 4.49

$$\frac{64512\,c^{60}x^{11}+354816\,bc^{59}x^{10}+887040\,b^2c^{58}x^9+1330560\,b^3c^{57}x^8+1330560\,b^4c^{56}x^7-394240\,c^{58}x^9+931392\,b^5c^{55}x^6-17}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/1024*(4*c*x^2 + 4*b*x + (b^2 - 4)/c)^5,x, algorithm="giac")
```

```
[Out] 1/709632*(64512*c^60*x^11 + 354816*b*c^59*x^10 + 887040*b^2*c^58*
x^9 + 1330560*b^3*c^57*x^8 + 1330560*b^4*c^56*x^7 - 394240*c^58*x
^9 + 931392*b^5*c^55*x^6 - 1774080*b*c^57*x^8 + 465696*b^6*c^54*x
^5 - 3548160*b^2*c^56*x^7 + 166320*b^7*c^53*x^4 - 4139520*b^3*c^5
5*x^6 + 41580*b^8*c^52*x^3 - 3104640*b^4*c^54*x^5 + 1013760*c^56*
x^7 + 6930*b^9*c^51*x^2 - 1552320*b^5*c^53*x^4 + 3548160*b*c^55*x
^6 + 693*b^10*c^50*x - 517440*b^6*c^52*x^3 + 5322240*b^2*c^54*x^5
 - 110880*b^7*c^51*x^2 + 4435200*b^3*c^53*x^4 - 13860*b^8*c^50*x
+ 2217600*b^4*c^52*x^3 - 1419264*c^54*x^5 + 665280*b^5*c^51*x^2 -
 3548160*b*c^53*x^4 + 110880*b^6*c^50*x - 3548160*b^2*c^52*x^3 -
1774080*b^3*c^51*x^2 - 443520*b^4*c^50*x + 1182720*c^52*x^3 + 177
4080*b*c^51*x^2 + 887040*b^2*c^50*x - 709632*c^50*x)/c^55
```

3.76 $\int \left(\frac{-9+b^2}{4c} + bx + cx^2\right)^5 dx$

Optimal. Leaf size=109

$$-\frac{(-b-2cx+3)^{11}}{22528c^6} + \frac{3(-b-2cx+3)^{10}}{2048c^6} - \frac{5(-b-2cx+3)^9}{256c^6} + \frac{135(-b-2cx+3)^8}{1024c^6} - \frac{405(-b-2cx+3)^7}{896c^6} + \frac{81(-b-2cx+3)^6}{128c^6}$$

```
[Out] (81*(3 - b - 2*c*x)^6)/(128*c^6) - (405*(3 - b - 2*c*x)^7)/(896*c
^6) + (135*(3 - b - 2*c*x)^8)/(1024*c^6) - (5*(3 - b - 2*c*x)^9)/
(256*c^6) + (3*(3 - b - 2*c*x)^10)/(2048*c^6) - (3 - b - 2*c*x)^1
1/(22528*c^6)
```

Rubi [A] time = 0.289377, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$-\frac{(-b-2cx+3)^{11}}{22528c^6} + \frac{3(-b-2cx+3)^{10}}{2048c^6} - \frac{5(-b-2cx+3)^9}{256c^6} + \frac{135(-b-2cx+3)^8}{1024c^6} - \frac{405(-b-2cx+3)^7}{896c^6} + \frac{81(-b-2cx+3)^6}{128c^6}$$

Antiderivative was successfully verified.

```
[In] Int[((-9 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] (81*(3 - b - 2*c*x)^6)/(128*c^6) - (405*(3 - b - 2*c*x)^7)/(896*c
^6) + (135*(3 - b - 2*c*x)^8)/(1024*c^6) - (5*(3 - b - 2*c*x)^9)/
(256*c^6) + (3*(3 - b - 2*c*x)^10)/(2048*c^6) - (3 - b - 2*c*x)^1
1/(22528*c^6)
```

Rubi in Sympy [A] time = 47.7095, size = 99, normalized size = 0.91

$$-\frac{(-b-2cx+3)^{11}}{22528c^6} + \frac{3\,(-b-2cx+3)^{10}}{2048c^6} - \frac{5\,(-b-2cx+3)^9}{256c^6} + \frac{135\,(-b-2cx+3)^8}{1024c^6} - \frac{405\,(-b-2cx+3)^7}{896c^6} + \frac{81\,(-b-2cx+3)^6}{128c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((1/4*(b**2-9)/c+b*x+c*x**2)**5,x)
```

```
[Out] -(-b - 2*c*x + 3)**11/(22528*c**6) + 3*(-b - 2*c*x + 3)**10/(2048
*c**6) - 5*(-b - 2*c*x + 3)**9/(256*c**6) + 135*(-b - 2*c*x + 3)*
*8/(1024*c**6) - 405*(-b - 2*c*x + 3)**7/(896*c**6) + 81*(-b - 2*
c*x + 3)**6/(128*c**6)
```

Mathematica [A] time = 0.0522433, size = 199, normalized size = 1.83

$$\frac{15}{8}\left(b^3-3b\right)c^2x^8+\frac{\left(b^2-9\right)^5x}{1024c^5}+\frac{5b\left(b^2-9\right)^4x^2}{512c^4}+\frac{5}{4}\left(b^2-1\right)c^3x^9+\frac{15\left(b^2-9\right)^3\left(b^2-1\right)x^3}{256c^3}+\frac{15b\left(b^2-9\right)^2\left(b^2-3\right)x^4}{64c^2}+\frac{15}{56}\left(7b^4-42b^2+27\right)cx^7+\frac{3\left(b^2-9\right)\left(7b^4-42b^2+27\right)x^5}{32c}+\frac{3}{16}b\left(7b^4-70b^2+135\right)x^6+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-9 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] ((-9 + b^2)^5*x)/(1024*c^5) + (5*b*(-9 + b^2)^4*x^2)/(512*c^4) +
(15*(-9 + b^2)^3*(-1 + b^2)*x^3)/(256*c^3) + (15*b*(-9 + b^2)^2*(
-3 + b^2)*x^4)/(64*c^2) + (3*(-9 + b^2)*(27 - 42*b^2 + 7*b^4)*x^5
)/(32*c) + (3*b*(135 - 70*b^2 + 7*b^4)*x^6)/16 + (15*(27 - 42*b^2
 + 7*b^4)*c*x^7)/56 + (15*(-3*b + b^3)*c^2*x^8)/8 + (5*(-1 + b^2)
*c^3*x^9)/4 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

Maple [B] time = 0.006, size = 636, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1/4*(b^2-9)/c+b*x+c*x^2)^5,x)
```

```
[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-9)*c^3+4*b^2*c^3+c*(2*
(3/2*b^2-9/2)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-9)*c^2*b+b*(2*(3/2*b^
2-9/2)*c^2+4*b^2*c^2)+c*((b^2-9)*c*b+4*(3/2*b^2-9/2)*b*c))*x^8+1/
7*(1/4*(b^2-9)/c*(2*(3/2*b^2-9/2)*c^2+4*b^2*c^2)+b*((b^2-9)*c*b+4
*(3/2*b^2-9/2)*b*c)+c*(1/8*(b^2-9)^2+2*(b^2-9)*b^2+(3/2*b^2-9/2)^
2))*x^7+1/6*(1/4*(b^2-9)/c*((b^2-9)*c*b+4*(3/2*b^2-9/2)*b*c)+b*(1
/8*(b^2-9)^2+2*(b^2-9)*b^2+(3/2*b^2-9/2)^2)+c*(1/4*(b^2-9)^2/c*b+
(b^2-9)/c*b*(3/2*b^2-9/2)))*x^6+1/5*(1/4*(b^2-9)/c*(1/8*(b^2-9)^2
+2*(b^2-9)*b^2+(3/2*b^2-9/2)^2)+b*(1/4*(b^2-9)^2/c*b+(b^2-9)/c*b*
(3/2*b^2-9/2))+c*(1/8*(b^2-9)^2/c^2*(3/2*b^2-9/2)+1/4*(b^2-9)^2/c
^2*b^2))*x^5+1/4*(1/4*(b^2-9)/c*(1/4*(b^2-9)^2/c*b+(b^2-9)/c*b*(3
```

```
/2*b^2-9/2))+b*(1/8*(b^2-9)^2/c^2*(3/2*b^2-9/2)+1/4*(b^2-9)^2/c^2
*b^2)+1/16/c^2*(b^2-9)^3*b)*x^4+1/3*(1/4*(b^2-9)/c*(1/8*(b^2-9)^2
/c^2*(3/2*b^2-9/2)+1/4*(b^2-9)^2/c^2*b^2)+1/16*b^2*(b^2-9)^3/c^3+
1/256/c^3*(b^2-9)^4)*x^3+5/512*(b^2-9)^4/c^4*b*x^2+1/1024*(b^2-9)
^5/c^5*x
```

Maxima [A] time = 0.732302, size = 316, normalized size = 2.9

$$\frac{1}{11}c^5x^{11}+\frac{1}{2}bc^4x^{10}+\frac{10}{9}b^2c^3x^9+\frac{5}{4}b^3c^2x^8+\frac{5}{7}b^4cx^7+\frac{1}{6}b^5x^6+\frac{5\left(2cx^3+3bx^2\right)\left(b^2-9\right)^4}{1536c^4}+\frac{\left(6c^2x^5+15bcx^4+10b^2x^3\right)\left(b^2-9\right)^3}{192c^3}+\frac{\left(20c^3x^7+70bc^2x^6+84b^2cx^5+35b^3x^4\right)\left(b^2-9\right)^2}{224c^2}+\frac{\left(70c^4x^9+315bc^3x^8+540b^2c^2x^7+420b^3cx^6+126b^4x^5\right)\left(b^2-9\right)}{504c}+\frac{\left(b^2-9\right)^5x}{1024c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/1024*(4*c*x^2 + 4*b*x + (b^2 - 9)/c)^5,x, algorithm="maxima")
```

```
[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x
^8 + 5/7*b^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^
2 - 9)^4/c^4 + 1/192*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2 -
 9)^3/c^3 + 1/224*(20*c^3*x^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 35*
b^3*x^4)*(b^2 - 9)^2/c^2 + 1/504*(70*c^4*x^9 + 315*b*c^3*x^8 + 54
0*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b^2 - 9)/c + 1/1024
*(b^2 - 9)^5*x/c^5
```

Fricas [A] time = 0.211517, size = 306, normalized size = 2.81

$$\frac{7168\,c^{10}x^{11}+39424\,bc^9x^{10}+98560\left(b^2-1\right)c^8x^9+147840\left(b^3-3\,b\right)c^7x^8+21120\left(7\,b^4-42\,b^2+27\right)c^6x^7+14784\left(7\,b^5-7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/1024*(4*c*x^2 + 4*b*x + (b^2 - 9)/c)^5,x, algorithm="fricas")
```

```
[Out] 1/78848*(7168*c^10*x^11 + 39424*b*c^9*x^10 + 98560*(b^2 - 1)*c^8*
x^9 + 147840*(b^3 - 3*b)*c^7*x^8 + 21120*(7*b^4 - 42*b^2 + 27)*c^
6*x^7 + 14784*(7*b^5 - 70*b^3 + 135*b)*c^5*x^6 + 7392*(7*b^6 - 10
5*b^4 + 405*b^2 - 243)*c^4*x^5 + 18480*(b^7 - 21*b^5 + 135*b^3 -
243*b)*c^3*x^4 + 4620*(b^8 - 28*b^6 + 270*b^4 - 972*b^2 + 729)*c^
2*x^3 + 770*(b^9 - 36*b^7 + 486*b^5 - 2916*b^3 + 6561*b)*c*x^2 +
77*(b^10 - 45*b^8 + 810*b^6 - 7290*b^4 + 32805*b^2 - 59049)*x)/c^
```

5

Sympy [A] time = 0.431249, size = 253, normalized size = 2.32

$$\frac{bc^4x^{10}}{2} + \frac{c^5x^{11}}{11} + x^9\left(\frac{5b^2c^3}{4} - \frac{5c^3}{4}\right) + x^8\left(\frac{15b^3c^2}{8} - \frac{45bc^2}{8}\right) + x^7\left(\frac{15b^4c}{8} - \frac{45b^2c}{4} + \frac{405c}{56}\right) + x^6\left(\frac{21b^5}{16} - \frac{105b^3}{8} + \frac{405b}{16}\right) + \frac{x^5\left(21b^6 - 315b^4 + 1215b^2 - 729\right)}{32c} + \frac{x^4\left(15b^7 - 315b^5 + 2025b^3 - 3645b\right)}{64c^2} + \frac{x^3\left(15b^8 - 420b^6 + 4050b^4 - 14580b^2 + 10935\right)}{256c^3} + \frac{x^2\left(5b^9 - 180b^7 + 2430b^5 - 14580b^3 + 32805b\right)}{512c^4} + \frac{x\left(b^{10} - 45b^8 + 810b^6 - 7290b^4 + 32805b^2 - 59049\right)}{1024c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/4*(b**2-9)/c+b*x+c*x**2)**5,x)
```

```
[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 5*c**3/4)
+ x**8*(15*b**3*c**2/8 - 45*b*c**2/8) + x**7*(15*b**4*c/8 - 45*b*
*2*c/4 + 405*c/56) + x**6*(21*b**5/16 - 105*b**3/8 + 405*b/16) +
x**5*(21*b**6 - 315*b**4 + 1215*b**2 - 729)/(32*c) + x**4*(15*b**
7 - 315*b**5 + 2025*b**3 - 3645*b)/(64*c**2) + x**3*(15*b**8 - 42
0*b**6 + 4050*b**4 - 14580*b**2 + 10935)/(256*c**3) + x**2*(5*b**
9 - 180*b**7 + 2430*b**5 - 14580*b**3 + 32805*b)/(512*c**4) + x*(
b**10 - 45*b**8 + 810*b**6 - 7290*b**4 + 32805*b**2 - 59049)/(102
4*c**5)
```

GIAC/XCAS [A] time = 0.20882, size = 489, normalized size = 4.49

$$7168\,c^{60}x^{11} + 39424\,bc^{59}x^{10} + 98560\,b^2c^{58}x^9 + 147840\,b^3c^{57}x^8 + 147840\,b^4c^{56}x^7 - 98560\,c^{58}x^9 + 103488\,b^5c^{55}x^6 - 443520\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/1024*(4*c*x^2 + 4*b*x + (b^2 - 9)/c)^5,x, algorithm="giac")
```

```
[Out] 1/78848*(7168*c^60*x^11 + 39424*b*c^59*x^10 + 98560*b^2*c^58*x^9
+ 147840*b^3*c^57*x^8 + 147840*b^4*c^56*x^7 - 98560*c^58*x^9 + 10
3488*b^5*c^55*x^6 - 443520*b*c^57*x^8 + 51744*b^6*c^54*x^5 - 8870
40*b^2*c^56*x^7 + 18480*b^7*c^53*x^4 - 1034880*b^3*c^55*x^6 + 462
0*b^8*c^52*x^3 - 776160*b^4*c^54*x^5 + 570240*c^56*x^7 + 770*b^9*
```

```
c^51*x^2 - 388080*b^5*c^53*x^4 + 1995840*b*c^55*x^6 + 77*b^10*c^5
0*x - 129360*b^6*c^52*x^3 + 2993760*b^2*c^54*x^5 - 27720*b^7*c^51
*x^2 + 2494800*b^3*c^53*x^4 - 3465*b^8*c^50*x + 1247400*b^4*c^52*
x^3 - 1796256*c^54*x^5 + 374220*b^5*c^51*x^2 - 4490640*b*c^53*x^4
 + 62370*b^6*c^50*x - 4490640*b^2*c^52*x^3 - 2245320*b^3*c^51*x^2
 - 561330*b^4*c^50*x + 3367980*c^52*x^3 + 5051970*b*c^51*x^2 + 25
25985*b^2*c^50*x - 4546773*c^50*x)/c^55
```

3.77 $\int \left(\frac{-16+b^2}{4c}+bx+cx^2\right)^5 \, dx$

Optimal. Leaf size=109

$$-\frac{(-b-2cx+4)^{11}}{22528c^6}+\frac{(-b-2cx+4)^{10}}{512c^6}-\frac{5(-b-2cx+4)^9}{144c^6}+\frac{5(-b-2cx+4)^8}{16c^6}-\frac{10(-b-2cx+4)^7}{7c^6}+\frac{8(-b-2cx+4)^6}{3c^6}$$

```
[Out] (8*(4 - b - 2*c*x)^6)/(3*c^6) - (10*(4 - b - 2*c*x)^7)/(7*c^6) +
(5*(4 - b - 2*c*x)^8)/(16*c^6) - (5*(4 - b - 2*c*x)^9)/(144*c^6)
+ (4 - b - 2*c*x)^10/(512*c^6) - (4 - b - 2*c*x)^11/(22528*c^6)
```

Rubi [A] time = 0.29194, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$-\frac{(-b-2cx+4)^{11}}{22528c^6}+\frac{(-b-2cx+4)^{10}}{512c^6}-\frac{5(-b-2cx+4)^9}{144c^6}+\frac{5(-b-2cx+4)^8}{16c^6}-\frac{10(-b-2cx+4)^7}{7c^6}+\frac{8(-b-2cx+4)^6}{3c^6}$$

Antiderivative was successfully verified.

```
[In]  Int[((-16 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] (8*(4 - b - 2*c*x)^6)/(3*c^6) - (10*(4 - b - 2*c*x)^7)/(7*c^6) +
(5*(4 - b - 2*c*x)^8)/(16*c^6) - (5*(4 - b - 2*c*x)^9)/(144*c^6)
+ (4 - b - 2*c*x)^10/(512*c^6) - (4 - b - 2*c*x)^11/(22528*c^6)
```

Rubi in Sympy [A] time = 44.2048, size = 97, normalized size = 0.89

$$-\frac{(-b-2cx+4)^{11}}{22528c^6}+\frac{(-b-2cx+4)^{10}}{512c^6}-\frac{5\left(-b-2cx+4\right)^9}{144c^6}+\frac{5\left(-b-2cx+4\right)^8}{16c^6}-\frac{10\left(-b-2cx+4\right)^7}{7c^6}+\frac{8\left(-b-2cx+4\right)^6}{3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((1/4*(b**2-16)/c+b*x+c*x**2)**5,x)
```

```
[Out] -(-b - 2*c*x + 4)**11/(22528*c**6) + (-b - 2*c*x + 4)**10/(512*c*
*6) - 5*(-b - 2*c*x + 4)**9/(144*c**6) + 5*(-b - 2*c*x + 4)**8/(1
6*c**6) - 10*(-b - 2*c*x + 4)**7/(7*c**6) + 8*(-b - 2*c*x + 4)**6
/(3*c**6)
```

Mathematica [A] time = 0.0776292, size = 207, normalized size = 1.9

$$\frac{5}{8}\left(3b^3-16b\right)c^2x^8+\frac{\left(b^2-16\right)^5x}{1024c^5}+\frac{5b\left(b^2-16\right)^4x^2}{512c^4}+\frac{5}{36}\left(9b^2-16\right)c^3x^9+\frac{5\left(b^2-16\right)^3\left(9b^2-16\right)x^3}{768c^3}+\frac{5b\left(b^2-16\right)^2\left(3b^2-16\right)x^4}{64c^2}+\frac{5}{56}\left(21b^4-224b^2+256\right)cx^7+\frac{\left(b^2-16\right)\left(21b^4-224b^2+256\right)x^5}{32c}+\frac{1}{48}b\left(63b^4-1120b^2+3840\right)x^6+\frac{1}{2}bc^4x^{10}+\frac{c^5x^{11}}{11}$$

Antiderivative was successfully verified.

```
[In]  Integrate[((-16 + b^2)/(4*c) + b*x + c*x^2)^5,x]
```

```
[Out] ((-16 + b^2)^5*x)/(1024*c^5) + (5*b*(-16 + b^2)^4*x^2)/(512*c^4)
+ (5*(-16 + b^2)^3*(-16 + 9*b^2)*x^3)/(768*c^3) + (5*b*(-16 + b^2
)^2*(-16 + 3*b^2)*x^4)/(64*c^2) + ((-16 + b^2)*(256 - 224*b^2 + 2
1*b^4)*x^5)/(32*c) + (b*(3840 - 1120*b^2 + 63*b^4)*x^6)/48 + (5*(
256 - 224*b^2 + 21*b^4)*c*x^7)/56 + (5*(-16*b + 3*b^3)*c^2*x^8)/8
 + (5*(-16 + 9*b^2)*c^3*x^9)/36 + (b*c^4*x^10)/2 + (c^5*x^11)/11
```

Maple [B] time = 0.006, size = 636, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((1/4*(b^2-16)/c+b*x+c*x^2)^5,x)
```

```
[Out] 1/11*c^5*x^11+1/2*b*c^4*x^10+1/9*(1/4*(b^2-16)*c^3+4*b^2*c^3+c*(2
*(3/2*b^2-8)*c^2+4*b^2*c^2))*x^9+1/8*((b^2-16)*c^2*b+b*(2*(3/2*b^
2-8)*c^2+4*b^2*c^2)+c*((b^2-16)*c*b+4*(3/2*b^2-8)*b*c))*x^8+1/7*(
1/4*(b^2-16)/c*(2*(3/2*b^2-8)*c^2+4*b^2*c^2)+b*((b^2-16)*c*b+4*(3
/2*b^2-8)*b*c)+c*(1/8*(b^2-16)^2+2*(b^2-16)*b^2+(3/2*b^2-8)^2))*x
^7+1/6*(1/4*(b^2-16)/c*((b^2-16)*c*b+4*(3/2*b^2-8)*b*c)+b*(1/8*(b
^2-16)^2+2*(b^2-16)*b^2+(3/2*b^2-8)^2)+c*(1/4*(b^2-16)^2/c*b+(b^2
-16)/c*b*(3/2*b^2-8)))*x^6+1/5*(1/4*(b^2-16)/c*(1/8*(b^2-16)^2+2*
(b^2-16)*b^2+(3/2*b^2-8)^2)+b*(1/4*(b^2-16)^2/c*b+(b^2-16)/c*b*(3
/2*b^2-8))+c*(1/8*(b^2-16)^2/c^2*(3/2*b^2-8)+1/4*(b^2-16)^2/c^2*b
^2))*x^5+1/4*(1/4*(b^2-16)/c*(1/4*(b^2-16)^2/c*b+(b^2-16)/c*b*(3/
```

```
2*b^2-8))+b*(1/8*(b^2-16)^2/c^2*(3/2*b^2-8)+1/4*(b^2-16)^2/c^2*b^
2)+1/16/c^2*(b^2-16)^3*b)*x^4+1/3*(1/4*(b^2-16)/c*(1/8*(b^2-16)^2
/c^2*(3/2*b^2-8)+1/4*(b^2-16)^2/c^2*b^2)+1/16*b^2*(b^2-16)^3/c^3+
1/256/c^3*(b^2-16)^4)*x^3+5/512*(b^2-16)^4/c^4*b*x^2+1/1024*(b^2-
16)^5/c^5*x
```

Maxima [A] time = 0.726255, size = 316, normalized size = 2.9

$$\frac{1}{11}\,c^5x^{11}+\frac{1}{2}\,bc^4x^{10}+\frac{10}{9}\,b^2c^3x^9+\frac{5}{4}\,b^3c^2x^8+\frac{5}{7}\,b^4cx^7+\frac{1}{6}\,b^5x^6+\frac{5\left(2\,cx^3+3\,bx^2\right)\left(b^2-16\right)^4}{1536\,c^4}+\frac{\left(6\,c^2x^5+15\,bcx^4+10\,b^2x^3\right)\left(b^2-16\right)^3}{192\,c^3}+\frac{\left(20\,c^3x^7+70\,bc^2x^6+84\,b^2cx^5+35\,b^3x^4\right)\left(b^2-16\right)^2}{224\,c^2}+\frac{\left(70\,c^4x^9+315\,bc^3x^8+540\,b^2c^2x^7+420\,b^3cx^6+126\,b^4x^5\right)\left(b^2-16\right)}{504\,c}+\frac{\left(b^2-16\right)^5x}{1024\,c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/1024*(4*c*x^2 + 4*b*x + (b^2 - 16)/c)^5,x, algorithm="maxima")
```

```
[Out] 1/11*c^5*x^11 + 1/2*b*c^4*x^10 + 10/9*b^2*c^3*x^9 + 5/4*b^3*c^2*x
^8 + 5/7*b^4*c*x^7 + 1/6*b^5*x^6 + 5/1536*(2*c*x^3 + 3*b*x^2)*(b^
2 - 16)^4/c^4 + 1/192*(6*c^2*x^5 + 15*b*c*x^4 + 10*b^2*x^3)*(b^2
- 16)^3/c^3 + 1/224*(20*c^3*x^7 + 70*b*c^2*x^6 + 84*b^2*c*x^5 + 3
5*b^3*x^4)*(b^2 - 16)^2/c^2 + 1/504*(70*c^4*x^9 + 315*b*c^3*x^8 +
 540*b^2*c^2*x^7 + 420*b^3*c*x^6 + 126*b^4*x^5)*(b^2 - 16)/c + 1/
1024*(b^2 - 16)^5*x/c^5
```

Fricas [A] time = 0.217172, size = 317, normalized size = 2.91

$$\frac{64512\,c^{10}x^{11}+354816\,bc^9x^{10}+98560\left(9\,b^2-16\right)c^8x^9+443520\left(3\,b^3-16\,b\right)c^7x^8+63360\left(21\,b^4-224\,b^2+256\right)c^6x^7+147}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/1024*(4*c*x^2 + 4*b*x + (b^2 - 16)/c)^5,x, algorithm="fricas")
```

```
[Out] 1/709632*(64512*c^10*x^11 + 354816*b*c^9*x^10 + 98560*(9*b^2 - 16
)*c^8*x^9 + 443520*(3*b^3 - 16*b)*c^7*x^8 + 63360*(21*b^4 - 224*b
^2 + 256)*c^6*x^7 + 14784*(63*b^5 - 1120*b^3 + 3840*b)*c^5*x^6 +
22176*(21*b^6 - 560*b^4 + 3840*b^2 - 4096)*c^4*x^5 + 55440*(3*b^7
 - 112*b^5 + 1280*b^3 - 4096*b)*c^3*x^4 + 4620*(9*b^8 - 448*b^6 +
 7680*b^4 - 49152*b^2 + 65536)*c^2*x^3 + 6930*(b^9 - 64*b^7 + 153
6*b^5 - 16384*b^3 + 65536*b)*c*x^2 + 693*(b^10 - 80*b^8 + 2560*b^
```

```
6 - 40960*b^4 + 327680*b^2 - 1048576)*x)/c^5
```

Sympy [A] time = 0.411293, size = 248, normalized size = 2.28

$$\frac{bc^4x^{10}}{2} + \frac{c^5x^{11}}{11} + x^9\left(\frac{5b^2c^3}{4} - \frac{20c^3}{9}\right) + x^8\left(\frac{15b^3c^2}{8} - 10bc^2\right) + x^7\left(\frac{15b^4c}{8} - 20b^2c + \frac{160c}{7}\right) + x^6\left(\frac{21b^5}{16} - \frac{70b^3}{3} + 80b\right) + \frac{x^5\left(21b^6 - 560b^4 + 3840b^2 - 4096\right)}{32c} + \frac{x^4\left(15b^7 - 560b^5 + 6400b^3 - 20480b\right)}{64c^2} + \frac{x^3\left(45b^8 - 2240b^6 + 38400b^4 - 245760b^2 + 327680\right)}{768c^3} + \frac{x^2\left(5b^9 - 320b^7 + 7680b^5 - 81920b^3 + 327680b\right)}{512c^4} + \frac{x\left(b^{10} - 80b^8 + 2560b^6 - 40960b^4 + 327680b^2 - 1048576\right)}{1024c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((1/4*(b**2-16)/c+b*x+c*x**2)**5,x)
```

```
[Out] b*c**4*x**10/2 + c**5*x**11/11 + x**9*(5*b**2*c**3/4 - 20*c**3/9)
 + x**8*(15*b**3*c**2/8 - 10*b*c**2) + x**7*(15*b**4*c/8 - 20*b**
2*c + 160*c/7) + x**6*(21*b**5/16 - 70*b**3/3 + 80*b) + x**5*(21*
b**6 - 560*b**4 + 3840*b**2 - 4096)/(32*c) + x**4*(15*b**7 - 560*
b**5 + 6400*b**3 - 20480*b)/(64*c**2) + x**3*(45*b**8 - 2240*b**6
 + 38400*b**4 - 245760*b**2 + 327680)/(768*c**3) + x**2*(5*b**9 -
 320*b**7 + 7680*b**5 - 81920*b**3 + 327680*b)/(512*c**4) + x*(b*
*10 - 80*b**8 + 2560*b**6 - 40960*b**4 + 327680*b**2 - 1048576)/(
1024*c**5)
```

GIAC/XCAS [A] time = 0.208897, size = 489, normalized size = 4.49

$$\frac{64512\,c^{60}x^{11} + 354816\,bc^{59}x^{10} + 887040\,b^2c^{58}x^9 + 1330560\,b^3c^{57}x^8 + 1330560\,b^4c^{56}x^7 - 1576960\,c^{58}x^9 + 931392\,b^5c^{55}x^6 - 7}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/1024*(4*c*x^2 + 4*b*x + (b^2 - 16)/c)^5,x, algorithm="giac")
```

```
[Out] 1/709632*(64512*c^60*x^11 + 354816*b*c^59*x^10 + 887040*b^2*c^58*
x^9 + 1330560*b^3*c^57*x^8 + 1330560*b^4*c^56*x^7 - 1576960*c^58*
x^9 + 931392*b^5*c^55*x^6 - 7096320*b*c^57*x^8 + 465696*b^6*c^54*
x^5 - 14192640*b^2*c^56*x^7 + 166320*b^7*c^53*x^4 - 16558080*b^3*
c^55*x^6 + 41580*b^8*c^52*x^3 - 12418560*b^4*c^54*x^5 + 16220160*
```

c^56*x^7 + 6930*b^9*c^51*x^2 - 6209280*b^5*c^53*x^4 + 56770560*b*
c^55*x^6 + 693*b^10*c^50*x - 2069760*b^6*c^52*x^3 + 85155840*b^2*
c^54*x^5 - 443520*b^7*c^51*x^2 + 70963200*b^3*c^53*x^4 - 55440*b^
8*c^50*x + 35481600*b^4*c^52*x^3 - 90832896*c^54*x^5 + 10644480*b
^5*c^51*x^2 - 227082240*b*c^53*x^4 + 1774080*b^6*c^50*x - 2270822
40*b^2*c^52*x^3 - 113541120*b^3*c^51*x^2 - 28385280*b^4*c^50*x +
302776320*c^52*x^3 + 454164480*b*c^51*x^2 + 227082240*b^2*c^50*x
- 726663168*c^50*x)/c^55

3.78 $\int \frac{1}{2+4x+3x^2}\,dx$

Optimal. Leaf size=18

$$\frac{\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{2}}$$

```
[Out] ArcTan[(2 + 3*x)/Sqrt[2]]/Sqrt[2]
```

Rubi [A] time = 0.0269848, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 4*x + 3*x^2)^(-1),x]

[Out] ArcTan[(2 + 3*x)/Sqrt[2]]/Sqrt[2]
```

Rubi in Sympy [A] time = 1.41611, size = 19, normalized size = 1.06

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3x}{2}+1\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2+4*x+2),x)

[Out] sqrt(2)*atan(sqrt(2)*(3*x/2 + 1))/2
```

Mathematica [A] time = 0.00811029, size = 18, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 4*x + 3*x^2)^(-1),x]
```

```
[Out] ArcTan[(2 + 3*x)/Sqrt[2]]/Sqrt[2]
```

Maple [A] time = 0.005, size = 17, normalized size = 0.9

$$\frac{\sqrt{2}}{2}\arctan\left(\frac{(6x+4)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2+4*x+2),x)
```

```
[Out] 1/2*2^(1/2)*arctan(1/4*(6*x+4)*2^(1/2))
```

Maxima [A] time = 0.794678, size = 22, normalized size = 1.22

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x^2 + 4*x + 2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

Fricas [A] time = 0.223852, size = 22, normalized size = 1.22

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x^2 + 4*x + 2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

Sympy [A] time = 0.22521, size = 22, normalized size = 1.22

$$\frac{\sqrt{2}\operatorname{atan}\left(\frac{3\sqrt{2}x}{2}+\sqrt{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2+4*x+2),x)

[Out] sqrt(2)*atan(3*sqrt(2)*x/2 + sqrt(2))/2
```

GIAC/XCAS [A] time = 0.206447, size = 22, normalized size = 1.22

$$\frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x^2 + 4*x + 2),x, algorithm="giac")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2))
```

3.79 $\int \frac{1}{4-2\sqrt{3}x+x^2}\,dx$

Optimal. Leaf size=12

$$-\tan^{-1}\left(\sqrt{3}-x\right)$$

```
[Out] -ArcTan[Sqrt[3] - x]
```

Rubi [A] time = 0.0168321, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133

$$-\tan^{-1}\left(\sqrt{3}-x\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(4 - 2*Sqrt[3]*x + x^2)^(-1),x]

[Out] -ArcTan[Sqrt[3] - x]
```

Rubi in Sympy [A] time = 1.35754, size = 7, normalized size = 0.58

$$\operatorname{atan}\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(4+x**2-2*x*3**(1/2)),x)

[Out] atan(x - sqrt(3))
```

Mathematica [A] time = 0.0230932, size = 12, normalized size = 1.

$$-\tan^{-1}\left(\sqrt{3}-x\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(4 - 2*Sqrt[3]*x + x^2)^(-1),x]
```

```
[Out] -ArcTan[Sqrt[3] - x]
```

Maple [A] time = 0.018, size = 9, normalized size = 0.8

$$\arctan\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(4+x^2-2*x*3^(1/2)),x)

[Out] arctan(x-3^(1/2))
```

Maxima [A] time = 0.792211, size = 11, normalized size = 0.92

$$\arctan\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2 - 2*sqrt(3)*x + 4),x, algorithm="maxima")

[Out] arctan(x - sqrt(3))
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x^2 - 2*sqrt(3)*x + 4),x, algorithm="fricas")

[Out] Exception raised: NotImplementedError
```

Sympy [A] time = 0.392541, size = 7, normalized size = 0.58

$$\operatorname{atan}\left(x-\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4+x**2-2*x*3**(1/2)),x)
```

```
[Out] atan(x - sqrt(3))
```

GIAC/XCAS [A] time = 0.20952, size = 11, normalized size = 0.92

$$\arctan\left(x - \sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 - 2*sqrt(3)*x + 4),x, algorithm="giac")
```

```
[Out] arctan(x - sqrt(3))
```

3.80 $\int \frac{1}{2+4x-3x^2}\,dx$

Optimal. Leaf size=19

$$-\frac{\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{10}}$$

```
[Out] -(ArcTanh[(2 - 3*x)/Sqrt[10]]/Sqrt[10])
```

Rubi [A] time = 0.0358445, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 4*x - 3*x^2)^(-1),x]

[Out] -(ArcTanh[(2 - 3*x)/Sqrt[10]]/Sqrt[10])
```

Rubi in Sympy [A] time = 1.43728, size = 22, normalized size = 1.16

$$-\frac{\sqrt{10}\,\operatorname{atanh}\left(\sqrt{10}\left(-\frac{3x}{10}+\frac{1}{5}\right)\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+4*x+2),x)

[Out] -sqrt(10)*atanh(sqrt(10)*(-3*x/10 + 1/5))/10
```

Mathematica [A] time = 0.0352976, size = 34, normalized size = 1.79

$$\frac{\log\left(3x+\sqrt{10}-2\right)-\log\left(-3x+\sqrt{10}+2\right)}{2\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 4*x - 3*x^2)^(-1),x]
```

```
[Out] (-Log[2 + Sqrt[10] - 3*x] + Log[-2 + Sqrt[10] + 3*x])/(2*Sqrt[10]
)
```

Maple [A] time = 0.002, size = 17, normalized size = 0.9

$$\frac{\sqrt{10}}{10} Artanh\left(\frac{(6x-4)\sqrt{10}}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+4*x+2),x)
```

```
[Out] 1/10*10^(1/2)*arctanh(1/20*(6*x-4)*10^(1/2))
```

Maxima [A] time = 0.819524, size = 36, normalized size = 1.89

$$-\frac{1}{20}\sqrt{10}\log\left(\frac{3x-\sqrt{10}-2}{3x+\sqrt{10}-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(3*x^2 - 4*x - 2),x, algorithm="maxima")
```

```
[Out] -1/20*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2))
```

Fricas [A] time = 0.219648, size = 51, normalized size = 2.68

$$\frac{1}{20}\sqrt{10}\log\left(\frac{\sqrt{10}\left(9x^2-12x+14\right)+60x-40}{3x^2-4x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(3*x^2 - 4*x - 2),x, algorithm="fricas")
```

```
[Out] 1/20*sqrt(10)*log((sqrt(10)*(9*x^2 - 12*x + 14) + 60*x - 40)/(3*x
^2 - 4*x - 2))
```

Sympy [A] time = 0.220102, size = 39, normalized size = 2.05

$$\frac{\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{20}-\frac{\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+4*x+2),x)
```

```
[Out] sqrt(10)*log(x - 2/3 + sqrt(10)/3)/20 - sqrt(10)*log(x - sqrt(10)
/3 - 2/3)/20
```

GIAC/XCAS [A] time = 0.210915, size = 42, normalized size = 2.21

$$-\frac{1}{20}\sqrt{10}\ln\left(\frac{\left|6\,x-2\,\sqrt{10}-4\right|}{\left|6\,x+2\,\sqrt{10}-4\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(3*x^2 - 4*x - 2),x, algorithm="giac")
```

```
[Out] -1/20*sqrt(10)*ln(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10)
- 4))
```

3.81 $\int \frac{1}{2+5x+3x^2}\,dx$

Optimal. Leaf size=13

$$\log(3x+2)-\log(x+1)$$

```
[Out] -Log[1 + x] + Log[2 + 3*x]
```

Rubi [A] time = 0.0123619, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$

$$\log(3x+2)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 5*x + 3*x^2)^(-1),x]

[Out] -Log[1 + x] + Log[2 + 3*x]
```

Rubi in Sympy [A] time = 1.51275, size = 10, normalized size = 0.77

$$-\log(x+1)+\log(3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2+5*x+2),x)

[Out] -log(x + 1) + log(3*x + 2)
```

Mathematica [A] time = 0.00473063, size = 13, normalized size = 1.

$$\log(3x+2)-\log(x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 5*x + 3*x^2)^(-1),x]

[Out] -Log[1 + x] + Log[2 + 3*x]
```

Maple [A] time = 0.007, size = 14, normalized size = 1.1

$$-\ln(1+x)+\ln(2+3x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2+5*x+2),x)
```

```
[Out] -ln(1+x)+ln(2+3*x)
```

Maxima [A] time = 0.738752, size = 18, normalized size = 1.38

$$\log(3x+2)-\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x^2 + 5*x + 2),x, algorithm="maxima")
```

```
[Out] log(3*x + 2) - log(x + 1)
```

Fricas [A] time = 0.213511, size = 18, normalized size = 1.38

$$\log(3x+2)-\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x^2 + 5*x + 2),x, algorithm="fricas")
```

```
[Out] log(3*x + 2) - log(x + 1)
```

Sympy [A] time = 0.190448, size = 10, normalized size = 0.77

$$\log\left(x+\frac{2}{3}\right)-\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2+5*x+2),x)

[Out] log(x + 2/3) - log(x + 1)
```

GIAC/XCAS [A] time = 0.207589, size = 20, normalized size = 1.54

$$\ln(|3x + 2|) - \ln(|x + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x^2 + 5*x + 2),x, algorithm="giac")

[Out] ln(abs(3*x + 2)) - ln(abs(x + 1))
```

3.82 $\int \frac{1}{2+5x-3x^2}\,dx$

Optimal. Leaf size=21

$$\frac{1}{7}\log(3x+1)-\frac{1}{7}\log(2-x)$$

```
[Out] -Log[2 - x]/7 + Log[1 + 3*x]/7
```

Rubi [A] time = 0.0144738, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{1}{7}\log(3x+1)-\frac{1}{7}\log(2-x)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 5*x - 3*x^2)^(-1),x]

[Out] -Log[2 - x]/7 + Log[1 + 3*x]/7
```

Rubi in Sympy [A] time = 1.65507, size = 14, normalized size = 0.67

$$-\frac{\log\left(-x+2\right)}{7}+\frac{\log\left(3x+1\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+5*x+2),x)

[Out] -log(-x + 2)/7 + log(3*x + 1)/7
```

Mathematica [A] time = 0.0052762, size = 21, normalized size = 1.

$$\frac{1}{7}\log(3x+1)-\frac{1}{7}\log(2-x)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 5*x - 3*x^2)^(-1),x]
```

```
[Out] -Log[2 - x]/7 + Log[1 + 3*x]/7
```

Maple [A] time = 0.008, size = 16, normalized size = 0.8

$$\frac{\ln(3x+1)}{7} - \frac{\ln(x-2)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+5*x+2),x)
```

```
[Out] 1/7*ln(3*x+1)-1/7*ln(x-2)
```

Maxima [A] time = 0.728245, size = 20, normalized size = 0.95

$$\frac{1}{7}\log(3x+1) - \frac{1}{7}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(3*x^2 - 5*x - 2),x, algorithm="maxima")
```

```
[Out] 1/7*log(3*x + 1) - 1/7*log(x - 2)
```

Fricas [A] time = 0.213459, size = 20, normalized size = 0.95

$$\frac{1}{7}\log(3x+1) - \frac{1}{7}\log(x-2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(3*x^2 - 5*x - 2),x, algorithm="fricas")
```

```
[Out] 1/7*log(3*x + 1) - 1/7*log(x - 2)
```

Sympy [A] time = 0.211821, size = 14, normalized size = 0.67

$$-\frac{\log(x-2)}{7} + \frac{\log\left(x+\frac{1}{3}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+5*x+2),x)
```

```
[Out] -log(x - 2)/7 + log(x + 1/3)/7
```

GIAC/XCAS [A] time = 0.210156, size = 23, normalized size = 1.1

$$\frac{1}{7}\ln\left(|3\,x+1|\right)-\frac{1}{7}\ln\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(3*x^2 - 5*x - 2),x, algorithm="giac")
```

```
[Out] 1/7*ln(abs(3*x + 1)) - 1/7*ln(abs(x - 2))
```

3.83 $\int \frac{1}{3+4x+x^2} \, dx$

Optimal. Leaf size=6

$$-\tanh^{-1}(x+2)$$

```
[Out] -ArcTanh[2 + x]
```

Rubi [B] time = 0.0118157, antiderivative size = 17, normalized size of antiderivative = 2.83, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{1}{2}\log(x+1) - \frac{1}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x + x^2)^(-1),x]

[Out] Log[1 + x]/2 - Log[3 + x]/2
```

Rubi in Sympy [A] time = 1.42147, size = 12, normalized size = 2.

$$\frac{\log(x+1)}{2} - \frac{\log(x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+4*x+3),x)

[Out] log(x + 1)/2 - log(x + 3)/2
```

Mathematica [B] time = 0.00484806, size = 17, normalized size = 2.83

$$\frac{1}{2}\log(x+1) - \frac{1}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 4*x + x^2)^(-1),x]
```

```
[Out] Log[1 + x]/2 - Log[3 + x]/2
```

Maple [B] time = 0.008, size = 14, normalized size = 2.3

$$\frac{\ln(1+x)}{2} - \frac{\ln(3+x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+4*x+3),x)
```

```
[Out] 1/2*ln(1+x)-1/2*ln(3+x)
```

Maxima [A] time = 0.727156, size = 18, normalized size = 3.

$$-\frac{1}{2}\log(x+3) + \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 + 4*x + 3),x, algorithm="maxima")
```

```
[Out] -1/2*log(x + 3) + 1/2*log(x + 1)
```

Fricas [A] time = 0.225825, size = 18, normalized size = 3.

$$-\frac{1}{2}\log(x+3) + \frac{1}{2}\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 + 4*x + 3),x, algorithm="fricas")
```

```
[Out] -1/2*log(x + 3) + 1/2*log(x + 1)
```

Sympy [A] time = 0.17887, size = 12, normalized size = 2.

$$\frac{\log(x+1)}{2} - \frac{\log(x+3)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+4*x+3),x)
```

```
[Out] log(x + 1)/2 - log(x + 3)/2
```

GIAC/XCAS [A] time = 0.208598, size = 20, normalized size = 3.33

$$-\frac{1}{2}\ln\left(|x+3|\right)+\frac{1}{2}\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 + 4*x + 3),x, algorithm="giac")
```

```
[Out] -1/2*ln(abs(x + 3)) + 1/2*ln(abs(x + 1))
```

3.84 $\int \frac{1}{1+\pi x+2x^2} \, dx$

Optimal. Leaf size=27

$$-\frac{2 \tanh^{-1}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

```
[Out] (-2*ArcTanh[(Pi + 4*x)/Sqrt[-8 + Pi^2]])/Sqrt[-8 + Pi^2]
```

Rubi [A] time = 0.0396846, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{2 \tanh^{-1}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Pi*x + 2*x^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(Pi + 4*x)/Sqrt[-8 + Pi^2]])/Sqrt[-8 + Pi^2]
```

Rubi in Sympy [A] time = 3.15625, size = 26, normalized size = 0.96

$$-\frac{2 \operatorname{atanh}\left(\frac{4x+\pi}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(pi*x+2*x**2+1),x)
```

```
[Out] -2*atanh((4*x + pi)/sqrt(-8 + pi**2))/sqrt(-8 + pi**2)
```

Mathematica [A] time = 0.0138082, size = 27, normalized size = 1.

$$-\frac{2 \tanh^{-1}\left(\frac{4x+\pi}{\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Pi*x + 2*x^2)^(-1),x]
```

```
[Out] (-2*ArcTanh[(Pi + 4*x)/Sqrt[-8 + Pi^2]])/Sqrt[-8 + Pi^2]
```

Maple [A] time = 0.005, size = 24, normalized size = 0.9

$$-2\frac{1}{\sqrt{\pi^2-8}}Artanh\left(\frac{\pi+4x}{\sqrt{\pi^2-8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(Pi*x+2*x^2+1),x)
```

```
[Out] -2*arctanh((Pi+4*x)/(Pi^2-8)^(1/2))/(Pi^2-8)^(1/2)
```

Maxima [A] time = 0.741532, size = 51, normalized size = 1.89

$$\frac{\log\left(\frac{\pi+4x-\sqrt{\pi^2-8}}{\pi+4x+\sqrt{\pi^2-8}}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(pi*x + 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] log((pi + 4*x - sqrt(pi^2 - 8))/(pi + 4*x + sqrt(pi^2 - 8)))/sqrt
(pi^2 - 8)
```

Fricas [A] time = 0.244069, size = 82, normalized size = 3.04

$$\frac{\log\left(\frac{8\pi-\pi^3-4(\pi^2-8)x+(\pi^2+4\pi x+8x^2-4)\sqrt{\pi^2-8}}{\pi x+2x^2+1}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(pi*x + 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] log((8*pi - pi^3 - 4*(pi^2 - 8)*x + (pi^2 + 4*pi*x + 8*x^2 - 4)*s
qrt(pi^2 - 8))/(pi*x + 2*x^2 + 1))/sqrt(pi^2 - 8)
```

Sympy [A] time = 0.525891, size = 76, normalized size = 2.81

$$\frac{\log\left(x-\frac{\pi^2}{4\sqrt{-8+\pi^2}}+\frac{\pi}{4}+\frac{2}{\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}-\frac{\log\left(x-\frac{2}{\sqrt{-8+\pi^2}}+\frac{\pi}{4}+\frac{\pi^2}{4\sqrt{-8+\pi^2}}\right)}{\sqrt{-8+\pi^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(pi*x+2*x**2+1),x)
```

```
[Out] log(x - pi**2/(4*sqrt(-8 + pi**2)) + pi/4 + 2/sqrt(-8 + pi**2))/s
qrt(-8 + pi**2) - log(x - 2/sqrt(-8 + pi**2) + pi/4 + pi**2/(4*sq
rt(-8 + pi**2)))/sqrt(-8 + pi**2)
```

GIAC/XCAS [A] time = 0.209669, size = 54, normalized size = 2.

$$\frac{\ln\left(\frac{\left|\pi+4x-\sqrt{\pi^2-8}\right|}{\left|\pi+4x+\sqrt{\pi^2-8}\right|}\right)}{\sqrt{\pi^2-8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(pi*x + 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] ln(abs(pi + 4*x - sqrt(pi^2 - 8))/abs(pi + 4*x + sqrt(pi^2 - 8)))
/sqrt(pi^2 - 8)
```

3.85 $\int \frac{1}{1+\pi x-2x^2}\,dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

```
[Out] (-2*ArcTanh[(Pi - 4*x)/Sqrt[8 + Pi^2]])/Sqrt[8 + Pi^2]
```

Rubi [A] time = 0.0405191, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-\frac{2\tanh^{-1}\left(\frac{\pi-4x}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Pi*x - 2*x^2)^(-1),x]

[Out] (-2*ArcTanh[(Pi - 4*x)/Sqrt[8 + Pi^2]])/Sqrt[8 + Pi^2]
```

Rubi in Sympy [A] time = 3.08479, size = 26, normalized size = 0.96

$$-\frac{2\,\mathrm{atanh}\left(\frac{-4x+\pi}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(pi*x-2*x**2+1),x)

[Out] -2*atanh((-4*x + pi)/sqrt(8 + pi**2))/sqrt(8 + pi**2)
```

Mathematica [A] time = 0.0141532, size = 29, normalized size = 1.07

$$\frac{2\tanh^{-1}\left(\frac{4x-\pi}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Pi*x - 2*x^2)^(-1),x]
```

```
[Out] (2*ArcTanh[(-Pi + 4*x)/Sqrt[8 + Pi^2]])/Sqrt[8 + Pi^2]
```

Maple [A] time = 0.004, size = 26, normalized size = 1.

$$2\frac{1}{\sqrt{\pi^2+8}}\mathit{Artanh}\left(\frac{4x-\pi}{\sqrt{\pi^2+8}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(Pi*x-2*x^2+1),x)
```

```
[Out] 2/(Pi^2+8)^(1/2)*arctanh((4*x-Pi)/(Pi^2+8)^(1/2))
```

Maxima [A] time = 0.733548, size = 53, normalized size = 1.96

$$-\frac{\log\left(\frac{\pi-4x+\sqrt{\pi^2+8}}{\pi-4x-\sqrt{\pi^2+8}}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(pi*x - 2*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -log((pi - 4*x + sqrt(pi^2 + 8))/(pi - 4*x - sqrt(pi^2 + 8)))/sqr
t(pi^2 + 8)
```

Fricas [A] time = 0.218713, size = 81, normalized size = 3.

$$\frac{\log\left(\frac{8\pi+\pi^3-4(\pi^2+8)x-(\pi^2-4\pi x+8x^2+4)\sqrt{\pi^2+8}}{\pi x-2x^2+1}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(pi*x - 2*x^2 + 1),x, algorithm="fricas")
```

```
[Out] log((8*pi + pi^3 - 4*(pi^2 + 8)*x - (pi^2 - 4*pi*x + 8*x^2 + 4)*s
qrt(pi^2 + 8))/(pi*x - 2*x^2 + 1))/sqrt(pi^2 + 8)
```

Sympy [A] time = 0.565906, size = 76, normalized size = 2.81

$$-\frac{\log\left(x-\frac{\pi}{4}-\frac{\pi^2}{4\sqrt{8+\pi^2}}-\frac{2}{\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}+\frac{\log\left(x-\frac{\pi}{4}+\frac{2}{\sqrt{8+\pi^2}}+\frac{\pi^2}{4\sqrt{8+\pi^2}}\right)}{\sqrt{8+\pi^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(pi*x-2*x**2+1),x)
```

```
[Out] -log(x - pi/4 - pi**2/(4*sqrt(8 + pi**2)) - 2/sqrt(8 + pi**2))/sq
rt(8 + pi**2) + log(x - pi/4 + 2/sqrt(8 + pi**2) + pi**2/(4*sqrt(
8 + pi**2)))/sqrt(8 + pi**2)
```

GIAC/XCAS [A] time = 0.211031, size = 61, normalized size = 2.26

$$-\frac{\ln\left(\frac{\left|-\pi+4\,x-\sqrt{\pi^2+8}\right|}{\left|-\pi+4\,x+\sqrt{\pi^2+8}\right|}\right)}{\sqrt{\pi^2+8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(pi*x - 2*x^2 + 1),x, algorithm="giac")
```

```
[Out] -ln(abs(-pi + 4*x - sqrt(pi^2 + 8))/abs(-pi + 4*x + sqrt(pi^2 + 8
)))/sqrt(pi^2 + 8)
```

3.86 $\int \frac{1}{1+\pi x+3x^2} \, dx$

Optimal. Leaf size=31

$$\frac{2\tan^{-1}\left(\frac{6x+\pi}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

```
[Out] (2*ArcTan[(Pi + 6*x)/Sqrt[12 - Pi^2]])/Sqrt[12 - Pi^2]
```

Rubi [A] time = 0.0461908, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$\frac{2\tan^{-1}\left(\frac{6x+\pi}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Pi*x + 3*x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(Pi + 6*x)/Sqrt[12 - Pi^2]])/Sqrt[12 - Pi^2]
```

Rubi in Sympy [A] time = 2.8625, size = 24, normalized size = 0.77

$$\frac{2\,\mathrm{atan}\left(\frac{6x+\pi}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(pi*x+3*x**2+1),x)
```

```
[Out] 2*atan((6*x + pi)/sqrt(-pi**2 + 12))/sqrt(-pi**2 + 12)
```

Mathematica [A] time = 0.0144901, size = 31, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{6x+\pi}{\sqrt{12-\pi^2}}\right)}{\sqrt{12-\pi^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Pi*x + 3*x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(Pi + 6*x)/Sqrt[12 - Pi^2]])/Sqrt[12 - Pi^2]
```

Maple [A] time = 0.004, size = 28, normalized size = 0.9

$$2\frac{1}{\sqrt{-\pi^2+12}}\arctan\left(\frac{\pi+6x}{\sqrt{-\pi^2+12}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(Pi*x+3*x^2+1),x)
```

```
[Out] 2*arctan((Pi+6*x)/(-Pi^2+12)^(1/2))/(-Pi^2+12)^(1/2)
```

Maxima [A] time = 0.728159, size = 36, normalized size = 1.16

$$\frac{2\arctan\left(\frac{\pi+6x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(pi*x + 3*x^2 + 1),x, algorithm="maxima")
```

```
[Out] 2*arctan((pi + 6*x)/sqrt(-pi^2 + 12))/sqrt(-pi^2 + 12)
```

Fricas [A] time = 0.229734, size = 46, normalized size = 1.48

$$-\frac{2\arctan\left(\frac{(\pi+6x)\sqrt{-\pi^2+12}}{\pi^2-12}\right)}{\sqrt{-\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(pi*x + 3*x^2 + 1),x, algorithm="fricas")
```

```
[Out] -2*arctan((pi + 6*x)*sqrt(-pi^2 + 12)/(pi^2 - 12))/sqrt(-pi^2 + 1
2)
```

Sympy [A] time = 0.441801, size = 87, normalized size = 2.81

$$-\frac{i\log\left(x+\frac{\pi}{6}-\frac{2i}{\sqrt{-\pi^2+12}}+\frac{i\pi^2}{6\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}+\frac{i\log\left(x+\frac{\pi}{6}-\frac{i\pi^2}{6\sqrt{-\pi^2+12}}+\frac{2i}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(pi*x+3*x**2+1),x)
```

```
[Out] -I*log(x + pi/6 - 2*I/sqrt(-pi**2 + 12) + I*pi**2/(6*sqrt(-pi**2
+ 12)))/sqrt(-pi**2 + 12) + I*log(x + pi/6 - I*pi**2/(6*sqrt(-pi*
*2 + 12)) + 2*I/sqrt(-pi**2 + 12))/sqrt(-pi**2 + 12)
```

GIAC/XCAS [A] time = 0.207752, size = 36, normalized size = 1.16

$$\frac{2\arctan\left(\frac{\pi+6x}{\sqrt{-\pi^2+12}}\right)}{\sqrt{-\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(pi*x + 3*x^2 + 1),x, algorithm="giac")
```

```
[Out] 2*arctan((pi + 6*x)/sqrt(-pi^2 + 12))/sqrt(-pi^2 + 12)
```

3.87 $\int \frac{1}{1+\pi x-3x^2}\,dx$

Optimal. Leaf size=27

$$-\frac{2\tanh^{-1}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

```
[Out] (-2*ArcTanh[(Pi - 6*x)/Sqrt[12 + Pi^2]])/Sqrt[12 + Pi^2]
```

Rubi [A] time = 0.0393691, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{2\tanh^{-1}\left(\frac{\pi-6x}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + Pi*x - 3*x^2)^(-1),x]

[Out] (-2*ArcTanh[(Pi - 6*x)/Sqrt[12 + Pi^2]])/Sqrt[12 + Pi^2]
```

Rubi in Sympy [A] time = 2.92276, size = 26, normalized size = 0.96

$$-\frac{2\,\mathrm{atanh}\left(\frac{-6x+\pi}{\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(pi*x-3*x**2+1),x)

[Out] -2*atanh((-6*x + pi)/sqrt(pi**2 + 12))/sqrt(pi**2 + 12)
```

Mathematica [A] time = 0.0141199, size = 29, normalized size = 1.07

$$\frac{2\tanh^{-1}\left(\frac{6x-\pi}{\sqrt{12+\pi^2}}\right)}{\sqrt{12+\pi^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + Pi*x - 3*x^2)^(-1),x]
```

```
[Out] (2*ArcTanh[(-Pi + 6*x)/Sqrt[12 + Pi^2]])/Sqrt[12 + Pi^2]
```

Maple [A] time = 0.006, size = 26, normalized size = 1.

$$2\frac{1}{\sqrt{\pi^2+12}}\,\mathit{Artanh}\left(\frac{6x-\pi}{\sqrt{\pi^2+12}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(Pi*x-3*x^2+1),x)
```

```
[Out] 2/(Pi^2+12)^(1/2)*arctanh((6*x-Pi)/(Pi^2+12)^(1/2))
```

Maxima [A] time = 0.715809, size = 53, normalized size = 1.96

$$-\frac{\log\left(\frac{\pi-6x+\sqrt{\pi^2+12}}{\pi-6x-\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(pi*x - 3*x^2 + 1),x, algorithm="maxima")
```

```
[Out] -log((pi - 6*x + sqrt(pi^2 + 12))/(pi - 6*x - sqrt(pi^2 + 12)))/s
qrt(pi^2 + 12)
```

Fricas [A] time = 0.223624, size = 81, normalized size = 3.

$$\frac{\log\left(\frac{12\pi+\pi^3-6(\pi^2+12)x-(\pi^2-6\pi x+18x^2+6)\sqrt{\pi^2+12}}{\pi x-3x^2+1}\right)}{\sqrt{\pi^2+12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(pi*x - 3*x^2 + 1),x, algorithm="fricas")
```

```
[Out] log((12*pi + pi^3 - 6*(pi^2 + 12)*x - (pi^2 - 6*pi*x + 18*x^2 + 6
)*sqrt(pi^2 + 12))/(pi*x - 3*x^2 + 1))/sqrt(pi^2 + 12)
```

Sympy [A] time = 0.584178, size = 76, normalized size = 2.81

$$\frac{\log \left(x - \frac{\pi}{6} + \frac{\pi^2}{6\sqrt{\pi^2+12}} + \frac{2}{\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2 + 12}} - \frac{\log \left(x - \frac{\pi}{6} - \frac{2}{\sqrt{\pi^2+12}} - \frac{\pi^2}{6\sqrt{\pi^2+12}}\right)}{\sqrt{\pi^2 + 12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(pi*x-3*x**2+1),x)
```

```
[Out] log(x - pi/6 + pi**2/(6*sqrt(pi**2 + 12)) + 2/sqrt(pi**2 + 12))/s
qrt(pi**2 + 12) - log(x - pi/6 - 2/sqrt(pi**2 + 12) - pi**2/(6*sq
rt(pi**2 + 12)))/sqrt(pi**2 + 12)
```

GIAC/XCAS [A] time = 0.209133, size = 61, normalized size = 2.26

$$-\frac{\ln \left(\frac{\left|-\pi + 6\,x - \sqrt{\pi^2+12}\right|}{\left|-\pi + 6\,x + \sqrt{\pi^2+12}\right|}\right)}{\sqrt{\pi^2 + 12}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(pi*x - 3*x^2 + 1),x, algorithm="giac")
```

```
[Out] -ln(abs(-pi + 6*x - sqrt(pi^2 + 12))/abs(-pi + 6*x + sqrt(pi^2 +
12)))/sqrt(pi^2 + 12)
```

3.88 $\int \frac{1}{a+cx+bx^2}\,dx$

Optimal. Leaf size=38

$$\frac{2\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

```
[Out] (2*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/Sqrt[4*a*b - c^2]
```

Rubi [A] time = 0.0649527, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{2\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x + b*x^2)^(-1),x]

[Out] (2*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/Sqrt[4*a*b - c^2]
```

Rubi in Sympy [A] time = 4.59494, size = 34, normalized size = 0.89

$$-\frac{2\operatorname{atanh}\left(\frac{2bx+c}{\sqrt{-4ab+c^2}}\right)}{\sqrt{-4ab+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+c*x+a),x)

[Out] -2*atanh((2*b*x + c)/sqrt(-4*a*b + c**2))/sqrt(-4*a*b + c**2)
```

Mathematica [A] time = 0.0190511, size = 38, normalized size = 1.

$$\frac{2\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x + b*x^2)^(-1),x]
```

```
[Out] (2*ArcTan[(c + 2*b*x)/Sqrt[4*a*b - c^2]])/Sqrt[4*a*b - c^2]
```

Maple [A] time = 0.008, size = 35, normalized size = 0.9

$$2\,{\frac {1}{\sqrt {4\,ab-{c}^{2}}}\arctan \left({\frac {2\,bx+c}{\sqrt {4\,ab-{c}^{2}}}} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+c*x+a),x)
```

```
[Out] 2*arctan((2*b*x+c)/(4*a*b-c^2)^(1/2))/(4*a*b-c^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x^2 + c*x + a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.217455, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(\frac{4\,abc - c^{3} + 2\,{\left(4\,ab^{2} - bc^{2}\right)}x + {\left(2\,b^{2}x^{2} + 2\,bcx - 2\,ab + c^{2}\right)}\sqrt{-4\,ab + c^{2}}}{bx^{2} + cx + a}\right)}{\sqrt{-4\,ab + c^{2}}}, -\frac{2\,\arctan\left(-\frac{2\,bx + c}{\sqrt{4\,ab - c^{2}}}\right)}{\sqrt{4\,ab - c^{2}}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x^2 + c*x + a),x, algorithm="fricas")
```

```
[Out] [log((4*a*b*c - c^3 + 2*(4*a*b^2 - b*c^2)*x + (2*b^2*x^2 + 2*b*c*
x - 2*a*b + c^2)*sqrt(-4*a*b + c^2))/(b*x^2 + c*x + a))/sqrt(-4*a
*b + c^2), -2*arctan(-(2*b*x + c)/sqrt(4*a*b - c^2))/sqrt(4*a*b -
 c^2)]
```

Sympy [A] time = 0.552638, size = 124, normalized size = 3.26

$$-\sqrt{-\frac{1}{4ab-c^2}}\log\left(x+\frac{-4ab\sqrt{-\frac{1}{4ab-c^2}}+c^2\sqrt{-\frac{1}{4ab-c^2}}+c}{2b}\right)+\sqrt{-\frac{1}{4ab-c^2}}\log\left(x+\frac{4ab\sqrt{-\frac{1}{4ab-c^2}}-c^2\sqrt{-\frac{1}{4ab-c^2}}+c}{2b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+c*x+a),x)
```

```
[Out] -sqrt(-1/(4*a*b - c**2))*log(x + (-4*a*b*sqrt(-1/(4*a*b - c**2))
+ c**2*sqrt(-1/(4*a*b - c**2)) + c)/(2*b)) + sqrt(-1/(4*a*b - c**
2))*log(x + (4*a*b*sqrt(-1/(4*a*b - c**2)) - c**2*sqrt(-1/(4*a*b
- c**2)) + c)/(2*b))
```

GIAC/XCAS [A] time = 0.20853, size = 46, normalized size = 1.21

$$\frac{2\arctan\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{\sqrt{4ab-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x^2 + c*x + a),x, algorithm="giac")
```

```
[Out] 2*arctan((2*b*x + c)/sqrt(4*a*b - c^2))/sqrt(4*a*b - c^2)
```

3.89 $\int \frac{1}{b+2ax+bx^2}\,dx$

Optimal. Leaf size=35

$$-\frac{\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

```
[Out] -(ArcTanh[(a + b*x)/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2])
```

Rubi [A] time = 0.0653591, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154

$$-\frac{\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*a*x + b*x^2)^(-1),x]
```

```
[Out] -(ArcTanh[(a + b*x)/Sqrt[a^2 - b^2]]/Sqrt[a^2 - b^2])
```

Rubi in Sympy [A] time = 7.51656, size = 27, normalized size = 0.77

$$-\frac{\operatorname{atanh}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+2*a*x+b),x)
```

```
[Out] -atanh((a + b*x)/sqrt(a**2 - b**2))/sqrt(a**2 - b**2)
```

Mathematica [A] time = 0.0159774, size = 34, normalized size = 0.97

$$\frac{\tan^{-1}\left(\frac{a+bx}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*a*x + b*x^2)^(-1),x]
```

```
[Out] ArcTan[(a + b*x)/Sqrt[-a^2 + b^2]]/Sqrt[-a^2 + b^2]
```

Maple [A] time = 0.008, size = 35, normalized size = 1.

$$1\arctan\left(\frac{2\,bx+2\,a}{2}\frac{1}{\sqrt{-a^2+b^2}}\right)\frac{1}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+2*a*x+b),x)
```

```
[Out] 1/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*x+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x^2 + 2*a*x + b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.217598, size = 1, normalized size = 0.03

$$\left[\frac{\log\left(-\frac{2\,a^3-2\,ab^2+2\,(a^2b-b^3)\,x-(b^2x^2+2\,abx+2\,a^2-b^2)\sqrt{a^2-b^2}}{bx^2+2\,ax+b}\right)}{2\sqrt{a^2-b^2}},\ \frac{\arctan\left(-\frac{\sqrt{-a^2+b^2}(bx+a)}{a^2-b^2}\right)}{\sqrt{-a^2+b^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x^2 + 2*a*x + b),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(2*a^3 - 2*a*b^2 + 2*(a^2*b - b^3)*x - (b^2*x^2 + 2*a*b
*x + 2*a^2 - b^2)*sqrt(a^2 - b^2))/(b*x^2 + 2*a*x + b))/sqrt(a^2
- b^2), arctan(-sqrt(-a^2 + b^2)*(b*x + a)/(a^2 - b^2))/sqrt(-a^2
 + b^2)]
```

Sympy [A] time = 0.57804, size = 100, normalized size = 2.86

$$\frac{\sqrt{\frac{1}{(a-b)(a+b)}}\log\left(x+\frac{-a^2\sqrt{\frac{1}{(a-b)(a+b)}}+a+b^2\sqrt{\frac{1}{(a-b)(a+b)}}}{b}\right)}{2}-\frac{\sqrt{\frac{1}{(a-b)(a+b)}}\log\left(x+\frac{a^2\sqrt{\frac{1}{(a-b)(a+b)}}+a-b^2\sqrt{\frac{1}{(a-b)(a+b)}}}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+2*a*x+b),x)
```

```
[Out] sqrt(1/((a - b)*(a + b)))*log(x + (-a**2*sqrt(1/((a - b)*(a + b))
) + a + b**2*sqrt(1/((a - b)*(a + b))))/b)/2 - sqrt(1/((a - b)*(a
 + b)))*log(x + (a**2*sqrt(1/((a - b)*(a + b))) + a - b**2*sqrt(1
/((a - b)*(a + b))))/b)/2
```

GIAC/XCAS [A] time = 0.207708, size = 41, normalized size = 1.17

$$\frac{\arctan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x^2 + 2*a*x + b),x, algorithm="giac")
```

```
[Out] arctan((b*x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2)
```

3.90 $\int \frac{1}{b+2ax-bx^2}\,dx$

Optimal. Leaf size=32

$$-\frac{\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] -(ArcTanh[(a - b*x)/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

Rubi [A] time = 0.0514027, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*a*x - b*x^2)^(-1),x]

[Out] -(ArcTanh[(a - b*x)/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

Rubi in Sympy [A] time = 7.22237, size = 27, normalized size = 0.84

$$-\frac{\operatorname{atanh}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+2*a*x+b),x)

[Out] -atanh((a - b*x)/sqrt(a**2 + b**2))/sqrt(a**2 + b**2)
```

Mathematica [A] time = 0.0184163, size = 41, normalized size = 1.28

$$-\frac{\tan^{-1}\left(\frac{bx-a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*a*x - b*x^2)^(-1),x]
```

```
[Out] -(ArcTan[(-a + b*x)/Sqrt[-a^2 - b^2]]/Sqrt[-a^2 - b^2])
```

Maple [A] time = 0.005, size = 31, normalized size = 1.

$$1 Artanh\left(\frac{2\,bx-2\,a}{2}\frac{1}{\sqrt{a^2+b^2}}\right)\frac{1}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+2*a*x+b),x)
```

```
[Out] 1/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*x-2*a)/(a^2+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(b*x^2 - 2*a*x - b),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.215828, size = 115, normalized size = 3.59

$$\frac{\log\left(-\frac{2\,a^3+2\,ab^2-2\,(a^2b+b^3)\,x-(b^2x^2-2\,abx+2\,a^2+b^2)\,\sqrt{a^2+b^2}}{bx^2-2\,ax-b}\right)}{2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(b*x^2 - 2*a*x - b),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*a^3 + 2*a*b^2 - 2*(a^2*b + b^3)*x - (b^2*x^2 - 2*a*b*
x + 2*a^2 + b^2)*sqrt(a^2 + b^2))/(b*x^2 - 2*a*x - b))/sqrt(a^2 +
```

```
b^2)
```

Sympy [A] time = 0.653943, size = 102, normalized size = 3.19

$$-\frac{\sqrt{\frac{1}{a^2+b^2}}\log\left(x+\frac{-a^2\sqrt{\frac{1}{a^2+b^2}}-a-b^2\sqrt{\frac{1}{a^2+b^2}}}{b}\right)}{2}+\frac{\sqrt{\frac{1}{a^2+b^2}}\log\left(x+\frac{a^2\sqrt{\frac{1}{a^2+b^2}}-a+b^2\sqrt{\frac{1}{a^2+b^2}}}{b}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+2*a*x+b),x)
```

```
[Out] -sqrt(1/(a**2 + b**2))*log(x + (-a**2*sqrt(1/(a**2 + b**2)) - a -
 b**2*sqrt(1/(a**2 + b**2)))/b)/2 + sqrt(1/(a**2 + b**2))*log(x +
 (a**2*sqrt(1/(a**2 + b**2)) - a + b**2*sqrt(1/(a**2 + b**2)))/b)
/2
```

GIAC/XCAS [A] time = 0.21338, size = 74, normalized size = 2.31

$$-\frac{\ln\left(\frac{\left|2\,bx-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,bx-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(-1/(b*x^2 - 2*a*x - b),x, algorithm="giac")
```

```
[Out] -1/2*ln(abs(2*b*x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*x - 2*a + 2*
sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

3.91 $\int \frac{1}{(2+4x+3x^2)^2} \, dx$

Optimal. Leaf size=43

$$\frac{3x+2}{4(3x^2+4x+2)}+\frac{3\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{4\sqrt{2}}$$

```
[Out] (2 + 3*x)/(4*(2 + 4*x + 3*x^2)) + (3*ArcTan[(2 + 3*x)/Sqrt[2]])/(
4*Sqrt[2])
```

Rubi [A] time = 0.0373894, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{3x+2}{4(3x^2+4x+2)}+\frac{3\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 4*x + 3*x^2)^(-2),x]
```

```
[Out] (2 + 3*x)/(4*(2 + 4*x + 3*x^2)) + (3*ArcTan[(2 + 3*x)/Sqrt[2]])/(
4*Sqrt[2])
```

Rubi in Sympy [A] time = 1.86408, size = 36, normalized size = 0.84

$$\frac{6x+4}{8(3x^2+4x+2)}+\frac{3\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\left(\frac{3x}{2}+1\right)\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2+4*x+2)**2,x)
```

```
[Out] (6*x + 4)/(8*(3*x**2 + 4*x + 2)) + 3*sqrt(2)*atan(sqrt(2)*(3*x/2
+ 1))/8
```

Mathematica [A] time = 0.0398516, size = 43, normalized size = 1.

$$\frac{3x+2}{4(3x^2+4x+2)}+\frac{3\tan^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 4*x + 3*x^2)^(-2),x]
```

```
[Out] (2 + 3*x)/(4*(2 + 4*x + 3*x^2)) + (3*ArcTan[(2 + 3*x)/Sqrt[2]])/(
4*Sqrt[2])
```

Maple [A] time = 0.005, size = 37, normalized size = 0.9

$$\frac{6x+4}{24x^2+32x+16}+\frac{3\sqrt{2}}{8}\arctan\left(\frac{(6x+4)\sqrt{2}}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2+4*x+2)^2,x)
```

```
[Out] 1/8*(6*x+4)/(3*x^2+4*x+2)+3/8*2^(1/2)*arctan(1/4*(6*x+4)*2^(1/2))
```

Maxima [A] time = 0.790684, size = 49, normalized size = 1.14

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right)+\frac{3x+2}{4(3x^2+4x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 4*x + 2)^(-2),x, algorithm="maxima")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 1/4*(3*x + 2)/(3*x^2
+ 4*x + 2)
```

Fricas [A] time = 0.21222, size = 68, normalized size = 1.58

$$\frac{\sqrt{2}\left(3\left(3x^2+4x+2\right)\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right)+\sqrt{2}(3x+2)\right)}{8(3x^2+4x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 4*x + 2)^(-2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(2)*(3*(3*x^2 + 4*x + 2)*arctan(1/2*sqrt(2)*(3*x + 2)) +
sqrt(2)*(3*x + 2))/(3*x^2 + 4*x + 2)
```

Sympy [A] time = 0.335114, size = 39, normalized size = 0.91

$$\frac{3x+2}{12x^2+16x+8}+\frac{3\sqrt{2}\operatorname{atan}\left(\frac{3\sqrt{2}x}{2}+\sqrt{2}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2+4*x+2)**2,x)
```

```
[Out] (3*x + 2)/(12*x**2 + 16*x + 8) + 3*sqrt(2)*atan(3*sqrt(2)*x/2 + s
qrt(2))/8
```

GIAC/XCAS [A] time = 0.207236, size = 49, normalized size = 1.14

$$\frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}(3x+2)\right)+\frac{3x+2}{4(3x^2+4x+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 4*x + 2)^(-2),x, algorithm="giac")
```

```
[Out] 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*x + 2)) + 1/4*(3*x + 2)/(3*x^2
+ 4*x + 2)
```

3.92 $\int \frac{1}{(2+4x-3x^2)^2}\,dx$

Optimal. Leaf size=43

$$-\frac{2-3x}{20\left(-3x^2+4x+2\right)}-\frac{3\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

```
[Out] -(2 - 3*x)/(20*(2 + 4*x - 3*x^2)) - (3*ArcTanh[(2 - 3*x)/Sqrt[10]
])/(20*Sqrt[10])
```

Rubi [A] time = 0.0401217, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{2-3x}{20\left(-3x^2+4x+2\right)}-\frac{3\tanh^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{20\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 4*x - 3*x^2)^(-2),x]
```

```
[Out] -(2 - 3*x)/(20*(2 + 4*x - 3*x^2)) - (3*ArcTanh[(2 - 3*x)/Sqrt[10]
])/(20*Sqrt[10])
```

Rubi in Sympy [A] time = 1.86541, size = 39, normalized size = 0.91

$$-\frac{-6x+4}{40\left(-3x^2+4x+2\right)}-\frac{3\sqrt{10}\,\operatorname{atanh}\left(\sqrt{10}\left(-\frac{3x}{10}+\frac{1}{5}\right)\right)}{200}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+4*x+2)**2,x)
```

```
[Out] -(-6*x + 4)/(40*(-3*x**2 + 4*x + 2)) - 3*sqrt(10)*atanh(sqrt(10)*
(-3*x/10 + 1/5))/200
```

Mathematica [A] time = 0.0538298, size = 62, normalized size = 1.44

$$\frac{2-3x}{20\left(3x^2-4x-2\right)}-\frac{3\log\left(-3x+\sqrt{10}+2\right)}{40\sqrt{10}}+\frac{3\log\left(3x+\sqrt{10}-2\right)}{40\sqrt{10}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 4*x - 3*x^2)^(-2),x]
```

```
[Out] (2 - 3*x)/(20*(-2 - 4*x + 3*x^2)) - (3*Log[2 + Sqrt[10] - 3*x])/(
40*Sqrt[10]) + (3*Log[-2 + Sqrt[10] + 3*x])/(40*Sqrt[10])
```

Maple [A] time = 0.003, size = 37, normalized size = 0.9

$$-\frac{6x-4}{120x^2-160x-80}+\frac{3\sqrt{10}}{200}\mathit{Artanh}\left(\frac{(6x-4)\sqrt{10}}{20}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+4*x+2)^2,x)
```

```
[Out] -1/40*(6*x-4)/(3*x^2-4*x-2)+3/200*10^(1/2)*arctanh(1/20*(6*x-4)*1
0^(1/2))
```

Maxima [A] time = 0.784907, size = 63, normalized size = 1.47

$$-\frac{3}{400}\sqrt{10}\log\left(\frac{3x-\sqrt{10}-2}{3x+\sqrt{10}-2}\right)-\frac{3x-2}{20(3x^2-4x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - 4*x - 2)^(-2),x, algorithm="maxima")
```

```
[Out] -3/400*sqrt(10)*log((3*x - sqrt(10) - 2)/(3*x + sqrt(10) - 2)) -
1/20*(3*x - 2)/(3*x^2 - 4*x - 2)
```

Fricas [A] time = 0.215183, size = 99, normalized size = 2.3

$$\frac{\sqrt{10}\left(3\left(3x^2-4x-2\right)\log\left(\frac{\sqrt{10}\left(9x^2-12x+14\right)+60x-40}{3x^2-4x-2}\right)-2\sqrt{10}(3x-2)\right)}{400\left(3x^2-4x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - 4*x - 2)^(-2),x, algorithm="fricas")
```

```
[Out] 1/400*sqrt(10)*(3*(3*x^2 - 4*x - 2)*log((sqrt(10)*(9*x^2 - 12*x +
 14) + 60*x - 40)/(3*x^2 - 4*x - 2)) - 2*sqrt(10)*(3*x - 2))/(3*x
^2 - 4*x - 2)
```

Sympy [A] time = 0.325273, size = 58, normalized size = 1.35

$$-\frac{3x-2}{60x^2-80x-40}+\frac{3\sqrt{10}\log\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{400}-\frac{3\sqrt{10}\log\left(x-\frac{\sqrt{10}}{3}-\frac{2}{3}\right)}{400}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+4*x+2)**2,x)
```

```
[Out] -(3*x - 2)/(60*x**2 - 80*x - 40) + 3*sqrt(10)*log(x - 2/3 + sqrt(
10)/3)/400 - 3*sqrt(10)*log(x - sqrt(10)/3 - 2/3)/400
```

GIAC/XCAS [A] time = 0.208425, size = 69, normalized size = 1.6

$$-\frac{3}{400}\sqrt{10}\ln\left(\frac{\left|6x-2\sqrt{10}-4\right|}{\left|6x+2\sqrt{10}-4\right|}\right)-\frac{3x-2}{20\left(3x^2-4x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - 4*x - 2)^(-2),x, algorithm="giac")
```

```
[Out] -3/400*sqrt(10)*ln(abs(6*x - 2*sqrt(10) - 4)/abs(6*x + 2*sqrt(10)
 - 4)) - 1/20*(3*x - 2)/(3*x^2 - 4*x - 2)
```

3.93 $\int \frac{1}{(2+5x+3x^2)^2}\,dx$

Optimal. Leaf size=34

$$-\frac{6x+5}{3x^2+5x+2}+6\log(x+1)-6\log(3x+2)$$

```
[Out] -((5 + 6*x)/(2 + 5*x + 3*x^2)) + 6*Log[1 + x] - 6*Log[2 + 3*x]
```

Rubi [A] time = 0.0218507, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$-\frac{6x+5}{3x^2+5x+2}+6\log(x+1)-6\log(3x+2)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 5*x + 3*x^2)^(-2),x]
```

```
[Out] -((5 + 6*x)/(2 + 5*x + 3*x^2)) + 6*Log[1 + x] - 6*Log[2 + 3*x]
```

Rubi in Sympy [A] time = 1.95132, size = 29, normalized size = 0.85

$$-\frac{6x+5}{3x^2+5x+2}+6\log\left(x+1\right)-6\log\left(3x+2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2+5*x+2)**2,x)
```

```
[Out] -(6*x + 5)/(3*x**2 + 5*x + 2) + 6*log(x + 1) - 6*log(3*x + 2)
```

Mathematica [A] time = 0.0188732, size = 33, normalized size = 0.97

$$\frac{-6x-5}{3x^2+5x+2}+6\log(x+1)-6\log(3x+2)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 5*x + 3*x^2)^(-2),x]
```

```
[Out] (-5 - 6*x)/(2 + 5*x + 3*x^2) + 6*Log[1 + x] - 6*Log[2 + 3*x]
```

Maple [A] time = 0.014, size = 32, normalized size = 0.9

$$-3\ (2+3\,x)^{-1}-6\ \ln(2+3\,x)-(1+x)^{-1}+6\ \ln(1+x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2+5*x+2)^2,x)
```

```
[Out] -3/(2+3*x)-6*ln(2+3*x)-1/(1+x)+6*ln(1+x)
```

Maxima [A] time = 0.722376, size = 46, normalized size = 1.35

$$-\frac{6\,x+5}{3\,x^2+5\,x+2}-6\ \log(3\,x+2)+6\ \log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 5*x + 2)^(-2),x, algorithm="maxima")
```

```
[Out] -(6*x + 5)/(3*x^2 + 5*x + 2) - 6*log(3*x + 2) + 6*log(x + 1)
```

Fricas [A] time = 0.217436, size = 72, normalized size = 2.12

$$-\frac{6\left(3\,x^2+5\,x+2\right)\log(3\,x+2)-6\left(3\,x^2+5\,x+2\right)\log(x+1)+6\,x+5}{3\,x^2+5\,x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 5*x + 2)^(-2),x, algorithm="fricas")
```

```
[Out] -(6*(3*x^2 + 5*x + 2)*log(3*x + 2) - 6*(3*x^2 + 5*x + 2)*log(x +
1) + 6*x + 5)/(3*x^2 + 5*x + 2)
```

Sympy [A] time = 0.312059, size = 29, normalized size = 0.85

$$-\frac{6x+5}{3x^2+5x+2}-6\log\left(x+\frac{2}{3}\right)+6\log(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2+5*x+2)**2,x)
```

```
[Out] -(6*x + 5)/(3*x**2 + 5*x + 2) - 6*log(x + 2/3) + 6*log(x + 1)
```

GIAC/XCAS [A] time = 0.209539, size = 49, normalized size = 1.44

$$-\frac{6x+5}{3x^2+5x+2} - 6\ln\left(|3x+2|\right) + 6\ln\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 5*x + 2)^(-2),x, algorithm="giac")
```

```
[Out] -(6*x + 5)/(3*x^2 + 5*x + 2) - 6*ln(abs(3*x + 2)) + 6*ln(abs(x +
1))
```

3.94 $\int \frac{1}{(2+5x-3x^2)^2}\,dx$

Optimal. Leaf size=42

$$-\frac{5-6x}{49\left(-3x^2+5x+2\right)}-\frac{6}{343}\log(2-x)+\frac{6}{343}\log(3x+1)$$

```
[Out] -(5 - 6*x)/(49*(2 + 5*x - 3*x^2)) - (6*Log[2 - x])/343 + (6*Log[1
 + 3*x])/343
```

Rubi [A] time = 0.0260591, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{5-6x}{49\left(-3x^2+5x+2\right)}-\frac{6}{343}\log(2-x)+\frac{6}{343}\log(3x+1)$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 5*x - 3*x^2)^(-2),x]
```

```
[Out] -(5 - 6*x)/(49*(2 + 5*x - 3*x^2)) - (6*Log[2 - x])/343 + (6*Log[1
 + 3*x])/343
```

Rubi in Sympy [A] time = 2.1557, size = 32, normalized size = 0.76

$$-\frac{-6x+5}{49\left(-3x^2+5x+2\right)}-\frac{6\log\left(-x+2\right)}{343}+\frac{6\log\left(3x+1\right)}{343}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+5*x+2)**2,x)
```

```
[Out] -(-6*x + 5)/(49*(-3*x**2 + 5*x + 2)) - 6*log(-x + 2)/343 + 6*log(
3*x + 1)/343
```

Mathematica [A] time = 0.0231421, size = 42, normalized size = 1.

$$\frac{5-6x}{49\left(3x^2-5x-2\right)}-\frac{6}{343}\log(2-x)+\frac{6}{343}\log(3x+1)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 5*x - 3*x^2)^(-2),x]
```

```
[Out] (5 - 6*x)/(49*(-2 - 5*x + 3*x^2)) - (6*Log[2 - x])/343 + (6*Log[1
 + 3*x])/343
```

Maple [A] time = 0.013, size = 32, normalized size = 0.8

$$-\frac{3}{147\,x+49}+\frac{6\,\ln\left(3\,x+1\right)}{343}-\frac{1}{49\,x-98}-\frac{6\,\ln\left(x-2\right)}{343}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+5*x+2)^2,x)
```

```
[Out] -3/49/(3*x+1)+6/343*ln(3*x+1)-1/49/(x-2)-6/343*ln(x-2)
```

Maxima [A] time = 0.715969, size = 46, normalized size = 1.1

$$-\frac{6\,x-5}{49\,\left(3\,x^2-5\,x-2\right)}+\frac{6}{343}\,\log\left(3\,x+1\right)-\frac{6}{343}\,\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - 5*x - 2)^(-2),x, algorithm="maxima")
```

```
[Out] -1/49*(6*x - 5)/(3*x^2 - 5*x - 2) + 6/343*log(3*x + 1) - 6/343*lo
g(x - 2)
```

Fricas [A] time = 0.21972, size = 72, normalized size = 1.71

$$\frac{6\,\left(3\,x^2-5\,x-2\right)\,\log\left(3\,x+1\right)-6\,\left(3\,x^2-5\,x-2\right)\,\log\left(x-2\right)-42\,x+35}{343\,\left(3\,x^2-5\,x-2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - 5*x - 2)^(-2),x, algorithm="fricas")
```

```
[Out] 1/343*(6*(3*x^2 - 5*x - 2)*log(3*x + 1) - 6*(3*x^2 - 5*x - 2)*log
(x - 2) - 42*x + 35)/(3*x^2 - 5*x - 2)
```

Sympy [A] time = 0.324176, size = 32, normalized size = 0.76

$$-\frac{6x-5}{147x^2-245x-98}-\frac{6\log(x-2)}{343}+\frac{6\log\left(x+\frac{1}{3}\right)}{343}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+5*x+2)**2,x)
```

```
[Out] -(6*x - 5)/(147*x**2 - 245*x - 98) - 6*log(x - 2)/343 + 6*log(x +
 1/3)/343
```

GIAC/XCAS [A] time = 0.209558, size = 49, normalized size = 1.17

$$-\frac{6\,x-5}{49\,(3\,x^2-5\,x-2)}+\frac{6}{343}\ln\left(|3\,x+1|\right)-\frac{6}{343}\ln\left(|x-2|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 - 5*x - 2)^(-2),x, algorithm="giac")
```

```
[Out] -1/49*(6*x - 5)/(3*x^2 - 5*x - 2) + 6/343*ln(abs(3*x + 1)) - 6/34
3*ln(abs(x - 2))
```

3.95 $\int \frac{1}{(a+cx+bx^2)^2}\,dx$

Optimal. Leaf size=71

$$\frac{2bx+c}{(4ab-c^2)(a+bx^2+cx)}+\frac{4b\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{3/2}}$$

```
[Out] (c + 2*b*x)/((4*a*b - c^2)*(a + c*x + b*x^2)) + (4*b*ArcTan[(c +
2*b*x)/Sqrt[4*a*b - c^2]])/(4*a*b - c^2)^(3/2)
```

Rubi [A] time = 0.082331, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25

$$\frac{2bx+c}{(4ab-c^2)(a+bx^2+cx)}+\frac{4b\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + c*x + b*x^2)^(-2),x]
```

```
[Out] (c + 2*b*x)/((4*a*b - c^2)*(a + c*x + b*x^2)) + (4*b*ArcTan[(c +
2*b*x)/Sqrt[4*a*b - c^2]])/(4*a*b - c^2)^(3/2)
```

Rubi in Sympy [A] time = 7.5301, size = 60, normalized size = 0.85

$$\frac{4b\,\operatorname{atanh}\left(\frac{2bx+c}{\sqrt{-4ab+c^2}}\right)}{(-4ab+c^2)^{\frac{3}{2}}}-\frac{2bx+c}{(-4ab+c^2)(a+bx^2+cx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+c*x+a)**2,x)
```

```
[Out] 4*b*atanh((2*b*x + c)/sqrt(-4*a*b + c**2))/(-4*a*b + c**2)**(3/2)
 - (2*b*x + c)/((-4*a*b + c**2)*(a + b*x**2 + c*x))
```

Mathematica [A] time = 0.104929, size = 70, normalized size = 0.99

$$\frac{2bx+c}{(4ab-c^2)(a+x(bx+c))}+\frac{4b\tan^{-1}\left(\frac{2bx+c}{\sqrt{4ab-c^2}}\right)}{(4ab-c^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + c*x + b*x^2)^(-2),x]
```

```
[Out] (c + 2*b*x)/((4*a*b - c^2)*(a + x*(c + b*x))) + (4*b*ArcTan[(c +
2*b*x)/Sqrt[4*a*b - c^2]])/(4*a*b - c^2)^(3/2)
```

Maple [A] time = 0.005, size = 68, normalized size = 1.

$$\frac{2\,bx+c}{(4\,ab-c^2)(bx^2+cx+a)}+4\,\frac{b}{(4\,ab-c^2)^{3/2}}\arctan\left(\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+c*x+a)^2,x)
```

```
[Out] (2*b*x+c)/(4*a*b-c^2)/(b*x^2+c*x+a)+4*b*arctan((2*b*x+c)/(4*a*b-c
^2)^(1/2))/(4*a*b-c^2)^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + c*x + a)^(-2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.232097, size = 1, normalized size = 0.01

$$\left[-\frac{2\left(b^2x^2+bcx+ab\right)\log\left(-\frac{4\,abc-c^3+2\left(4\,ab^2-bc^2\right)x-\left(2\,b^2x^2+2\,bcx-2\,ab+c^2\right)\sqrt{-4\,ab+c^2}}{bx^2+cx+a}\right)-\sqrt{-4\,ab+c^2}(2\,bx+c)}{(4\,a^2b-ac^2+(4\,ab^2-bc^2)x^2+(4\,abc-c^3)x)\sqrt{-4\,ab+c^2}},\right.$$
$$\left.-\frac{4\left(b^2x^2+bcx+ab\right)\arctan\left(-\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)-\sqrt{4\,ab-c^2}(2\,bx+c)}{(4\,a^2b-ac^2+(4\,ab^2-bc^2)x^2+(4\,abc-c^3)x)\sqrt{4\,ab-c^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + c*x + a)^(-2),x, algorithm="fricas")
```

```
[Out] [-(2*(b^2*x^2 + b*c*x + a*b)*log(-(4*a*b*c - c^3 + 2*(4*a*b^2 - b
*c^2)*x - (2*b^2*x^2 + 2*b*c*x - 2*a*b + c^2)*sqrt(-4*a*b + c^2))
/(b*x^2 + c*x + a)) - sqrt(-4*a*b + c^2)*(2*b*x + c))/((4*a^2*b -
 a*c^2 + (4*a*b^2 - b*c^2)*x^2 + (4*a*b*c - c^3)*x)*sqrt(-4*a*b +
 c^2)), -(4*(b^2*x^2 + b*c*x + a*b)*arctan(-(2*b*x + c)/sqrt(4*a*
b - c^2)) - sqrt(4*a*b - c^2)*(2*b*x + c))/((4*a^2*b - a*c^2 + (4
*a*b^2 - b*c^2)*x^2 + (4*a*b*c - c^3)*x)*sqrt(4*a*b - c^2))]
```

Sympy [A] time = 2.78101, size = 265, normalized size = 3.73

$$-2b\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}\log\left(x+\frac{-32a^2b^3\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}+16ab^2c^2\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}-2bc^4\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}+2bc}{4b^2}\right)$$
$$+2b\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}\log\left(x+\frac{32a^2b^3\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}-16ab^2c^2\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}+2bc^4\sqrt{-\frac{1}{\left(4ab-c^2\right)^3}}+2bc}{4b^2}\right)$$
$$+\frac{2bx+c}{4a^2b-ac^2+x^2\left(4ab^2-bc^2\right)+x\left(4abc-c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+c*x+a)**2,x)
```

```
[Out] -2*b*sqrt(-1/(4*a*b - c**2)**3)*log(x + (-32*a**2*b**3*sqrt(-1/(4
*a*b - c**2)**3) + 16*a*b**2*c**2*sqrt(-1/(4*a*b - c**2)**3) - 2*
b*c**4*sqrt(-1/(4*a*b - c**2)**3) + 2*b*c)/(4*b**2)) + 2*b*sqrt(-
1/(4*a*b - c**2)**3)*log(x + (32*a**2*b**3*sqrt(-1/(4*a*b - c**2)
**3) - 16*a*b**2*c**2*sqrt(-1/(4*a*b - c**2)**3) + 2*b*c**4*sqrt(
-1/(4*a*b - c**2)**3) + 2*b*c)/(4*b**2)) + (2*b*x + c)/(4*a**2*b
```

```
- a*c**2 + x**2*(4*a*b**2 - b*c**2) + x*(4*a*b*c - c**3))
```

GIAC/XCAS [A] time = 0.207574, size = 90, normalized size = 1.27

$$\frac{4\,b\arctan\left(\frac{2\,bx+c}{\sqrt{4\,ab-c^2}}\right)}{(4\,ab-c^2)^{\frac{3}{2}}}+\frac{2\,bx+c}{(bx^2+cx+a)(4\,ab-c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + c*x + a)^(-2),x, algorithm="giac")
```

```
[Out] 4*b*arctan((2*b*x + c)/sqrt(4*a*b - c^2))/(4*a*b - c^2)^(3/2) + (
2*b*x + c)/((b*x^2 + c*x + a)*(4*a*b - c^2))
```

3.96 $\int \frac{1}{(b+2ax+bx^2)^2}\,dx$

Optimal. Leaf size=72

$$\frac{b\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}-\frac{a+bx}{2\left(a^2-b^2\right)\left(2ax+bx^2+b\right)}$$

```
[Out] -(a + b*x)/(2*(a^2 - b^2)*(b + 2*a*x + b*x^2)) + (b*ArcTanh[(a +
b*x)/Sqrt[a^2 - b^2]])/(2*(a^2 - b^2)^(3/2))
```

Rubi [A] time = 0.0962778, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231

$$\frac{b\tanh^{-1}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{3/2}}-\frac{a+bx}{2\left(a^2-b^2\right)\left(2ax+bx^2+b\right)}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*a*x + b*x^2)^(-2),x]
```

```
[Out] -(a + b*x)/(2*(a^2 - b^2)*(b + 2*a*x + b*x^2)) + (b*ArcTanh[(a +
b*x)/Sqrt[a^2 - b^2]])/(2*(a^2 - b^2)^(3/2))
```

Rubi in Sympy [A] time = 11.5646, size = 58, normalized size = 0.81

$$\frac{b\,\mathrm{atanh}\left(\frac{a+bx}{\sqrt{a^2-b^2}}\right)}{2\left(a^2-b^2\right)^{\frac{3}{2}}}-\frac{2a+2bx}{4\left(a^2-b^2\right)\left(2ax+bx^2+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(b*x**2+2*a*x+b)**2,x)
```

```
[Out] b*atanh((a + b*x)/sqrt(a**2 - b**2))/(2*(a**2 - b**2)**(3/2)) - (
2*a + 2*b*x)/(4*(a**2 - b**2)*(2*a*x + b*x**2 + b))
```

Mathematica [A] time = 0.0715335, size = 72, normalized size = 1.

$$\frac{a+bx}{2\left(b^2-a^2\right)\left(2ax+bx^2+b\right)}+\frac{b\tan^{-1}\left(\frac{a+bx}{\sqrt{b^2-a^2}}\right)}{2\left(b^2-a^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*a*x + b*x^2)^(-2),x]
```

```
[Out] (a + b*x)/(2*(-a^2 + b^2)*(b + 2*a*x + b*x^2)) + (b*ArcTan[(a + b
*x)/Sqrt[-a^2 + b^2]])/(2*(-a^2 + b^2)^(3/2))
```

Maple [A] time = 0.004, size = 86, normalized size = 1.2

$$\frac{2\,bx+2\,a}{\left(-4\,a^2+4\,b^2\right)\left(bx^2+2\,ax+b\right)}+2\,\frac{b}{\left(-4\,a^2+4\,b^2\right)\sqrt{-a^2+b^2}}\arctan\left(1/2\,\frac{2\,bx+2\,a}{\sqrt{-a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(b*x^2+2*a*x+b)^2,x)
```

```
[Out] (2*b*x+2*a)/(-4*a^2+4*b^2)/(b*x^2+2*a*x+b)+2*b/(-4*a^2+4*b^2)/(-a
^2+b^2)^(1/2)*arctan(1/2*(2*b*x+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 2*a*x + b)^(-2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.224718, size = 1, normalized size = 0.01

$$\left[-\frac{\left(b^2x^2+2\,abx+b^2\right)\log\left(-\frac{2\,a^3-2\,ab^2+2\,\left(a^2b-b^3\right)x-\left(b^2x^2+2\,abx+2\,a^2-b^2\right)\sqrt{a^2-b^2}}{bx^2+2\,ax+b}\right)+2\,\sqrt{a^2-b^2}(bx+a)}{4\,(a^2b-b^3+(a^2b-b^3)x^2+2\,(a^3-ab^2)x)\sqrt{a^2-b^2}},\right.$$
$$\left.-\frac{\left(b^2x^2+2\,abx+b^2\right)\arctan\left(-\frac{\sqrt{-a^2+b^2}(bx+a)}{a^2-b^2}\right)+\sqrt{-a^2+b^2}(bx+a)}{2\,(a^2b-b^3+(a^2b-b^3)x^2+2\,(a^3-ab^2)x)\sqrt{-a^2+b^2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 2*a*x + b)^(-2),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2*x^2 + 2*a*b*x + b^2)*log(-(2*a^3 - 2*a*b^2 + 2*(a^2*b
 - b^3)*x - (b^2*x^2 + 2*a*b*x + 2*a^2 - b^2)*sqrt(a^2 - b^2))/(b
*x^2 + 2*a*x + b)) + 2*sqrt(a^2 - b^2)*(b*x + a))/((a^2*b - b^3 +
 (a^2*b - b^3)*x^2 + 2*(a^3 - a*b^2)*x)*sqrt(a^2 - b^2)), -1/2*((
b^2*x^2 + 2*a*b*x + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*x + a)/(a^2
- b^2)) + sqrt(-a^2 + b^2)*(b*x + a))/((a^2*b - b^3 + (a^2*b - b^
3)*x^2 + 2*(a^3 - a*b^2)*x)*sqrt(-a^2 + b^2))]
```

Sympy [A] time = 2.58932, size = 228, normalized size = 3.17

$$-\frac{b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}\log\left(x+\frac{-a^4b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}+2a^2b^3\sqrt{\frac{1}{(a-b)^3(a+b)^3}}+ab-b^5\sqrt{\frac{1}{(a-b)^3(a+b)^3}}}{b^2}\right)}{4}$$
$$+\frac{b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}\log\left(x+\frac{a^4b\sqrt{\frac{1}{(a-b)^3(a+b)^3}}-2a^2b^3\sqrt{\frac{1}{(a-b)^3(a+b)^3}}+ab+b^5\sqrt{\frac{1}{(a-b)^3(a+b)^3}}}{b^2}\right)}{4}$$
$$-\frac{a+bx}{2a^2b-2b^3+x^2\left(2a^2b-2b^3\right)+x\left(4a^3-4ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(b*x**2+2*a*x+b)**2,x)
```

```
[Out] -b*sqrt(1/((a - b)**3*(a + b)**3))*log(x + (-a**4*b*sqrt(1/((a -
b)**3*(a + b)**3)) + 2*a**2*b**3*sqrt(1/((a - b)**3*(a + b)**3))
+ a*b - b**5*sqrt(1/((a - b)**3*(a + b)**3)))/b**2)/4 + b*sqrt(1/
((a - b)**3*(a + b)**3))*log(x + (a**4*b*sqrt(1/((a - b)**3*(a +
b)**3)) - 2*a**2*b**3*sqrt(1/((a - b)**3*(a + b)**3)) + a*b + b**
5*sqrt(1/((a - b)**3*(a + b)**3)))/b**2)/4 - (a + b*x)/(2*a**2*b
```

```
- 2*b**3 + x**2*(2*a**2*b - 2*b**3) + x*(4*a**3 - 4*a*b**2))
```

GIAC/XCAS [A] time = 0.209048, size = 101, normalized size = 1.4

$$-\frac{b\arctan\left(\frac{bx+a}{\sqrt{-a^2+b^2}}\right)}{2\,(a^2-b^2)\sqrt{-a^2+b^2}}-\frac{bx+a}{2\,(bx^2+2\,ax+b)(a^2-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 + 2*a*x + b)^(-2),x, algorithm="giac")
```

```
[Out] -1/2*b*arctan((b*x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2
+ b^2)) - 1/2*(b*x + a)/((b*x^2 + 2*a*x + b)*(a^2 - b^2))
```

3.97 $\int \frac{1}{(b+2ax-bx^2)^2} dx$

Optimal. Leaf size=69

$$-\frac{a-bx}{2(a^2+b^2)(2ax-bx^2+b)}-\frac{b\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}$$

```
[Out] -(a - b*x)/(2*(a^2 + b^2)*(b + 2*a*x - b*x^2)) - (b*ArcTanh[(a - 
b*x)/Sqrt[a^2 + b^2]])/(2*(a^2 + b^2)^(3/2))
```

Rubi [A] time = 0.0699646, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$-\frac{a-bx}{2(a^2+b^2)(2ax-bx^2+b)}-\frac{b\tanh^{-1}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(b + 2*a*x - b*x^2)^(-2),x]
```

```
[Out] -(a - b*x)/(2*(a^2 + b^2)*(b + 2*a*x - b*x^2)) - (b*ArcTanh[(a - 
b*x)/Sqrt[a^2 + b^2]])/(2*(a^2 + b^2)^(3/2))
```

Rubi in Sympy [A] time = 9.55234, size = 60, normalized size = 0.87

$$-\frac{b\,\text{atanh}\left(\frac{a-bx}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{\frac{3}{2}}}-\frac{2a-2bx}{4(a^2+b^2)(2ax-bx^2+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-b*x**2+2*a*x+b)**2,x)
```

```
[Out] -b*atanh((a - b*x)/sqrt(a**2 + b**2))/(2*(a**2 + b**2)**(3/2)) - 
(2*a - 2*b*x)/(4*(a**2 + b**2)*(2*a*x - b*x**2 + b))
```

Mathematica [A] time = 0.110337, size = 78, normalized size = 1.13

$$\frac{\frac{bx-a}{2ax-bx^2+b}-\frac{b\tan^{-1}\left(\frac{bx-a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}}}{2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(b + 2*a*x - b*x^2)^(-2),x]
```

```
[Out] ((-a + b*x)/(b + 2*a*x - b*x^2) - (b*ArcTan[(-a + b*x)/Sqrt[-a^2
- b^2]])/Sqrt[-a^2 - b^2])/(2*(a^2 + b^2))
```

Maple [A] time = 0.004, size = 84, normalized size = 1.2

$$\frac{2\,bx-2\,a}{\left(-4\,a^2-4\,b^2\right)\left(bx^2-2\,ax-b\right)}-2\,\frac{b}{\left(-4\,a^2-4\,b^2\right)\sqrt{a^2+b^2}}\mathit{Artanh}\left(1/2\,\frac{2\,bx-2\,a}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-b*x^2+2*a*x+b)^2,x)
```

```
[Out] (2*b*x-2*a)/(-4*a^2-4*b^2)/(b*x^2-2*a*x-b)-2*b/(-4*a^2-4*b^2)/(a^
2+b^2)^(1/2)*arctanh(1/2*(2*b*x-2*a)/(a^2+b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 - 2*a*x - b)^(-2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.237248, size = 216, normalized size = 3.13

$$-\frac{\left(b^2x^2-2\,abx-b^2\right)\log\left(-\frac{2\,a^3+2\,ab^2-2\,\left(a^2b+b^3\right)x-\left(b^2x^2-2\,abx+2\,a^2+b^2\right)\sqrt{a^2+b^2}}{bx^2-2\,ax-b}\right)-2\,\sqrt{a^2+b^2}(bx-a)}{4\left(a^2b+b^3-(a^2b+b^3)x^2+2\,(a^3+ab^2)x\right)\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 - 2*a*x - b)^(-2),x, algorithm="fricas")
```

```
[Out] -1/4*((b^2*x^2 - 2*a*b*x - b^2)*log(-(2*a^3 + 2*a*b^2 - 2*(a^2*b
+ b^3)*x - (b^2*x^2 - 2*a*b*x + 2*a^2 + b^2)*sqrt(a^2 + b^2))/(b*
x^2 - 2*a*x - b)) - 2*sqrt(a^2 + b^2)*(b*x - a))/((a^2*b + b^3 -
(a^2*b + b^3)*x^2 + 2*(a^3 + a*b^2)*x)*sqrt(a^2 + b^2))
```

Sympy [A] time = 2.67383, size = 218, normalized size = 3.16

$$-\frac{b\sqrt{\frac{1}{(a^2+b^2)^3}}\log\left(x+\frac{-a^4b\sqrt{\frac{1}{(a^2+b^2)^3}}-2a^2b^3\sqrt{\frac{1}{(a^2+b^2)^3}}-ab-b^5\sqrt{\frac{1}{(a^2+b^2)^3}}}{b^2}\right)}{4}+\frac{b\sqrt{\frac{1}{(a^2+b^2)^3}}\log\left(x+\frac{a^4b\sqrt{\frac{1}{(a^2+b^2)^3}}+2a^2b^3\sqrt{\frac{1}{(a^2+b^2)^3}}-ab+b^5\sqrt{\frac{1}{(a^2+b^2)^3}}}{b^2}\right)}{4}-\frac{-a+bx}{-2a^2b-2b^3+x^2(2a^2b+2b^3)+x(-4a^3-4ab^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-b*x**2+2*a*x+b)**2,x)
```

```
[Out] -b*sqrt((a**2 + b**2)**(-3))*log(x + (-a**4*b*sqrt((a**2 + b**2)*
*(-3)) - 2*a**2*b**3*sqrt((a**2 + b**2)**(-3)) - a*b - b**5*sqrt(
(a**2 + b**2)**(-3)))/b**2)/4 + b*sqrt((a**2 + b**2)**(-3))*log(x
 + (a**4*b*sqrt((a**2 + b**2)**(-3)) + 2*a**2*b**3*sqrt((a**2 + b
**2)**(-3)) - a*b + b**5*sqrt((a**2 + b**2)**(-3)))/b**2)/4 - (-a
 + b*x)/(-2*a**2*b - 2*b**3 + x**2*(2*a**2*b + 2*b**3) + x*(-4*a*
*3 - 4*a*b**2))
```

GIAC/XCAS [A] time = 0.21435, size = 122, normalized size = 1.77

$$-\frac{b\ln\left(\frac{\left|2\,bx-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,bx-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{4\,(a^2+b^2)^{\frac{3}{2}}}-\frac{bx-a}{2\,(bx^2-2\,ax-b)(a^2+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((b*x^2 - 2*a*x - b)^(-2),x, algorithm="giac")
```

```
[Out] -1/4*b*ln(abs(2*b*x - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*x - 2*a +
2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 1/2*(b*x - a)/((b*x^2 - 2
*a*x - b)*(a^2 + b^2))
```

3.98 $\int \frac{1}{\left(\frac{a}{b}\right)^{2/n}+x^2-2\left(\frac{a}{b}\right)^{\frac{1}{n}}x\cos\left(\frac{\pi-2k\pi}{n}\right)}\,dx$

Optimal. Leaf size=62

$$-\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\tan^{-1}\left(\cot\left(\frac{\pi-2\pi k}{n}\right)-x\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\right)$$

```
[Out] -((ArcTan[Cot[(Pi - 2*k*Pi)/n] - (x*Csc[(Pi - 2*k*Pi)/n])/(a/b)^n
^(-1)]*Csc[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1))
```

Rubi [A] time = 0.267981, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.05

$$-\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\tan^{-1}\left(\cot\left(\frac{\pi-2\pi k}{n}\right)-x\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[((a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2*k*Pi)/n])^(-
1),x]
```

```
[Out] -((ArcTan[Cot[(Pi - 2*k*Pi)/n] - (x*Csc[(Pi - 2*k*Pi)/n])/(a/b)^n
^(-1)]*Csc[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1))
```

Rubi in Sympy [A] time = 80.5349, size = 94, normalized size = 1.52

$$\frac{\left(\frac{a}{b}\right)^{-\frac{1}{n}}\operatorname{atan}\left(\frac{\left(\frac{a}{b}\right)^{-\frac{1}{n}}\left(x-\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{\pi(2k-1)}{n}\right)\right)}{\sqrt{-\cos\left(\frac{\pi(2k-1)}{n}\right)+1}\sqrt{\cos\left(\frac{\pi(2k-1)}{n}\right)+1}}\right)}{\sqrt{-\cos\left(\frac{\pi(2k-1)}{n}\right)+1}\sqrt{\cos\left(\frac{\pi(2k-1)}{n}\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/((a/b)**(2/n)+x**2-2*(a/b)**(1/n)*x*cos((-2*pi*k+pi)/n)),x)
```

```
[Out] (a/b)**(-1/n)*atan((a/b)**(-1/n)*(x - (a/b)**(1/n)*cos(pi*(2*k -
1)/n))/(sqrt(-cos(pi*(2*k - 1)/n) + 1)*sqrt(cos(pi*(2*k - 1)/n) +
 1)))/(sqrt(-cos(pi*(2*k - 1)/n) + 1)*sqrt(cos(pi*(2*k - 1)/n) +
```

```
1))
```

Mathematica [A] time = 0.154134, size = 65, normalized size = 1.05

$$\left(\frac{a}{b}\right)^{-1/n}\csc\left(\frac{\pi-2\pi k}{n}\right)\tan^{-1}\left(\frac{\tan\left(\frac{\pi-2\pi k}{2n}\right)\left(\left(\frac{a}{b}\right)^{\frac{1}{n}}+x\right)}{\left(\frac{a}{b}\right)^{\frac{1}{n}}-x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a/b)^(2/n) + x^2 - 2*(a/b)^n^(-1)*x*Cos[(Pi - 2*k*Pi)/n])^(-
1),x]
```

```
[Out] (ArcTan[(((a/b)^n^(-1) + x)*Tan[(Pi - 2*k*Pi)/(2*n)])/((a/b)^n^(-
1) - x)]*Csc[(Pi - 2*k*Pi)/n])/(a/b)^n^(-1)
```

Maple [A] time = 0.044, size = 111, normalized size = 1.8

$$1\arctan\left(\frac{1}{2}\left(2x-2\sqrt[n]{\frac{a}{b}}\cos\left(\frac{(2k-1)\pi}{n}\right)\right)\frac{1}{\sqrt{-\left(\sqrt[n]{\frac{a}{b}}\right)^{2}\left(\cos\left(\frac{(2k-1)\pi}{n}\right)\right)^{2}+\left(\frac{a}{b}\right)^{2n^{-1}}}}\right)\frac{1}{\sqrt{-\left(\sqrt[n]{\frac{a}{b}}\right)^{2}\left(\cos\left(\frac{(2k-1)\pi}{n}\right)\right)^{2}+\left(\frac{a}{b}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a/b)^(2/n)+x^2-2*(a/b)^(1/n)*x*cos((-2*Pi*k+Pi)/n)),x)
```

```
[Out] 1/(-((a/b)^(1/n))^2*cos(Pi*(2*k-1)/n)^2+(a/b)^(2/n))^(1/2)*arctan
(1/2*(2*x-2*(a/b)^(1/n)*cos(Pi*(2*k-1)/n))/(-((a/b)^(1/n))^2*cos(
Pi*(2*k-1)/n)^2+(a/b)^(2/n))^(1/2))
```

Maxima [A] time = 0.828108, size = 215, normalized size = 3.47

$$\frac{\left(\frac{a}{b}\right)^{-\frac{1}{n}}\log\left(\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+\sqrt{\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)^{2}-1}\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}-x}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)-\sqrt{\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)^{2}-1}\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}-x}\right)}{2\sqrt{\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)^{2}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(2*x*(a/b)^(1/n)*cos(-2*pi*k/n + pi/n) - x^2 - (a/b)^(2/n)),x, algo
```

```
[Out] 1/2*(a/b)^(-1/n)*log(((a/b)^(1/n)*cos(2*pi*k/n - pi/n) + sqrt(cos
(2*pi*k/n - pi/n)^2 - 1)*(a/b)^(1/n) - x)/((a/b)^(1/n)*cos(2*pi*k
/n - pi/n) - sqrt(cos(2*pi*k/n - pi/n)^2 - 1)*(a/b)^(1/n) - x))/s
qrt(cos(2*pi*k/n - pi/n)^2 - 1)
```

Fricas [A] time = 0.243957, size = 120, normalized size = 1.94

$$-\frac{\arctan\left(\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)-x}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\sin\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)}\right)}{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\sin\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(2*x*(a/b)^(1/n)*cos(-2*pi*k/n + pi/n) - x^2 - (a/b)^(2/n)),x, algo
```

```
[Out] -arctan(((a/b)^(1/n)*cos(2*pi*k/n - pi/n) - x)/((a/b)^(1/n)*sin(2
*pi*k/n - pi/n)))/((a/b)^(1/n)*sin(2*pi*k/n - pi/n))
```

Sympy [A] time = 2.99014, size = 212, normalized size = 3.42

$$-\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^2\left(\frac{\pi(2k-1)}{n}\right)-1}}\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)-\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^2\left(\frac{\pi(2k-1)}{n}\right)-1}}\left(-2\left(\frac{a}{b}\right)^{\frac{2}{n}}\cos^2\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+2\left(\frac{a}{b}\right)^{\frac{2}{n}}\right)}{2}\right)}{2}+\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^2\left(\frac{\pi(2k-1)}{n}\right)-1}}\log\left(x-\left(\frac{a}{b}\right)^{\frac{1}{n}}\cos\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+\frac{\sqrt{\frac{\left(\frac{a}{b}\right)^{-\frac{2}{n}}}{\cos^2\left(\frac{\pi(2k-1)}{n}\right)-1}}\left(-2\left(\frac{a}{b}\right)^{\frac{2}{n}}\cos^2\left(\frac{2\pi k}{n}-\frac{\pi}{n}\right)+2\left(\frac{a}{b}\right)^{\frac{2}{n}}\right)}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((a/b)**(2/n)+x**2-2*(a/b)**(1/n)*x*cos((-2*pi*k+pi)/n)),x)
```

```
[Out] -sqrt((a/b)**(-2/n)/(cos(pi*(2*k - 1)/n)**2 - 1))*log(x - (a/b)**
(1/n)*cos(2*pi*k/n - pi/n) - sqrt((a/b)**(-2/n)/(cos(pi*(2*k - 1)
/n)**2 - 1))*(-2*(a/b)**(2/n)*cos(2*pi*k/n - pi/n)**2 + 2*(a/b)**
(2/n))/2)/2 + sqrt((a/b)**(-2/n)/(cos(pi*(2*k - 1)/n)**2 - 1))*lo
g(x - (a/b)**(1/n)*cos(2*pi*k/n - pi/n) + sqrt((a/b)**(-2/n)/(cos
(pi*(2*k - 1)/n)**2 - 1))*(-2*(a/b)**(2/n)*cos(2*pi*k/n - pi/n)**
2 + 2*(a/b)**(2/n))/2)/2
```

GIAC/XCAS [A] time = 0.222173, size = 135, normalized size = 2.18

$$\frac{\arctan\left(-\frac{\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}\cos\left(-\frac{2\,\pi k}{n}+\frac{\pi}{n}\right)-x}{\sqrt{-\cos\left(\frac{2\,\pi k}{n}-\frac{\pi}{n}\right)^{2}+1}\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}}\right)}{\sqrt{-\cos\left(\frac{2\,\pi k}{n}-\frac{\pi}{n}\right)^{2}+1}\left(\frac{a}{b}\right)^{\left(\frac{1}{n}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(2*x*(a/b)^(1/n)*cos(-2*pi*k/n + pi/n) - x^2 - (a/b)^(2/n)),x, algo
```

```
[Out] arctan(-((a/b)^(1/n)*cos(-2*pi*k/n + pi/n) - x)/(sqrt(-cos(2*pi*k
/n - pi/n)^2 + 1)*(a/b)^(1/n)))/(sqrt(-cos(2*pi*k/n - pi/n)^2 + 1
)*(a/b)^(1/n))
```

3.99 $\int \frac{1}{ab+\sqrt{b^2-4ab^3}x-b^2x^2}\,dx$

Optimal. Leaf size=33

$$\frac{2\tanh^{-1}\left(\frac{2b^2x-\sqrt{b^2-4ab^3}}{b}\right)}{b}$$

```
[Out] (2*ArcTanh[(-Sqrt[b^2 - 4*a*b^3] + 2*b^2*x)/b])/b
```

Rubi [A] time = 0.0736025, antiderivative size = 58, normalized size of antiderivative = 1.76, number of steps used = 3, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$\frac{\log\left(-\sqrt{b^2-4ab^3}+2b^2x+b\right)}{b}-\frac{\log\left(\sqrt{b^2-4ab^3}-2b^2x+b\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*b + Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

```
[Out] -(Log[b + Sqrt[b^2 - 4*a*b^3] - 2*b^2*x]/b) + Log[b - Sqrt[b^2 -
4*a*b^3] + 2*b^2*x]/b
```

Rubi in Sympy [A] time = 4.47441, size = 49, normalized size = 1.48

$$-\frac{\log\left(-2b^2x+b+\sqrt{b^2\left(-4ab+1\right)}\right)}{b}+\frac{\log\left(2b^2x+b-\sqrt{b^2\left(-4ab+1\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a*b-b**2*x**2+x*(-4*a*b**3+b**2)**(1/2)),x)
```

```
[Out] -log(-2*b**2*x + b + sqrt(b**2*(-4*a*b + 1)))/b + log(2*b**2*x +
b - sqrt(b**2*(-4*a*b + 1)))/b
```

Mathematica [A] time = 0.0563528, size = 34, normalized size = 1.03

$$\frac{2\tanh^{-1}\left(\frac{2b^2x-\sqrt{-b^2(4ab-1)}}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*b + Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

```
[Out] (2*ArcTanh[(-Sqrt[-(b^2*(-1 + 4*a*b))] + 2*b^2*x)/b])/b
```

Maple [A] time = 0.013, size = 31, normalized size = 0.9

$$-2\frac{1}{b}Artanh\left(\frac{-2\,b^2x+\sqrt{-b^2\,(4\,ab-1)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a*b-b^2*x^2+x*(-4*a*b^3+b^2)^(1/2)),x)
```

```
[Out] -2/b*arctanh((-2*b^2*x+(-b^2*(4*a*b-1))^(1/2))/b)
```

Maxima [A] time = 0.703031, size = 88, normalized size = 2.67

$$-\frac{\log\left(\frac{2\,b^2x-\sqrt{-4\,ab^3+b^2}-\sqrt{b^2}}{2\,b^2x-\sqrt{-4\,ab^3+b^2}+\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b^2*x^2 - a*b - sqrt(-4*a*b^3 + b^2)*x),x, algorithm="maxima")
```

```
[Out] -log((2*b^2*x - sqrt(-4*a*b^3 + b^2) - sqrt(b^2))/(2*b^2*x - sqrt
(-4*a*b^3 + b^2) + sqrt(b^2)))/sqrt(b^2)
```

Fricas [A] time = 0.235365, size = 85, normalized size = 2.58

$$\frac{\log\left(\frac{2\,b^2x+b-\sqrt{-4\,ab^3+b^2}}{b}\right)-\log\left(\frac{2\,b^2x-b-\sqrt{-4\,ab^3+b^2}}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b^2*x^2 - a*b - sqrt(-4*a*b^3 + b^2)*x),x, algorithm="fricas")
```

```
[Out] (log((2*b^2*x + b - sqrt(-4*a*b^3 + b^2))/b) - log((2*b^2*x - b -
 sqrt(-4*a*b^3 + b^2))/b))/b
```

Sympy [A] time = 0.806361, size = 56, normalized size = 1.7

$$-\frac{\log\left(x-\frac{1}{2b}-\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)-\log\left(x+\frac{1}{2b}-\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*b-b**2*x**2+x*(-4*a*b**3+b**2)**(1/2)),x)
```

```
[Out] -(log(x - 1/(2*b) - sqrt(-4*a*b**3 + b**2)/(2*b**2)) - log(x + 1/
(2*b) - sqrt(-4*a*b**3 + b**2)/(2*b**2)))/b
```

GIAC/XCAS [A] time = 0.217982, size = 76, normalized size = 2.3

$$-\frac{\ln\left(\frac{\left|2\,b^2x-\sqrt{-4\,ab+1}\,|b|-|b|\right|}{\left|2\,b^2x-\sqrt{-4\,ab+1}\,|b|+|b|\right|}\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b^2*x^2 - a*b - sqrt(-4*a*b^3 + b^2)*x),x, algorithm="giac")
```

```
[Out] -ln(abs(2*b^2*x - sqrt(-4*a*b + 1)*abs(b) - abs(b))/abs(2*b^2*x -
 sqrt(-4*a*b + 1)*abs(b) + abs(b)))/abs(b)
```

3.100 $\int \frac{1}{ab-\sqrt{b^2-4ab^3}x-b^2x^2}\,dx$

Optimal. Leaf size=31

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{b^2-4ab^3}+2b^2x}{b}\right)}{b}$$

```
[Out] (2*ArcTanh[(Sqrt[b^2 - 4*a*b^3] + 2*b^2*x)/b])/b
```

Rubi [A] time = 0.0643668, antiderivative size = 58, normalized size of antiderivative = 1.87, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{\log\left(\sqrt{b^2-4ab^3}+2b^2x+b\right)}{b}-\frac{\log\left(-\sqrt{b^2-4ab^3}-2b^2x+b\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Int[(a*b - Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

```
[Out] -(Log[b - Sqrt[b^2 - 4*a*b^3] - 2*b^2*x]/b) + Log[b + Sqrt[b^2 -
4*a*b^3] + 2*b^2*x]/b
```

Rubi in Sympy [A] time = 4.64834, size = 49, normalized size = 1.58

$$-\frac{\log\left(-2b^2x+b-\sqrt{b^2\left(-4ab+1\right)}\right)}{b}+\frac{\log\left(2b^2x+b+\sqrt{b^2\left(-4ab+1\right)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(a*b-b**2*x**2-x*(-4*a*b**3+b**2)**(1/2)),x)
```

```
[Out] -log(-2*b**2*x + b - sqrt(b**2*(-4*a*b + 1)))/b + log(2*b**2*x +
b + sqrt(b**2*(-4*a*b + 1)))/b
```

Mathematica [A] time = 0.0510984, size = 32, normalized size = 1.03

$$\frac{2\tanh^{-1}\left(\frac{\sqrt{-b^2(4ab-1)}+2b^2x}{b}\right)}{b}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a*b - Sqrt[b^2 - 4*a*b^3]*x - b^2*x^2)^(-1),x]
```

```
[Out] (2*ArcTanh[(Sqrt[-(b^2*(-1 + 4*a*b))] + 2*b^2*x)/b])/b
```

Maple [A] time = 0.009, size = 31, normalized size = 1.

$$2\,\frac{1}{b}Artanh\left(\frac{2\,b^2x+\sqrt{-b^2\,(4\,ab-1)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(a*b-b^2*x^2-x*(-4*a*b^3+b^2)^(1/2)),x)
```

```
[Out] 2/b*arctanh((2*b^2*x+(-b^2*(4*a*b-1))^(1/2))/b)
```

Maxima [A] time = 0.755773, size = 82, normalized size = 2.65

$$-\frac{\log\left(\frac{2\,b^2x+\sqrt{-4\,ab^3+b^2}-\sqrt{b^2}}{2\,b^2x+\sqrt{-4\,ab^3+b^2}+\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b^2*x^2 - a*b + sqrt(-4*a*b^3 + b^2)*x),x, algorithm="maxima")
```

```
[Out] -log((2*b^2*x + sqrt(-4*a*b^3 + b^2) - sqrt(b^2))/(2*b^2*x + sqrt
(-4*a*b^3 + b^2) + sqrt(b^2)))/sqrt(b^2)
```

Fricas [A] time = 0.230087, size = 80, normalized size = 2.58

$$\frac{\log\left(\frac{2\,b^2x+b+\sqrt{-4\,ab^3+b^2}}{b}\right)-\log\left(\frac{2\,b^2x-b+\sqrt{-4\,ab^3+b^2}}{b}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b^2*x^2 - a*b + sqrt(-4*a*b^3 + b^2)*x),x, algorithm="fricas")
```

```
[Out] (log((2*b^2*x + b + sqrt(-4*a*b^3 + b^2))/b) - log((2*b^2*x - b +
 sqrt(-4*a*b^3 + b^2))/b))/b
```

Sympy [A] time = 0.794652, size = 56, normalized size = 1.81

$$-\frac{\log\left(x-\frac{1}{2b}+\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)-\log\left(x+\frac{1}{2b}+\frac{\sqrt{-4ab^3+b^2}}{2b^2}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(a*b-b**2*x**2-x*(-4*a*b**3+b**2)**(1/2)),x)
```

```
[Out] -(log(x - 1/(2*b) + sqrt(-4*a*b**3 + b**2)/(2*b**2)) - log(x + 1/
(2*b) + sqrt(-4*a*b**3 + b**2)/(2*b**2)))/b
```

GIAC/XCAS [A] time = 0.220036, size = 73, normalized size = 2.35

$$-\frac{\ln\left(\frac{\left|2\,b^2x+\sqrt{-4\,ab+1}|b|-|b|\right|}{\left|2\,b^2x+\sqrt{-4\,ab+1}|b|+|b|\right|}\right)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-1/(b^2*x^2 - a*b + sqrt(-4*a*b^3 + b^2)*x),x, algorithm="giac")
```

```
[Out] -ln(abs(2*b^2*x + sqrt(-4*a*b + 1)*abs(b) - abs(b))/abs(2*b^2*x +
 sqrt(-4*a*b + 1)*abs(b) + abs(b)))/abs(b)
```

3.101 $\int \frac{1}{1+x^2+2x\cos\left(\frac{1}{7}\right)}\,dx$

Optimal. Leaf size=17

$$\csc\left(\frac{1}{7}\right)\tan^{-1}\left(\csc\left(\frac{1}{7}\right)\left(x+\cos\left(\frac{1}{7}\right)\right)\right)$$

```
[Out] ArcTan[(x + Cos[1/7])*Csc[1/7]]*Csc[1/7]
```

Rubi [A] time = 0.0392757, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\csc\left(\frac{1}{7}\right)\tan^{-1}\left(\csc\left(\frac{1}{7}\right)\left(x+\cos\left(\frac{1}{7}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2 + 2*x*Cos[1/7])^(-1),x]

[Out] ArcTan[(x + Cos[1/7])*Csc[1/7]]*Csc[1/7]
```

Rubi in Sympy [A] time = 3.90748, size = 17, normalized size = 1.

$$\frac{\operatorname{atan}\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sin\left(\frac{1}{7}\right)}\right)}{\sin\left(\frac{1}{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x**2+2*x*cos(1/7)),x)

[Out] atan((x + cos(1/7))/sin(1/7))/sin(1/7)
```

Mathematica [A] time = 0.0337022, size = 19, normalized size = 1.12

$$\csc\left(\frac{1}{7}\right)\tan^{-1}\left(\frac{(x-1)\tan\left(\frac{1}{14}\right)}{x+1}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[(1 + x^2 + 2*x*Cos[1/7])^(-1),x]
```

```
[Out] ArcTan[((-1 + x)*Tan[1/14])/(1 + x)]*Csc[1/7]
```

Maple [B] time = 0.02, size = 33, normalized size = 1.9

$$\frac{1}{\sqrt{1-\left(\cos\left(\frac{1}{7}\right)\right)^{2}}}\arctan\left(\frac{2x+2\cos(1/7)}{2\sqrt{1-(\cos(1/7))^{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x^2+2*x*cos(1/7)),x)
```

```
[Out] 1/(1-cos(1/7)^2)^(1/2)*arctan(1/2*(2*x+2*cos(1/7))/(1-cos(1/7)^2)
^(1/2))
```

Maxima [A] time = 0.79504, size = 36, normalized size = 2.12

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^{2}+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 + 2*x*cos(1/7) + 1),x, algorithm="maxima")
```

```
[Out] arctan((x + cos(1/7))/sqrt(-cos(1/7)^2 + 1))/sqrt(-cos(1/7)^2 + 1
)
```

Fricas [A] time = 0.226155, size = 20, normalized size = 1.18

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sin\left(\frac{1}{7}\right)}\right)}{\sin\left(\frac{1}{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 + 2*x*cos(1/7) + 1),x, algorithm="fricas")
```

```
[Out] arctan((x + cos(1/7))/sin(1/7))/sin(1/7)
```

Sympy [A] time = 0.450432, size = 165, normalized size = 9.71

$$-\frac{i\log\left(x+\cos\left(\frac{1}{7}\right)-\frac{i}{\sqrt{-\cos\left(\frac{1}{7}\right)+1}\sqrt{\cos\left(\frac{1}{7}\right)+1}}+\frac{i\cos^{2}\left(\frac{1}{7}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)+1}\sqrt{\cos\left(\frac{1}{7}\right)+1}}\right)}{2\sqrt{-\cos\left(\frac{1}{7}\right)+1}\sqrt{\cos\left(\frac{1}{7}\right)+1}}+\frac{i\log\left(x+\cos\left(\frac{1}{7}\right)-\frac{i\cos^{2}\left(\frac{1}{7}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)+1}\sqrt{\cos\left(\frac{1}{7}\right)+1}}+\frac{i}{\sqrt{-\cos\left(\frac{1}{7}\right)+1}\sqrt{\cos\left(\frac{1}{7}\right)+1}}\right)}{2\sqrt{-\cos\left(\frac{1}{7}\right)+1}\sqrt{\cos\left(\frac{1}{7}\right)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x**2+2*x*cos(1/7)),x)
```

```
[Out] -I*log(x + cos(1/7) - I/(sqrt(-cos(1/7) + 1)*sqrt(cos(1/7) + 1))
+ I*cos(1/7)**2/(sqrt(-cos(1/7) + 1)*sqrt(cos(1/7) + 1)))/(2*sqrt
(-cos(1/7) + 1)*sqrt(cos(1/7) + 1)) + I*log(x + cos(1/7) - I*cos(
1/7)**2/(sqrt(-cos(1/7) + 1)*sqrt(cos(1/7) + 1)) + I/(sqrt(-cos(1
/7) + 1)*sqrt(cos(1/7) + 1)))/(2*sqrt(-cos(1/7) + 1)*sqrt(cos(1/7
) + 1))
```

GIAC/XCAS [A] time = 0.207219, size = 36, normalized size = 2.12

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^{2}+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\right)^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 + 2*x*cos(1/7) + 1),x, algorithm="giac")
```

```
[Out] arctan((x + cos(1/7))/sqrt(-cos(1/7)^2 + 1))/sqrt(-cos(1/7)^2 + 1
)
```

3.102 $\int \frac{1}{1+x^2+2x\cos\left(\frac{\pi}{7}\right)}\,dx$

Optimal. Leaf size=23

$$\csc\left(\frac{\pi}{7}\right)\tan^{-1}\left(x\csc\left(\frac{\pi}{7}\right)+\cot\left(\frac{\pi}{7}\right)\right)$$

```
[Out] ArcTan[Cot[Pi/7] + x*Csc[Pi/7]]*Csc[Pi/7]
```

Rubi [A] time = 0.0537731, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125

$$\csc\left(\frac{\pi}{7}\right)\tan^{-1}\left(\csc\left(\frac{\pi}{7}\right)\left(x+\cos\left(\frac{\pi}{7}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(1 + x^2 + 2*x*Cos[Pi/7])^(-1),x]

[Out] ArcTan[(x + Cos[Pi/7])*Csc[Pi/7]]*Csc[Pi/7]
```

Rubi in Sympy [A] time = 4.85653, size = 17, normalized size = 0.74

$$\frac{\operatorname{atan}\left(\frac{x+\cos\left(\frac{\pi}{7}\right)}{\sin\left(\frac{\pi}{7}\right)}\right)}{\sin\left(\frac{\pi}{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(1+x**2+2*x*cos(1/7*pi)),x)

[Out] atan((x + cos(pi/7))/sin(pi/7))/sin(pi/7)
```

Mathematica [B] time = 0.0606854, size = 56, normalized size = 2.43

$$\frac{2\tan^{-1}\left(\frac{2x-(-1)^{6/7}+\sqrt[7]{-1}}{\sqrt{2-(-1)^{2/7}+(-1)^{5/7}}}\right)}{\sqrt{2-(-1)^{2/7}+(-1)^{5/7}}}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(1 + x^2 + 2*x*Cos[Pi/7])^(-1),x]
```

```
[Out] (2*ArcTan[((-1)^(1/7) - (-1)^(6/7) + 2*x)/Sqrt[2 - (-1)^(2/7) + (
-1)^(5/7)]])/Sqrt[2 - (-1)^(2/7) + (-1)^(5/7)]
```

Maple [B] time = 0.044, size = 39, normalized size = 1.7

$$\frac{1}{\sqrt{1-\left(\cos\left(\frac{\pi}{7}\right)\right)^{2}}}\arctan\left(\frac{2\,x+2\,\cos\left(\pi/7\right)}{2\,\sqrt{1-\left(\cos\left(\pi/7\right)\right)^{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(1+x^2+2*x*cos(1/7*Pi)),x)
```

```
[Out] 1/(1-cos(1/7*Pi)^2)^(1/2)*arctan(1/2*(2*x+2*cos(1/7*Pi))/(1-cos(1
/7*Pi)^2)^(1/2))
```

Maxima [A] time = 0.787698, size = 45, normalized size = 1.96

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\,\pi\right)}{\sqrt{-\cos\left(\frac{1}{7}\,\pi\right)^{2}+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\,\pi\right)^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 + 2*x*cos(1/7*pi) + 1),x, algorithm="maxima")
```

```
[Out] arctan((x + cos(1/7*pi))/sqrt(-cos(1/7*pi)^2 + 1))/sqrt(-cos(1/7*
pi)^2 + 1)
```

Fricas [A] time = 0.230899, size = 28, normalized size = 1.22

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\,\pi\right)}{\sin\left(\frac{1}{7}\,\pi\right)}\right)}{\sin\left(\frac{1}{7}\,\pi\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 + 2*x*cos(1/7*pi) + 1),x, algorithm="fricas")
```

```
[Out] arctan((x + cos(1/7*pi))/sin(1/7*pi))/sin(1/7*pi)
```

Sympy [A] time = 2.25479, size = 70, normalized size = 3.04

$$-\frac{i\log\left(x+\cos\left(\frac{\pi}{7}\right)-\frac{i\left(-2\cos^{2}\left(\frac{\pi}{7}\right)+2\right)}{2\sin\left(\frac{\pi}{7}\right)}\right)}{2\sin\left(\frac{\pi}{7}\right)}+\frac{i\log\left(x+\cos\left(\frac{\pi}{7}\right)+\frac{i\left(-2\cos^{2}\left(\frac{\pi}{7}\right)+2\right)}{2\sin\left(\frac{\pi}{7}\right)}\right)}{2\sin\left(\frac{\pi}{7}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(1+x**2+2*x*cos(1/7*pi)),x)
```

```
[Out] -I*log(x + cos(pi/7) - I*(-2*cos(pi/7)**2 + 2)/(2*sin(pi/7)))/(2*
sin(pi/7)) + I*log(x + cos(pi/7) + I*(-2*cos(pi/7)**2 + 2)/(2*sin
(pi/7)))/(2*sin(pi/7))
```

GIAC/XCAS [A] time = 0.208199, size = 45, normalized size = 1.96

$$\frac{\arctan\left(\frac{x+\cos\left(\frac{1}{7}\pi\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^{2}+1}}\right)}{\sqrt{-\cos\left(\frac{1}{7}\pi\right)^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x^2 + 2*x*cos(1/7*pi) + 1),x, algorithm="giac")
```

```
[Out] arctan((x + cos(1/7*pi))/sqrt(-cos(1/7*pi)^2 + 1))/sqrt(-cos(1/7*
pi)^2 + 1)
```

3.103 $\int \sqrt{5-6x+9x^2}\,dx$

Optimal. Leaf size=38

$$\frac{2}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)-\frac{1}{6}(1-3x)\sqrt{9x^2-6x+5}$$

```
[Out] -((1 - 3*x)*Sqrt[5 - 6*x + 9*x^2])/6 + (2*ArcSinh[(-1 + 3*x)/2])/
3
```

Rubi [A] time = 0.0273342, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{2}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)-\frac{1}{6}(1-3x)\sqrt{9x^2-6x+5}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[5 - 6*x + 9*x^2],x]
```

```
[Out] -((1 - 3*x)*Sqrt[5 - 6*x + 9*x^2])/6 + (2*ArcSinh[(-1 + 3*x)/2])/
3
```

Rubi in Sympy [A] time = 2.02363, size = 44, normalized size = 1.16

$$-\frac{(-18x+6)\sqrt{9x^2-6x+5}}{36}+\frac{2\,\mathrm{atanh}\left(\frac{18x-6}{6\sqrt{9x^2-6x+5}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((9*x**2-6*x+5)**(1/2),x)
```

```
[Out] -(-18*x + 6)*sqrt(9*x**2 - 6*x + 5)/36 + 2*atanh((18*x - 6)/(6*sq
rt(9*x**2 - 6*x + 5)))/3
```

Mathematica [A] time = 0.0283147, size = 39, normalized size = 1.03

$$\sqrt{9x^2-6x+5}\left(\frac{x}{2}-\frac{1}{6}\right)+\frac{2}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[5 - 6*x + 9*x^2],x]
```

```
[Out] (-1/6 + x/2)*Sqrt[5 - 6*x + 9*x^2] + (2*ArcSinh[(-1 + 3*x)/2])/3
```

Maple [A] time = 0.005, size = 29, normalized size = 0.8

$$\frac{18\,x-6}{36}\sqrt{9\,x^{2}-6\,x+5}+\frac{2}{3}Arcsinh\left(-\frac{1}{2}+\frac{3\,x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((9*x^2-6*x+5)^(1/2),x)
```

```
[Out] 1/36*(18*x-6)*(9*x^2-6*x+5)^(1/2)+2/3*arcsinh(-1/2+3/2*x)
```

Maxima [A] time = 0.790748, size = 51, normalized size = 1.34

$$\frac{1}{2}\,\sqrt{9\,x^{2}-6\,x+5}x-\frac{1}{6}\,\sqrt{9\,x^{2}-6\,x+5}+\frac{2}{3}\,\operatorname{arsinh}\left(\frac{3}{2}\,x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(9*x^2 - 6*x + 5),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(9*x^2 - 6*x + 5)*x - 1/6*sqrt(9*x^2 - 6*x + 5) + 2/3*arc
sinh(3/2*x - 1/2)
```

Fricas [A] time = 0.215401, size = 177, normalized size = 4.66

$$-\frac{324\,x^{4}-432\,x^{3}+351\,x^{2}+16\left(9\,x^{2}-\sqrt{9\,x^{2}-6\,x+5}(3\,x-1)-6\,x+3\right)\log\left(-3\,x+\sqrt{9\,x^{2}-6\,x+5}+1\right)-\left(108\,x^{3}-108\right.}{24\left(9\,x^{2}-\sqrt{9\,x^{2}-6\,x+5}(3\,x-1)-6\,x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(9*x^2 - 6*x + 5),x, algorithm="fricas")
```

```
[Out] -1/24*(324*x^4 - 432*x^3 + 351*x^2 + 16*(9*x^2 - sqrt(9*x^2 - 6*x
 + 5)*(3*x - 1) - 6*x + 3)*log(-3*x + sqrt(9*x^2 - 6*x + 5) + 1)
- (108*x^3 - 108*x^2 + 57*x - 11)*sqrt(9*x^2 - 6*x + 5) - 138*x +
 17)/(9*x^2 - sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 6*x + 3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{9x^2 - 6x + 5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((9*x**2-6*x+5)**(1/2),x)
```

```
[Out] Integral(sqrt(9*x**2 - 6*x + 5), x)
```

GIAC/XCAS [A] time = 0.209735, size = 54, normalized size = 1.42

$$\frac{1}{6}\sqrt{9\,x^2 - 6\,x + 5}(3\,x - 1) - \frac{2}{3}\ln\left(-3\,x + \sqrt{9\,x^2 - 6\,x + 5} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(9*x^2 - 6*x + 5),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(9*x^2 - 6*x + 5)*(3*x - 1) - 2/3*ln(-3*x + sqrt(9*x^2 -
6*x + 5) + 1)
```

3.104 $\int \sqrt{3-4x-4x^2}\,dx$

Optimal. Leaf size=30

$$\frac{1}{4}\sqrt{-4x^2-4x+3}(2x+1)+\sin^{-1}\left(x+\frac{1}{2}\right)$$

```
[Out] ((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 + ArcSin[1/2 + x]
```

Rubi [A] time = 0.0239971, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{1}{4}\sqrt{-4x^2-4x+3}(2x+1)+\sin^{-1}\left(x+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[3 - 4*x - 4*x^2],x]
```

```
[Out] ((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 + ArcSin[1/2 + x]
```

Rubi in Sympy [A] time = 1.93607, size = 44, normalized size = 1.47

$$\frac{(8x+4)\sqrt{-4x^2-4x+3}}{16}+\operatorname{atan}\left(-\frac{-8x-4}{4\sqrt{-4x^2-4x+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-4*x**2-4*x+3)**(1/2),x)
```

```
[Out] (8*x + 4)*sqrt(-4*x**2 - 4*x + 3)/16 + atan(-(-8*x - 4)/(4*sqrt(-
4*x**2 - 4*x + 3)))
```

Mathematica [A] time = 0.0249404, size = 30, normalized size = 1.

$$\frac{1}{4}\sqrt{-4x^2-4x+3}(2x+1)+\sin^{-1}\left(x+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[3 - 4*x - 4*x^2],x]

[Out] ((1 + 2*x)*Sqrt[3 - 4*x - 4*x^2])/4 + ArcSin[1/2 + x]
```

Maple [A] time = 0.004, size = 25, normalized size = 0.8

$$-\frac{-8\,x-4}{16}\sqrt{-4\,x^{2}-4\,x+3}+\arcsin\left(\frac{1}{2}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-4*x^2-4*x+3)^(1/2),x)

[Out] -1/16*(-8*x-4)*(-4*x^2-4*x+3)^(1/2)+arcsin(1/2+x)
```

Maxima [A] time = 0.829307, size = 51, normalized size = 1.7

$$\frac{1}{2}\sqrt{-4\,x^{2}-4\,x+3}x+\frac{1}{4}\sqrt{-4\,x^{2}-4\,x+3}-\arcsin\left(-x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-4*x^2 - 4*x + 3),x, algorithm="maxima")

[Out] 1/2*sqrt(-4*x^2 - 4*x + 3)*x + 1/4*sqrt(-4*x^2 - 4*x + 3) - arcsi
n(-x - 1/2)
```

Fricas [A] time = 0.22149, size = 53, normalized size = 1.77

$$\frac{1}{4}\sqrt{-4\,x^{2}-4\,x+3}(2\,x+1)+\arctan\left(\frac{2\,x+1}{\sqrt{-4\,x^{2}-4\,x+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-4*x^2 - 4*x + 3),x, algorithm="fricas")

[Out] 1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) + arctan((2*x + 1)/sqrt(-4*x
^2 - 4*x + 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-4x^2 - 4x + 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x**2-4*x+3)**(1/2),x)
```

```
[Out] Integral(sqrt(-4*x**2 - 4*x + 3), x)
```

GIAC/XCAS [A] time = 0.209281, size = 32, normalized size = 1.07

$$\frac{1}{4}\,\sqrt{-4\,x^2 - 4\,x + 3}(2\,x + 1) + \arcsin\left(x + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-4*x^2 - 4*x + 3),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(-4*x^2 - 4*x + 3)*(2*x + 1) + arcsin(x + 1/2)
```

3.105 $\int \sqrt{-8+6x+9x^2}\,dx$

Optimal. Leaf size=49

$$\frac{1}{6}(3x+1)\sqrt{9x^2+6x-8}-\frac{3}{2}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

```
[Out] ((1 + 3*x)*Sqrt[-8 + 6*x + 9*x^2])/6 - (3*ArcTanh[(1 + 3*x)/Sqrt[
-8 + 6*x + 9*x^2]])/2
```

Rubi [A] time = 0.0275822, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{1}{6}(3x+1)\sqrt{9x^2+6x-8}-\frac{3}{2}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] ((1 + 3*x)*Sqrt[-8 + 6*x + 9*x^2])/6 - (3*ArcTanh[(1 + 3*x)/Sqrt[
-8 + 6*x + 9*x^2]])/2
```

Rubi in Sympy [A] time = 1.98259, size = 44, normalized size = 0.9

$$\frac{(18x+6)\sqrt{9x^2+6x-8}}{36}-\frac{3\,\mathrm{atanh}\left(\frac{18x+6}{6\sqrt{9x^2+6x-8}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((9*x**2+6*x-8)**(1/2),x)
```

```
[Out] (18*x + 6)*sqrt(9*x**2 + 6*x - 8)/36 - 3*atanh((18*x + 6)/(6*sqrt
(9*x**2 + 6*x - 8)))/2
```

Mathematica [A] time = 0.0270837, size = 49, normalized size = 1.

$$\left(\frac{x}{2}+\frac{1}{6}\right)\sqrt{9x^2+6x-8}-\frac{3}{2}\log\left(\sqrt{9x^2+6x-8}+3x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-8 + 6*x + 9*x^2],x]
```

```
[Out] (1/6 + x/2)*Sqrt[-8 + 6*x + 9*x^2] - (3*Log[1 + 3*x + Sqrt[-8 + 6
*x + 9*x^2]])/2
```

Maple [A] time = 0.006, size = 50, normalized size = 1.

$$\frac{18\,x+6}{36}\sqrt{9\,x^{2}+6\,x-8}-\frac{\sqrt{9}}{2}\ln\left(\frac{(9\,x+3)\sqrt{9}}{9}+\sqrt{9\,x^{2}+6\,x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((9*x^2+6*x-8)^(1/2),x)
```

```
[Out] 1/36*(18*x+6)*(9*x^2+6*x-8)^(1/2)-1/2*ln(1/9*(9*x+3)*9^(1/2)+(9*x
^2+6*x-8)^(1/2))*9^(1/2)
```

Maxima [A] time = 0.804273, size = 70, normalized size = 1.43

$$\frac{1}{2}\sqrt{9\,x^{2}+6\,x-8}x+\frac{1}{6}\sqrt{9\,x^{2}+6\,x-8}-\frac{3}{2}\log\left(18\,x+6\sqrt{9\,x^{2}+6\,x-8}+6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(9*x^2 + 6*x - 8),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(9*x^2 + 6*x - 8)*x + 1/6*sqrt(9*x^2 + 6*x - 8) - 3/2*log
(18*x + 6*sqrt(9*x^2 + 6*x - 8) + 6)
```

Fricas [A] time = 0.216237, size = 177, normalized size = 3.61

$$-\frac{216\,x^{4}+288\,x^{3}-78\,x^{2}-12\left(18\,x^{2}-2\sqrt{9\,x^{2}+6\,x-8}(3\,x+1)+12\,x-7\right)\log\left(-3\,x+\sqrt{9\,x^{2}+6\,x-8}-1\right)-2\left(36\,x^{3}+\right.}{8\left(18\,x^{2}-2\sqrt{9\,x^{2}+6\,x-8}(3\,x+1)+12\,x-7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(9*x^2 + 6*x - 8),x, algorithm="fricas")
```

```
[Out] -1/8*(216*x^4 + 288*x^3 - 78*x^2 - 12*(18*x^2 - 2*sqrt(9*x^2 + 6*
x - 8)*(3*x + 1) + 12*x - 7)*log(-3*x + sqrt(9*x^2 + 6*x - 8) - 1
) - 2*(36*x^3 + 36*x^2 - 7*x - 5)*sqrt(9*x^2 + 6*x - 8) - 116*x -
 19)/(18*x^2 - 2*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 12*x - 7)
```

Sympy [F]   time = 0., size = 0, normalized size = 0.

$$\int \sqrt{9x^2 + 6x - 8}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((9*x**2+6*x-8)**(1/2),x)
```

```
[Out] Integral(sqrt(9*x**2 + 6*x - 8), x)
```

GIAC/XCAS [A]   time = 0.20901, size = 55, normalized size = 1.12

$$\frac{1}{6}\sqrt{9\,x^2 + 6\,x - 8}(3\,x + 1) + \frac{3}{2}\ln\left(\left|-3\,x + \sqrt{9\,x^2 + 6\,x - 8} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(9*x^2 + 6*x - 8),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(9*x^2 + 6*x - 8)*(3*x + 1) + 3/2*ln(abs(-3*x + sqrt(9*x^
2 + 6*x - 8) - 1))
```

3.106 $\int \sqrt{2+4x+3x^2}\,dx$

Optimal. Leaf size=45

$$\frac{1}{6}\sqrt{3x^2+4x+2}(3x+2)+\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{3\sqrt{3}}$$

```
[Out] ((2 + 3*x)*Sqrt[2 + 4*x + 3*x^2])/6 + ArcSinh[(2 + 3*x)/Sqrt[2]]/
(3*Sqrt[3])
```

Rubi [A] time = 0.0361754, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{1}{6}\sqrt{3x^2+4x+2}(3x+2)+\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + 4*x + 3*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[2 + 4*x + 3*x^2])/6 + ArcSinh[(2 + 3*x)/Sqrt[2]]/
(3*Sqrt[3])
```

Rubi in Sympy [A] time = 2.05403, size = 53, normalized size = 1.18

$$\frac{(6x+4)\sqrt{3x^2+4x+2}}{12}+\frac{\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}(6x+4)}{6\sqrt{3x^2+4x+2}}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+4*x+2)**(1/2),x)
```

```
[Out] (6*x + 4)*sqrt(3*x**2 + 4*x + 2)/12 + sqrt(3)*atanh(sqrt(3)*(6*x
+ 4)/(6*sqrt(3*x**2 + 4*x + 2)))/9
```

Mathematica [A] time = 0.0299878, size = 46, normalized size = 1.02

$$\sqrt{3x^2+4x+2}\left(\frac{x}{2}+\frac{1}{3}\right)+\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + 4*x + 3*x^2],x]
```

```
[Out] (1/3 + x/2)*Sqrt[2 + 4*x + 3*x^2] + ArcSinh[(2 + 3*x)/Sqrt[2]]/(3
*Sqrt[3])
```

Maple [A] time = 0.003, size = 35, normalized size = 0.8

$$\frac{6x+4}{12}\sqrt{3x^2+4x+2}+\frac{\sqrt{3}}{9}\mathit{Arcsinh}\left(\frac{3\sqrt{2}}{2}\left(x+\frac{2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+4*x+2)^(1/2),x)
```

```
[Out] 1/12*(6*x+4)*(3*x^2+4*x+2)^(1/2)+1/9*3^(1/2)*arcsinh(3/2*2^(1/2)*
(x+2/3))
```

Maxima [A] time = 0.816412, size = 62, normalized size = 1.38

$$\frac{1}{2}\sqrt{3x^2+4x+2}x+\frac{1}{9}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{2}(3x+2)\right)+\frac{1}{3}\sqrt{3x^2+4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 + 4*x + 2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 + 4*x + 2)*x + 1/9*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*
x + 2)) + 1/3*sqrt(3*x^2 + 4*x + 2)
```

Fricas [A] time = 0.230105, size = 85, normalized size = 1.89

$$\frac{1}{18}\sqrt{3}\left(\sqrt{3}\sqrt{3x^2+4x+2}(3x+2)+\log\left(-\sqrt{3}\left(9x^2+12x+5\right)-3\sqrt{3x^2+4x+2}(3x+2)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 + 4*x + 2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(sqrt(3)*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) + log(-sqrt
(3)*(9*x^2 + 12*x + 5) - 3*sqrt(3*x^2 + 4*x + 2)*(3*x + 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3x^2 + 4x + 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 + 4*x + 2), x)
```

GIAC/XCAS [A] time = 0.21201, size = 72, normalized size = 1.6

$$\frac{1}{6}\sqrt{3\,x^2 + 4\,x + 2}(3\,x + 2) - \frac{1}{9}\sqrt{3}\ln\left(-\sqrt{3}\left(\sqrt{3}x - \sqrt{3\,x^2 + 4\,x + 2}\right) - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 + 4*x + 2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3*x^2 + 4*x + 2)*(3*x + 2) - 1/9*sqrt(3)*ln(-sqrt(3)*(sq
rt(3)*x - sqrt(3*x^2 + 4*x + 2)) - 2)
```

3.107 $\int \sqrt{2+4x-3x^2}\,dx$

Optimal. Leaf size=45

$$-\frac{1}{6}\sqrt{-3x^2+4x+2}(2-3x)-\frac{5\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

```
[Out] -((2 - 3*x)*Sqrt[2 + 4*x - 3*x^2])/6 - (5*ArcSin[(2 - 3*x)/Sqrt[1
0]])/(3*Sqrt[3])
```

Rubi [A] time = 0.0425203, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$-\frac{1}{6}\sqrt{-3x^2+4x+2}(2-3x)-\frac{5\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + 4*x - 3*x^2],x]
```

```
[Out] -((2 - 3*x)*Sqrt[2 + 4*x - 3*x^2])/6 - (5*ArcSin[(2 - 3*x)/Sqrt[1
0]])/(3*Sqrt[3])
```

Rubi in Sympy [A] time = 2.02748, size = 56, normalized size = 1.24

$$-\frac{(-6x+4)\sqrt{-3x^2+4x+2}}{12}-\frac{5\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}(-6x+4)}{6\sqrt{-3x^2+4x+2}}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3*x**2+4*x+2)**(1/2),x)
```

```
[Out] -(-6*x + 4)*sqrt(-3*x**2 + 4*x + 2)/12 - 5*sqrt(3)*atan(sqrt(3)*(
-6*x + 4)/(6*sqrt(-3*x**2 + 4*x + 2)))/9
```

Mathematica [A] time = 0.0345582, size = 46, normalized size = 1.02

$$\left(\frac{x}{2}-\frac{1}{3}\right)\sqrt{-3x^2+4x+2}-\frac{5\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + 4*x - 3*x^2],x]
```

```
[Out] (-1/3 + x/2)*Sqrt[2 + 4*x - 3*x^2] - (5*ArcSin[(2 - 3*x)/Sqrt[10]
])/(3*Sqrt[3])
```

Maple [A] time = 0.005, size = 35, normalized size = 0.8

$$-\frac{-6x+4}{12}\sqrt{-3x^2+4x+2}+\frac{5\sqrt{3}}{9}\arcsin\left(\frac{3\sqrt{10}}{10}\left(x-\frac{2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3*x^2+4*x+2)^(1/2),x)
```

```
[Out] -1/12*(-6*x+4)*(-3*x^2+4*x+2)^(1/2)+5/9*3^(1/2)*arcsin(3/10*10^(1
/2)*(x-2/3))
```

Maxima [A] time = 0.831942, size = 62, normalized size = 1.38

$$\frac{1}{2}\sqrt{-3x^2+4x+2}x-\frac{5}{9}\sqrt{3}\arcsin\left(-\frac{1}{10}\sqrt{10}(3x-2)\right)-\frac{1}{3}\sqrt{-3x^2+4x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-3*x^2 + 4*x + 2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-3*x^2 + 4*x + 2)*x - 5/9*sqrt(3)*arcsin(-1/10*sqrt(10)*
(3*x - 2)) - 1/3*sqrt(-3*x^2 + 4*x + 2)
```

Fricas [A] time = 0.221357, size = 70, normalized size = 1.56

$$\frac{1}{18}\sqrt{3}\left(\sqrt{3}\sqrt{-3x^2+4x+2}(3x-2)+10\arctan\left(\frac{\sqrt{3}(3x-2)}{3\sqrt{-3x^2+4x+2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-3*x^2 + 4*x + 2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(sqrt(3)*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2) + 10*arcta
n(1/3*sqrt(3)*(3*x - 2)/sqrt(-3*x^2 + 4*x + 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-3x^2 + 4x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-3*x**2 + 4*x + 2), x)
```

GIAC/XCAS [A] time = 0.20963, size = 49, normalized size = 1.09

$$\frac{1}{6}\sqrt{-3\,x^2 + 4\,x + 2}(3\,x - 2) + \frac{5}{9}\sqrt{3}\arcsin\left(\frac{1}{10}\sqrt{10}(3\,x - 2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-3*x^2 + 4*x + 2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(-3*x^2 + 4*x + 2)*(3*x - 2) + 5/9*sqrt(3)*arcsin(1/10*sq
rt(10)*(3*x - 2))
```

3.108 $\int \sqrt{2+5x+3x^2}\,dx$

Optimal. Leaf size=62

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x+2}-\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{24\sqrt{3}}$$

```
[Out] ((5 + 6*x)*Sqrt[2 + 5*x + 3*x^2])/12 - ArcTanh[(5 + 6*x)/(2*Sqrt[
3]*Sqrt[2 + 5*x + 3*x^2])]/(24*Sqrt[3])
```

Rubi [A] time = 0.0335726, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x+2}-\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + 5*x + 3*x^2],x]
```

```
[Out] ((5 + 6*x)*Sqrt[2 + 5*x + 3*x^2])/12 - ArcTanh[(5 + 6*x)/(2*Sqrt[
3]*Sqrt[2 + 5*x + 3*x^2])]/(24*Sqrt[3])
```

Rubi in Sympy [A] time = 2.01324, size = 53, normalized size = 0.85

$$\frac{(6x+5)\sqrt{3x^2+5x+2}}{12}-\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}(6x+5)}{6\sqrt{3x^2+5x+2}}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+5*x+2)**(1/2),x)
```

```
[Out] (6*x + 5)*sqrt(3*x**2 + 5*x + 2)/12 - sqrt(3)*atanh(sqrt(3)*(6*x
+ 5)/(6*sqrt(3*x**2 + 5*x + 2)))/72
```

Mathematica [A] time = 0.046443, size = 55, normalized size = 0.89

$$\frac{1}{72}\left(6(6x+5)\sqrt{3x^2+5x+2}-\sqrt{3}\log\left(2\sqrt{9x^2+15x+6}+6x+5\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + 5*x + 3*x^2],x]
```

```
[Out] (6*(5 + 6*x)*Sqrt[2 + 5*x + 3*x^2] - Sqrt[3]*Log[5 + 6*x + 2*Sqrt
[6 + 15*x + 9*x^2]])/72
```

Maple [A] time = 0.004, size = 50, normalized size = 0.8

$$\frac{5+6x}{12}\sqrt{3x^2+5x+2}-\frac{\sqrt{3}}{72}\ln\left(\frac{\sqrt{3}}{3}\left(\frac{5}{2}+3x\right)+\sqrt{3x^2+5x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+5*x+2)^(1/2),x)
```

```
[Out] 1/12*(5+6*x)*(3*x^2+5*x+2)^(1/2)-1/72*ln(1/3*(5/2+3*x)*3^(1/2)+(3
*x^2+5*x+2)^(1/2))*3^(1/2)
```

Maxima [A] time = 0.847109, size = 78, normalized size = 1.26

$$\frac{1}{2}\sqrt{3x^2+5x+2}x-\frac{1}{72}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x+2}+6x+5\right)+\frac{5}{12}\sqrt{3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 + 5*x + 2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 + 5*x + 2)*x - 1/72*sqrt(3)*log(2*sqrt(3)*sqrt(3*x
^2 + 5*x + 2) + 6*x + 5) + 5/12*sqrt(3*x^2 + 5*x + 2)
```

Fricas [A] time = 0.222748, size = 85, normalized size = 1.37

$$\frac{1}{144}\sqrt{3}\left(4\sqrt{3}\sqrt{3x^2+5x+2}(6x+5)+\log\left(\sqrt{3}\left(72x^2+120x+49\right)-12\sqrt{3x^2+5x+2}(6x+5)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 + 5*x + 2),x, algorithm="fricas")
```

```
[Out] 1/144*sqrt(3)*(4*sqrt(3)*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + log(sq
rt(3)*(72*x^2 + 120*x + 49) - 12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5))
)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3x^2 + 5x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 + 5*x + 2), x)
```

GIAC/XCAS [A] time = 0.212968, size = 73, normalized size = 1.18

$$\frac{1}{12}\sqrt{3\,x^2 + 5\,x + 2}(6\,x + 5) + \frac{1}{72}\sqrt{3}\ln\left(\left|-2\sqrt{3}\left(\sqrt{3}x - \sqrt{3\,x^2 + 5\,x + 2}\right) - 5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 + 5*x + 2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3*x^2 + 5*x + 2)*(6*x + 5) + 1/72*sqrt(3)*ln(abs(-2*sqr
t(3)*(sqrt(3)*x - sqrt(3*x^2 + 5*x + 2)) - 5))
```

3.109 $\int \sqrt{2+5x-3x^2}\,dx$

Optimal. Leaf size=43

$$-\frac{1}{12}\sqrt{-3x^2+5x+2}(5-6x)-\frac{49\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}$$

```
[Out] -((5 - 6*x)*Sqrt[2 + 5*x - 3*x^2])/12 - (49*ArcSin[(5 - 6*x)/7])/
(24*Sqrt[3])
```

Rubi [A] time = 0.0288746, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$-\frac{1}{12}\sqrt{-3x^2+5x+2}(5-6x)-\frac{49\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[2 + 5*x - 3*x^2],x]
```

```
[Out] -((5 - 6*x)*Sqrt[2 + 5*x - 3*x^2])/12 - (49*ArcSin[(5 - 6*x)/7])/
(24*Sqrt[3])
```

Rubi in Sympy [A] time = 2.07932, size = 56, normalized size = 1.3

$$-\frac{(-6x+5)\sqrt{-3x^2+5x+2}}{12}-\frac{49\sqrt{3}\,\text{atan}\left(\frac{\sqrt{3}(-6x+5)}{6\sqrt{-3x^2+5x+2}}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3*x**2+5*x+2)**(1/2),x)
```

```
[Out] -(-6*x + 5)*sqrt(-3*x**2 + 5*x + 2)/12 - 49*sqrt(3)*atan(sqrt(3)*
(-6*x + 5)/(6*sqrt(-3*x**2 + 5*x + 2)))/72
```

Mathematica [A] time = 0.0336152, size = 44, normalized size = 1.02

$$\left(\frac{x}{2}-\frac{5}{12}\right)\sqrt{-3x^2+5x+2}-\frac{49\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[2 + 5*x - 3*x^2],x]
```

```
[Out] (-5/12 + x/2)*Sqrt[2 + 5*x - 3*x^2] - (49*ArcSin[(5 - 6*x)/7])/(2
4*Sqrt[3])
```

Maple [A] time = 0.004, size = 32, normalized size = 0.7

$$\frac{49\sqrt{3}}{72}\arcsin\left(-\frac{5}{7}+\frac{6x}{7}\right)-\frac{5-6x}{12}\sqrt{-3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3*x^2+5*x+2)^(1/2),x)
```

```
[Out] 49/72*arcsin(-5/7+6/7*x)*3^(1/2)-1/12*(5-6*x)*(-3*x^2+5*x+2)^(1/2
)
```

Maxima [A] time = 0.820811, size = 55, normalized size = 1.28

$$\frac{1}{2}\sqrt{-3x^2+5x+2}x-\frac{49}{72}\sqrt{3}\arcsin\left(-\frac{6}{7}x+\frac{5}{7}\right)-\frac{5}{12}\sqrt{-3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-3*x^2 + 5*x + 2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-3*x^2 + 5*x + 2)*x - 49/72*sqrt(3)*arcsin(-6/7*x + 5/7)
 - 5/12*sqrt(-3*x^2 + 5*x + 2)
```

Fricas [A] time = 0.224465, size = 72, normalized size = 1.67

$$\frac{1}{72}\sqrt{3}\left(2\sqrt{3}\sqrt{-3x^2+5x+2}(6x-5)+49\arctan\left(\frac{\sqrt{3}(6x-5)}{6\sqrt{-3x^2+5x+2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-3*x^2 + 5*x + 2),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(3)*(2*sqrt(3)*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5) + 49*arc
tan(1/6*sqrt(3)*(6*x - 5)/sqrt(-3*x^2 + 5*x + 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-3x^2 + 5x + 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(sqrt(-3*x**2 + 5*x + 2), x)
```

GIAC/XCAS [A] time = 0.21008, size = 42, normalized size = 0.98

$$\frac{1}{12}\sqrt{-3\,x^2 + 5\,x + 2}(6\,x - 5) + \frac{49}{72}\sqrt{3}\arcsin\left(\frac{6}{7}\,x - \frac{5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-3*x^2 + 5*x + 2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(-3*x^2 + 5*x + 2)*(6*x - 5) + 49/72*sqrt(3)*arcsin(6/7*
x - 5/7)
```

3.110 $\int \sqrt{-2+4x+3x^2}\,dx$

Optimal. Leaf size=59

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2}-\frac{5\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{3\sqrt{3}}$$

```
[Out] ((2 + 3*x)*Sqrt[-2 + 4*x + 3*x^2])/6 - (5*ArcTanh[(2 + 3*x)/(Sqrt
[3]*Sqrt[-2 + 4*x + 3*x^2])])/(3*Sqrt[3])
```

Rubi [A] time = 0.0322648, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2}-\frac{5\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-2 + 4*x + 3*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[-2 + 4*x + 3*x^2])/6 - (5*ArcTanh[(2 + 3*x)/(Sqrt
[3]*Sqrt[-2 + 4*x + 3*x^2])])/(3*Sqrt[3])
```

Rubi in Sympy [A] time = 2.00368, size = 54, normalized size = 0.92

$$\frac{(6x+4)\sqrt{3x^2+4x-2}}{12}-\frac{5\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}(6x+4)}{6\sqrt{3x^2+4x-2}}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+4*x-2)**(1/2),x)
```

```
[Out] (6*x + 4)*sqrt(3*x**2 + 4*x - 2)/12 - 5*sqrt(3)*atanh(sqrt(3)*(6*
x + 4)/(6*sqrt(3*x**2 + 4*x - 2)))/9
```

Mathematica [A] time = 0.0381253, size = 53, normalized size = 0.9

$$\frac{1}{6}(3x+2)\sqrt{3x^2+4x-2}-\frac{5\log\left(\sqrt{9x^2+12x-6}+3x+2\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-2 + 4*x + 3*x^2],x]
```

```
[Out] ((2 + 3*x)*Sqrt[-2 + 4*x + 3*x^2])/6 - (5*Log[2 + 3*x + Sqrt[-6 +
 12*x + 9*x^2]])/(3*Sqrt[3])
```

Maple [A] time = 0.005, size = 50, normalized size = 0.9

$$\frac{6x+4}{12}\sqrt{3x^2+4x-2}-\frac{5\sqrt{3}}{9}\ln\left(\frac{(2+3x)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+4*x-2)^(1/2),x)
```

```
[Out] 1/12*(6*x+4)*(3*x^2+4*x-2)^(1/2)-5/9*ln(1/3*(2+3*x)*3^(1/2)+(3*x^
2+4*x-2)^(1/2))*3^(1/2)
```

Maxima [A] time = 0.826497, size = 78, normalized size = 1.32

$$\frac{1}{2}\sqrt{3x^2+4x-2}x-\frac{5}{9}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+4x-2}+6x+4\right)+\frac{1}{3}\sqrt{3x^2+4x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 + 4*x - 2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 + 4*x - 2)*x - 5/9*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^
2 + 4*x - 2) + 6*x + 4) + 1/3*sqrt(3*x^2 + 4*x - 2)
```

Fricas [A] time = 0.226902, size = 86, normalized size = 1.46

$$\frac{1}{18}\sqrt{3}\left(\sqrt{3}\sqrt{3x^2+4x-2}(3x+2)+5\log\left(\sqrt{3}\left(9x^2+12x-1\right)-3\sqrt{3x^2+4x-2}(3x+2)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 + 4*x - 2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(sqrt(3)*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 5*log(sqr
t(3)*(9*x^2 + 12*x - 1) - 3*sqrt(3*x^2 + 4*x - 2)*(3*x + 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3x^2 + 4x - 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 + 4*x - 2), x)
```

GIAC/XCAS [A] time = 0.212271, size = 73, normalized size = 1.24

$$\frac{1}{6}\sqrt{3\,x^2 + 4\,x - 2}(3\,x + 2) + \frac{5}{9}\sqrt{3}\ln\left(\left|-\sqrt{3}\left(\sqrt{3}x - \sqrt{3\,x^2 + 4\,x - 2}\right) - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 + 4*x - 2),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3*x^2 + 4*x - 2)*(3*x + 2) + 5/9*sqrt(3)*ln(abs(-sqrt(3)
*(sqrt(3)*x - sqrt(3*x^2 + 4*x - 2)) - 2))
```

3.111 $\int \sqrt{-2+4x-3x^2}\,dx$

Optimal. Leaf size=59

$$\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{3\sqrt{3}}-\frac{1}{6}(2-3x)\sqrt{-3x^2+4x-2}$$

```
[Out] -((2 - 3*x)*Sqrt[-2 + 4*x - 3*x^2])/6 + ArcTan[(2 - 3*x)/(Sqrt[3]
*Sqrt[-2 + 4*x - 3*x^2])]/(3*Sqrt[3])
```

Rubi [A] time = 0.0328287, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{3\sqrt{3}}-\frac{1}{6}(2-3x)\sqrt{-3x^2+4x-2}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-2 + 4*x - 3*x^2],x]
```

```
[Out] -((2 - 3*x)*Sqrt[-2 + 4*x - 3*x^2])/6 + ArcTan[(2 - 3*x)/(Sqrt[3]
*Sqrt[-2 + 4*x - 3*x^2])]/(3*Sqrt[3])
```

Rubi in Sympy [A] time = 2.00817, size = 53, normalized size = 0.9

$$-\frac{(-6x+4)\sqrt{-3x^2+4x-2}}{12}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}(-6x+4)}{6\sqrt{-3x^2+4x-2}}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3*x**2+4*x-2)**(1/2),x)
```

```
[Out] -(-6*x + 4)*sqrt(-3*x**2 + 4*x - 2)/12 + sqrt(3)*atan(sqrt(3)*(-6
*x + 4)/(6*sqrt(-3*x**2 + 4*x - 2)))/9
```

Mathematica [A] time = 0.0487513, size = 54, normalized size = 0.92

$$\frac{1}{6}\sqrt{-3x^2+4x-2}(3x-2)+\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{-9x^2+12x-6}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-2 + 4*x - 3*x^2],x]
```

```
[Out] ((-2 + 3*x)*Sqrt[-2 + 4*x - 3*x^2])/6 + ArcTan[(2 - 3*x)/Sqrt[-6
+ 12*x - 9*x^2]]/(3*Sqrt[3])
```

Maple [A] time = 0.007, size = 46, normalized size = 0.8

$$-\frac{-6x+4}{12}\sqrt{-3x^2+4x-2}-\frac{\sqrt{3}}{9}\arctan\left(\sqrt{3}\left(x-\frac{2}{3}\right)\frac{1}{\sqrt{-3x^2+4x-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3*x^2+4*x-2)^(1/2),x)
```

```
[Out] -1/12*(-6*x+4)*(-3*x^2+4*x-2)^(1/2)-1/9*3^(1/2)*arctan(3^(1/2)*(x
-2/3)/(-3*x^2+4*x-2)^(1/2))
```

Maxima [A] time = 0.917032, size = 62, normalized size = 1.05

$$\frac{1}{2}\sqrt{-3x^2+4x-2}x+\frac{1}{9}i\sqrt{3}\,\text{arsinh}\left(\frac{1}{2}\sqrt{2}(3x-2)\right)-\frac{1}{3}\sqrt{-3x^2+4x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-3*x^2 + 4*x - 2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-3*x^2 + 4*x - 2)*x + 1/9*I*sqrt(3)*arcsinh(1/2*sqrt(2)*
(3*x - 2)) - 1/3*sqrt(-3*x^2 + 4*x - 2)
```

Fricas [A] time = 0.220191, size = 115, normalized size = 1.95

$$\frac{1}{18}\sqrt{3}\left(\sqrt{3}\sqrt{-3x^2+4x-2}(3x-2)+i\log\left(\frac{2i\sqrt{3}\sqrt{-3x^2+4x-2}-6x+4}{x}\right)-i\log\left(\frac{-2i\sqrt{3}\sqrt{-3x^2+4x-2}-6x+4}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-3*x^2 + 4*x - 2),x, algorithm="fricas")
```

```
[Out] 1/18*sqrt(3)*(sqrt(3)*sqrt(-3*x^2 + 4*x - 2)*(3*x - 2) + I*log((2
*I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) - 6*x + 4)/x) - I*log((-2*I*sqr
t(3)*sqrt(-3*x^2 + 4*x - 2) - 6*x + 4)/x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-3x^2 + 4x - 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(-3*x**2 + 4*x - 2), x)
```

GIAC/XCAS [A] time = 0.211172, size = 51, normalized size = 0.86

$$\frac{1}{9}\,\sqrt{3}i\arcsin\left(\frac{1}{2}\,\sqrt{2}i(3\,x - 2)\right) + \frac{1}{6}\,\sqrt{-3\,x^2 + 4\,x - 2}(3\,x - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-3*x^2 + 4*x - 2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(3)*i*arcsin(1/2*sqrt(2)*i*(3*x - 2)) + 1/6*sqrt(-3*x^2 +
 4*x - 2)*(3*x - 2)
```

3.112 $\int \sqrt{-2+5x+3x^2}\,dx$

Optimal. Leaf size=62

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x-2}-\frac{49\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{24\sqrt{3}}$$

```
[Out] ((5 + 6*x)*Sqrt[-2 + 5*x + 3*x^2])/12 - (49*ArcTanh[(5 + 6*x)/(2*
Sqrt[3]*Sqrt[-2 + 5*x + 3*x^2])])/(24*Sqrt[3])
```

Rubi [A] time = 0.0334027, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{1}{12}(6x+5)\sqrt{3x^2+5x-2}-\frac{49\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] ((5 + 6*x)*Sqrt[-2 + 5*x + 3*x^2])/12 - (49*ArcTanh[(5 + 6*x)/(2*
Sqrt[3]*Sqrt[-2 + 5*x + 3*x^2])])/(24*Sqrt[3])
```

Rubi in Sympy [A] time = 2.10196, size = 54, normalized size = 0.87

$$\frac{(6x+5)\sqrt{3x^2+5x-2}}{12}-\frac{49\sqrt{3}\,\text{atanh}\left(\frac{\sqrt{3}(6x+5)}{6\sqrt{3x^2+5x-2}}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+5*x-2)**(1/2),x)
```

```
[Out] (6*x + 5)*sqrt(3*x**2 + 5*x - 2)/12 - 49*sqrt(3)*atanh(sqrt(3)*(6
*x + 5)/(6*sqrt(3*x**2 + 5*x - 2)))/72
```

Mathematica [A] time = 0.0473255, size = 55, normalized size = 0.89

$$\frac{1}{72}\left(6(6x+5)\sqrt{3x^2+5x-2}-49\sqrt{3}\log\left(2\sqrt{9x^2+15x-6}+6x+5\right)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] (6*(5 + 6*x)*Sqrt[-2 + 5*x + 3*x^2] - 49*Sqrt[3]*Log[5 + 6*x + 2*
Sqrt[-6 + 15*x + 9*x^2]])/72
```

Maple [A] time = 0.005, size = 50, normalized size = 0.8

$$\frac{5+6x}{12}\sqrt{3x^2+5x-2}-\frac{49\sqrt{3}}{72}\ln\left(\frac{\sqrt{3}}{3}\left(\frac{5}{2}+3x\right)+\sqrt{3x^2+5x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+5*x-2)^(1/2),x)
```

```
[Out] 1/12*(5+6*x)*(3*x^2+5*x-2)^(1/2)-49/72*ln(1/3*(5/2+3*x)*3^(1/2)+(
3*x^2+5*x-2)^(1/2))*3^(1/2)
```

Maxima [A] time = 0.833348, size = 78, normalized size = 1.26

$$\frac{1}{2}\sqrt{3x^2+5x-2}x-\frac{49}{72}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x-2}+6x+5\right)+\frac{5}{12}\sqrt{3x^2+5x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 + 5*x - 2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(3*x^2 + 5*x - 2)*x - 49/72*sqrt(3)*log(2*sqrt(3)*sqrt(3*
x^2 + 5*x - 2) + 6*x + 5) + 5/12*sqrt(3*x^2 + 5*x - 2)
```

Fricas [A] time = 0.223969, size = 88, normalized size = 1.42

$$\frac{1}{144}\sqrt{3}\left(4\sqrt{3}\sqrt{3x^2+5x-2}(6x+5)+49\log\left(\sqrt{3}\left(72x^2+120x+1\right)-12\sqrt{3x^2+5x-2}(6x+5)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 + 5*x - 2),x, algorithm="fricas")
```

```
[Out] 1/144*sqrt(3)*(4*sqrt(3)*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49*log
(sqrt(3)*(72*x^2 + 120*x + 1) - 12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5
)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{3x^2 + 5x - 2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(3*x**2 + 5*x - 2), x)
```

GIAC/XCAS [A] time = 0.212002, size = 73, normalized size = 1.18

$$\frac{1}{12}\sqrt{3\,x^2 + 5\,x - 2}(6\,x + 5) + \frac{49}{72}\sqrt{3}\ln\left(\left|-2\,\sqrt{3}\left(\sqrt{3}x - \sqrt{3\,x^2 + 5\,x - 2}\right) - 5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(3*x^2 + 5*x - 2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(3*x^2 + 5*x - 2)*(6*x + 5) + 49/72*sqrt(3)*ln(abs(-2*sq
rt(3)*(sqrt(3)*x - sqrt(3*x^2 + 5*x - 2)) - 5))
```

3.113 $\int \sqrt{-2+5x-3x^2}\,dx$

Optimal. Leaf size=39

$$-\frac{1}{12}\sqrt{-3x^2+5x-2}(5-6x)-\frac{\sin^{-1}(5-6x)}{24\sqrt{3}}$$

```
[Out] -((5 - 6*x)*Sqrt[-2 + 5*x - 3*x^2])/12 - ArcSin[5 - 6*x]/(24*Sqrt
[3])
```

Rubi [A] time = 0.0222279, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214

$$-\frac{1}{12}\sqrt{-3x^2+5x-2}(5-6x)-\frac{\sin^{-1}(5-6x)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[Sqrt[-2 + 5*x - 3*x^2],x]
```

```
[Out] -((5 - 6*x)*Sqrt[-2 + 5*x - 3*x^2])/12 - ArcSin[5 - 6*x]/(24*Sqrt
[3])
```

Rubi in Sympy [A] time = 2.06051, size = 54, normalized size = 1.38

$$-\frac{(-6x+5)\sqrt{-3x^2+5x-2}}{12}-\frac{\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{3}(-6x+5)}{6\sqrt{-3x^2+5x-2}}\right)}{72}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3*x**2+5*x-2)**(1/2),x)
```

```
[Out] -(-6*x + 5)*sqrt(-3*x**2 + 5*x - 2)/12 - sqrt(3)*atan(sqrt(3)*(-6
*x + 5)/(6*sqrt(-3*x**2 + 5*x - 2)))/72
```

Mathematica [A] time = 0.0330053, size = 40, normalized size = 1.03

$$\left(\frac{x}{2}-\frac{5}{12}\right)\sqrt{-3x^2+5x-2}-\frac{\sin^{-1}(5-6x)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[Sqrt[-2 + 5*x - 3*x^2],x]
```

```
[Out] (-5/12 + x/2)*Sqrt[-2 + 5*x - 3*x^2] - ArcSin[5 - 6*x]/(24*Sqrt[3
])
```

Maple [A] time = 0.005, size = 32, normalized size = 0.8

$$\frac{\arcsin\left(-5+6\,x\right)\sqrt{3}}{72}-\frac{5-6\,x}{12}\sqrt{-3\,x^{2}+5\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3*x^2+5*x-2)^(1/2),x)
```

```
[Out] 1/72*arcsin(-5+6*x)*3^(1/2)-1/12*(5-6*x)*(-3*x^2+5*x-2)^(1/2)
```

Maxima [A] time = 0.864663, size = 55, normalized size = 1.41

$$\frac{1}{2}\,\sqrt{-3\,x^{2}+5\,x-2}x+\frac{1}{72}\,\sqrt{3}\arcsin\left(6\,x-5\right)-\frac{5}{12}\,\sqrt{-3\,x^{2}+5\,x-2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-3*x^2 + 5*x - 2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-3*x^2 + 5*x - 2)*x + 1/72*sqrt(3)*arcsin(6*x - 5) - 5/1
2*sqrt(-3*x^2 + 5*x - 2)
```

Fricas [A] time = 0.223178, size = 69, normalized size = 1.77

$$\frac{1}{72}\,\sqrt{3}\left(2\,\sqrt{3}\sqrt{-3\,x^{2}+5\,x-2}(6\,x-5)+\arctan\left(\frac{\sqrt{3}(6\,x-5)}{6\,\sqrt{-3\,x^{2}+5\,x-2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-3*x^2 + 5*x - 2),x, algorithm="fricas")
```

```
[Out] 1/72*sqrt(3)*(2*sqrt(3)*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) + arctan
(1/6*sqrt(3)*(6*x - 5)/sqrt(-3*x^2 + 5*x - 2)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-3x^2 + 5x - 2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(sqrt(-3*x**2 + 5*x - 2), x)
```

GIAC/XCAS [A] time = 0.209828, size = 42, normalized size = 1.08

$$\frac{1}{12}\,\sqrt{-3\,x^2 + 5\,x - 2}(6\,x - 5) + \frac{1}{72}\,\sqrt{3}\arcsin\left(6\,x - 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(sqrt(-3*x^2 + 5*x - 2),x, algorithm="giac")
```

```
[Out] 1/12*sqrt(-3*x^2 + 5*x - 2)*(6*x - 5) + 1/72*sqrt(3)*arcsin(6*x -
 5)
```

3.114 $\int \frac{1}{\sqrt{5-6x+9x^2}}\,dx$

Optimal. Leaf size=14

$$\frac{1}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)$$

```
[Out] ArcSinh[(-1 + 3*x)/2]/3
```

Rubi [A] time = 0.0167329, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[5 - 6*x + 9*x^2],x]

[Out] ArcSinh[(-1 + 3*x)/2]/3
```

Rubi in Sympy [A] time = 1.50775, size = 22, normalized size = 1.57

$$\frac{\operatorname{atanh}\left(\frac{18x-6}{6\sqrt{9x^2-6x+5}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(9*x**2-6*x+5)**(1/2),x)

[Out] atanh((18*x - 6)/(6*sqrt(9*x**2 - 6*x + 5)))/3
```

Mathematica [A] time = 0.00894224, size = 14, normalized size = 1.

$$\frac{1}{3}\sinh^{-1}\left(\frac{1}{2}(3x-1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[5 - 6*x + 9*x^2],x]

[Out] ArcSinh[(-1 + 3*x)/2]/3
```

Maple [A] time = 0.004, size = 9, normalized size = 0.6

$$\frac{1}{3}Arcsinh\left(-\frac{1}{2}+\frac{3x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(9*x^2-6*x+5)^(1/2),x)

[Out] 1/3*arcsinh(-1/2+3/2*x)
```

Maxima [A] time = 0.870758, size = 11, normalized size = 0.79

$$\frac{1}{3}\,\operatorname{arsinh}\left(\frac{3}{2}\,x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(9*x^2 - 6*x + 5),x, algorithm="maxima")

[Out] 1/3*arcsinh(3/2*x - 1/2)
```

Fricas [A] time = 0.21785, size = 27, normalized size = 1.93

$$-\frac{1}{3}\,\log\left(-3\,x+\sqrt{9\,x^2-6\,x+5}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(9*x^2 - 6*x + 5),x, algorithm="fricas")

[Out] -1/3*log(-3*x + sqrt(9*x^2 - 6*x + 5) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{9x^2 - 6x + 5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(9*x**2-6*x+5)**(1/2),x)
```

```
[Out] Integral(1/sqrt(9*x**2 - 6*x + 5), x)
```

GIAC/XCAS [A] time = 0.212001, size = 27, normalized size = 1.93

$$-\frac{1}{3}\ln\left(-3\,x + \sqrt{9\,x^2 - 6\,x + 5} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(9*x^2 - 6*x + 5),x, algorithm="giac")
```

```
[Out] -1/3*ln(-3*x + sqrt(9*x^2 - 6*x + 5) + 1)
```

3.115 $\int \frac{1}{\sqrt{3-4x-4x^2}}\,dx$

Optimal. Leaf size=10

$$\frac{1}{2}\sin^{-1}\left(x+\frac{1}{2}\right)$$

```
[Out] ArcSin[1/2 + x]/2
```

Rubi [A] time = 0.0146968, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{2}\sin^{-1}\left(x+\frac{1}{2}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[3 - 4*x - 4*x^2],x]

[Out] ArcSin[1/2 + x]/2
```

Rubi in Sympy [A] time = 1.5281, size = 26, normalized size = 2.6

$$\frac{\operatorname{atan}\left(-\frac{-8x-4}{4\sqrt{-4x^2-4x+3}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-4*x**2-4*x+3)**(1/2),x)

[Out] atan(-(-8*x - 4)/(4*sqrt(-4*x**2 - 4*x + 3)))/2
```

Mathematica [A] time = 0.0100993, size = 14, normalized size = 1.4

$$-\frac{1}{2}\sin^{-1}\left(\frac{1}{2}(-2x-1)\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[3 - 4*x - 4*x^2],x]
```

```
[Out] -ArcSin[(-1 - 2*x)/2]/2
```

Maple [A] time = 0.003, size = 7, normalized size = 0.7

$$\frac{1}{2}\arcsin\left(\frac{1}{2}+x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-4*x^2-4*x+3)^(1/2),x)
```

```
[Out] 1/2*arcsin(1/2+x)
```

Maxima [A] time = 0.804628, size = 11, normalized size = 1.1

$$-\frac{1}{2}\arcsin\left(-x-\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-4*x^2 - 4*x + 3),x, algorithm="maxima")
```

```
[Out] -1/2*arcsin(-x - 1/2)
```

Fricas [A] time = 0.228652, size = 28, normalized size = 2.8

$$\frac{1}{2}\arctan\left(\frac{2\,x+1}{\sqrt{-4\,x^2-4\,x+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-4*x^2 - 4*x + 3),x, algorithm="fricas")
```

```
[Out] 1/2*arctan((2*x + 1)/sqrt(-4*x^2 - 4*x + 3))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-4x^2 - 4x + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-4*x**2-4*x+3)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-4*x**2 - 4*x + 3), x)
```

GIAC/XCAS [A] time = 0.211983, size = 8, normalized size = 0.8

$$\frac{1}{2} \arcsin\left(x + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-4*x^2 - 4*x + 3),x, algorithm="giac")
```

```
[Out] 1/2*arcsin(x + 1/2)
```

3.116 $\int \frac{1}{\sqrt{-8+6x+9x^2}}\,dx$

Optimal. Leaf size=25

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

```
[Out] ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x + 9*x^2]]/3
```

Rubi [A] time = 0.0155982, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{1}{3}\tanh^{-1}\left(\frac{3x+1}{\sqrt{9x^2+6x-8}}\right)$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-8 + 6*x + 9*x^2],x]

[Out] ArcTanh[(1 + 3*x)/Sqrt[-8 + 6*x + 9*x^2]]/3
```

Rubi in Sympy [A] time = 1.48554, size = 22, normalized size = 0.88

$$\frac{\operatorname{atanh}\left(\frac{18x+6}{6\sqrt{9x^2+6x-8}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(9*x**2+6*x-8)**(1/2),x)

[Out] atanh((18*x + 6)/(6*sqrt(9*x**2 + 6*x - 8)))/3
```

Mathematica [A] time = 0.00978668, size = 24, normalized size = 0.96

$$\frac{1}{3}\log\left(\sqrt{9x^2+6x-8}+3x+1\right)$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-8 + 6*x + 9*x^2],x]

[Out] Log[1 + 3*x + Sqrt[-8 + 6*x + 9*x^2]]/3
```

Maple [A] time = 0.005, size = 30, normalized size = 1.2

$$\frac{\sqrt{9}}{9}\ln\left(\frac{(9x+3)\sqrt{9}}{9}+\sqrt{9x^2+6x-8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(9*x^2+6*x-8)^(1/2),x)

[Out] 1/9*ln(1/9*(9*x+3)*9^(1/2)+(9*x^2+6*x-8)^(1/2))*9^(1/2)
```

Maxima [A] time = 0.872102, size = 30, normalized size = 1.2

$$\frac{1}{3}\log\left(18x+6\sqrt{9x^2+6x-8}+6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(9*x^2 + 6*x - 8),x, algorithm="maxima")

[Out] 1/3*log(18*x + 6*sqrt(9*x^2 + 6*x - 8) + 6)
```

Fricas [A] time = 0.233257, size = 27, normalized size = 1.08

$$-\frac{1}{3}\log\left(-3x+\sqrt{9x^2+6x-8}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(9*x^2 + 6*x - 8),x, algorithm="fricas")

[Out] -1/3*log(-3*x + sqrt(9*x^2 + 6*x - 8) - 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{9x^2+6x-8}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(9*x**2+6*x-8)**(1/2),x)
```

```
[Out] Integral(1/sqrt(9*x**2 + 6*x - 8), x)
```

GIAC/XCAS [A] time = 0.211914, size = 28, normalized size = 1.12

$$-\frac{1}{3}\,\ln\left(\left|-3\,x+\sqrt{9\,x^2+6\,x-8}-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(9*x^2 + 6*x - 8),x, algorithm="giac")
```

```
[Out] -1/3*ln(abs(-3*x + sqrt(9*x^2 + 6*x - 8) - 1))
```

3.117 $\int \frac{1}{\sqrt{2+4x+3x^2}}\,dx$

Optimal. Leaf size=18

$$\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{3}}$$

```
[Out] ArcSinh[(2 + 3*x)/Sqrt[2]]/Sqrt[3]
```

Rubi [A] time = 0.022426, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 4*x + 3*x^2],x]

[Out] ArcSinh[(2 + 3*x)/Sqrt[2]]/Sqrt[3]
```

Rubi in Sympy [A] time = 1.48068, size = 32, normalized size = 1.78

$$\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}(6x+4)}{6\sqrt{3x^2+4x+2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2+4*x+2)**(1/2),x)

[Out] sqrt(3)*atanh(sqrt(3)*(6*x + 4)/(6*sqrt(3*x**2 + 4*x + 2)))/3
```

Mathematica [A] time = 0.01124, size = 18, normalized size = 1.

$$\frac{\sinh^{-1}\left(\frac{3x+2}{\sqrt{2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 4*x + 3*x^2],x]
```

```
[Out] ArcSinh[(2 + 3*x)/Sqrt[2]]/Sqrt[3]
```

Maple [A] time = 0.004, size = 15, normalized size = 0.8

$$\frac{\sqrt{3}}{3}Arcsinh\left(\frac{3\sqrt{2}}{2}\left(x+\frac{2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2+4*x+2)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*arcsinh(3/2*2^(1/2)*(x+2/3))
```

Maxima [A] time = 0.828608, size = 22, normalized size = 1.22

$$\frac{1}{3}\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\sqrt{2}(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^2 + 4*x + 2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x + 2))
```

Fricas [A] time = 0.231959, size = 55, normalized size = 3.06

$$\frac{1}{6}\sqrt{3}\log\left(-\sqrt{3}\left(9x^2+12x+5\right)-3\sqrt{3x^2+4x+2}(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^2 + 4*x + 2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(-sqrt(3)*(9*x^2 + 12*x + 5) - 3*sqrt(3*x^2 + 4*x
+ 2)*(3*x + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^2 + 4x + 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**2 + 4*x + 2), x)
```

GIAC/XCAS [A] time = 0.214246, size = 45, normalized size = 2.5

$$-\frac{1}{3}\,\sqrt{3}\ln\left(-\sqrt{3}\left(\sqrt{3}x - \sqrt{3\,x^2 + 4\,x + 2}\right) - 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^2 + 4*x + 2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*ln(-sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 4*x + 2)) - 2)
```

3.118 $\int \frac{1}{\sqrt{2+4x-3x^2}}\,dx$

Optimal. Leaf size=19

$$-\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcSin[(2 - 3*x)/Sqrt[10]]/Sqrt[3])
```

Rubi [A] time = 0.0263589, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 4*x - 3*x^2],x]

[Out] -(ArcSin[(2 - 3*x)/Sqrt[10]]/Sqrt[3])
```

Rubi in Sympy [A] time = 1.53222, size = 34, normalized size = 1.79

$$-\frac{\sqrt{3}\,\text{atan}\left(\frac{\sqrt{3}(-6x+4)}{6\sqrt{-3x^2+4x+2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+4*x+2)**(1/2),x)

[Out] -sqrt(3)*atan(sqrt(3)*(-6*x + 4)/(6*sqrt(-3*x**2 + 4*x + 2)))/3
```

Mathematica [A] time = 0.0162651, size = 19, normalized size = 1.

$$-\frac{\sin^{-1}\left(\frac{2-3x}{\sqrt{10}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 4*x - 3*x^2],x]

[Out] -(ArcSin[(2 - 3*x)/Sqrt[10]]/Sqrt[3])
```

Maple [A] time = 0.004, size = 15, normalized size = 0.8

$$\frac{\sqrt{3}}{3}\arcsin\left(\frac{3\sqrt{10}}{10}\left(x-\frac{2}{3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+4*x+2)^(1/2),x)

[Out] 1/3*3^(1/2)*arcsin(3/10*10^(1/2)*(x-2/3))
```

Maxima [A] time = 0.883958, size = 22, normalized size = 1.16

$$-\frac{1}{3}\sqrt{3}\arcsin\left(-\frac{1}{10}\sqrt{10}(3x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^2 + 4*x + 2),x, algorithm="maxima")

[Out] -1/3*sqrt(3)*arcsin(-1/10*sqrt(10)*(3*x - 2))
```

Fricas [A] time = 0.232517, size = 38, normalized size = 2.

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(3x-2)}{3\sqrt{-3x^2+4x+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^2 + 4*x + 2),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(3*x - 2)/sqrt(-3*x^2 + 4*x + 2))
```

<u>Sympy</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^2 + 4x + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+4*x+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**2 + 4*x + 2), x)
```

<u>GIAC/XCAS</u> [A] time = 0.215309, size = 22, normalized size = 1.16

$$\frac{1}{3}\,\sqrt{3}\arcsin\left(\frac{1}{10}\,\sqrt{10}(3\,x - 2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^2 + 4*x + 2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arcsin(1/10*sqrt(10)*(3*x - 2))
```

3.119 $\int \frac{1}{\sqrt{2+5x+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{\sqrt{3}}$$

```
[Out] ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2 + 5*x + 3*x^2])]/Sqrt[3]
```

Rubi [A] time = 0.0199993, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x+2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x + 3*x^2],x]

[Out] ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[2 + 5*x + 3*x^2])]/Sqrt[3]
```

Rubi in Sympy [A] time = 1.48974, size = 32, normalized size = 0.91

$$\frac{\sqrt{3}\,\operatorname{atanh}\left(\frac{\sqrt{3}(6x+5)}{6\sqrt{3x^2+5x+2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2+5*x+2)**(1/2),x)

[Out] sqrt(3)*atanh(sqrt(3)*(6*x + 5)/(6*sqrt(3*x**2 + 5*x + 2)))/3
```

Mathematica [A] time = 0.0167892, size = 28, normalized size = 0.8

$$\frac{\log\left(2\sqrt{9x^2+15x+6}+6x+5\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 5*x + 3*x^2],x]
```

```
[Out] Log[5 + 6*x + 2*Sqrt[6 + 15*x + 9*x^2]]/Sqrt[3]
```

Maple [A] time = 0.004, size = 30, normalized size = 0.9

$$\frac{\sqrt{3}}{3}\ln\left(\frac{\sqrt{3}}{3}\left(\frac{5}{2}+3x\right)+\sqrt{3x^2+5x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2+5*x+2)^(1/2),x)
```

```
[Out] 1/3*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2)
```

Maxima [A] time = 0.84768, size = 38, normalized size = 1.09

$$\frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+5x+2}+6x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^2 + 5*x + 2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x + 2) + 6*x + 5)
```

Fricas [A] time = 0.231341, size = 54, normalized size = 1.54

$$\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\left(72x^2+120x+49\right)+12\sqrt{3x^2+5x+2}(6x+5)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^2 + 5*x + 2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(sqrt(3)*(72*x^2 + 120*x + 49) + 12*sqrt(3*x^2 + 5
*x + 2)*(6*x + 5))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^2+5x+2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**2 + 5*x + 2), x)
```

GIAC/XCAS [A] time = 0.215823, size = 46, normalized size = 1.31

$$-\frac{1}{3}\,\sqrt{3}\ln\left(\left|-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+5\,x+2}\right)-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^2 + 5*x + 2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*ln(abs(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 5*x + 2)
) - 5))
```

3.120 $\int \frac{1}{\sqrt{2+5x-3x^2}}\,dx$

Optimal. Leaf size=17

$$-\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

```
[Out] -(ArcSin[(5 - 6*x)/7]/Sqrt[3])
```

Rubi [A] time = 0.0166977, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[2 + 5*x - 3*x^2],x]

[Out] -(ArcSin[(5 - 6*x)/7]/Sqrt[3])
```

Rubi in Sympy [A] time = 1.49799, size = 34, normalized size = 2.

$$-\frac{\sqrt{3}\,\text{atan}\left(\frac{\sqrt{3}(-6x+5)}{6\sqrt{-3x^2+5x+2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+5*x+2)**(1/2),x)

[Out] -sqrt(3)*atan(sqrt(3)*(-6*x + 5)/(6*sqrt(-3*x**2 + 5*x + 2)))/3
```

Mathematica [A] time = 0.0103534, size = 17, normalized size = 1.

$$-\frac{\sin^{-1}\left(\frac{1}{7}(5-6x)\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[2 + 5*x - 3*x^2],x]
```

```
[Out] -(ArcSin[(5 - 6*x)/7]/Sqrt[3])
```

Maple [A] time = 0.005, size = 12, normalized size = 0.7

$$\frac{\sqrt{3}}{3}\arcsin\left(-\frac{5}{7}+\frac{6x}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+5*x+2)^(1/2),x)
```

```
[Out] 1/3*arcsin(-5/7+6/7*x)*3^(1/2)
```

Maxima [A] time = 0.848237, size = 15, normalized size = 0.88

$$-\frac{1}{3}\sqrt{3}\arcsin\left(-\frac{6}{7}x+\frac{5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^2 + 5*x + 2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*arcsin(-6/7*x + 5/7)
```

Fricas [A] time = 0.220069, size = 38, normalized size = 2.24

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(6x-5)}{6\sqrt{-3x^2+5x+2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^2 + 5*x + 2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/6*sqrt(3)*(6*x - 5)/sqrt(-3*x^2 + 5*x + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^2 + 5x + 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+5*x+2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**2 + 5*x + 2), x)
```

GIAC/XCAS [A] time = 0.21623, size = 15, normalized size = 0.88

$$\frac{1}{3}\sqrt{3}\arcsin\left(\frac{6}{7}\,x - \frac{5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^2 + 5*x + 2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arcsin(6/7*x - 5/7)
```

3.121 $\int \frac{1}{\sqrt{-2+4x+3x^2}}\,dx$

Optimal. Leaf size=32

$$\frac{\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{\sqrt{3}}$$

```
[Out] ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x + 3*x^2])]/Sqrt[3]
```

Rubi [A] time = 0.020452, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{\tanh^{-1}\left(\frac{3x+2}{\sqrt{3}\sqrt{3x^2+4x-2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 + 4*x + 3*x^2],x]

[Out] ArcTanh[(2 + 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x + 3*x^2])]/Sqrt[3]
```

Rubi in Sympy [A] time = 1.48087, size = 32, normalized size = 1.

$$\frac{\sqrt{3}\operatorname{atanh}\left(\frac{\sqrt{3}(6x+4)}{6\sqrt{3x^2+4x-2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2+4*x-2)**(1/2),x)

[Out] sqrt(3)*atanh(sqrt(3)*(6*x + 4)/(6*sqrt(3*x**2 + 4*x - 2)))/3
```

Mathematica [A] time = 0.0102251, size = 26, normalized size = 0.81

$$\frac{\log\left(\sqrt{9x^2+12x-6}+3x+2\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 + 4*x + 3*x^2],x]
```

```
[Out] Log[2 + 3*x + Sqrt[-6 + 12*x + 9*x^2]]/Sqrt[3]
```

Maple [A] time = 0.004, size = 30, normalized size = 0.9

$$\frac{\sqrt{3}}{3}\ln\left(\frac{(2+3x)\sqrt{3}}{3}+\sqrt{3x^2+4x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2+4*x-2)^(1/2),x)
```

```
[Out] 1/3*ln(1/3*(2+3*x)*3^(1/2)+(3*x^2+4*x-2)^(1/2))*3^(1/2)
```

Maxima [A] time = 0.834002, size = 38, normalized size = 1.19

$$\frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+4x-2}+6x+4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^2 + 4*x - 2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 4*x - 2) + 6*x + 4)
```

Fricas [A] time = 0.223004, size = 54, normalized size = 1.69

$$\frac{1}{6}\sqrt{3}\log\left(\sqrt{3}\left(9x^2+12x-1\right)+3\sqrt{3x^2+4x-2}(3x+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^2 + 4*x - 2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(sqrt(3)*(9*x^2 + 12*x - 1) + 3*sqrt(3*x^2 + 4*x -
 2)*(3*x + 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^2+4x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**2 + 4*x - 2), x)
```

GIAC/XCAS [A] time = 0.214449, size = 46, normalized size = 1.44

$$-\frac{1}{3}\sqrt{3}\ln\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+4\,x-2}\right)-2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^2 + 4*x - 2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*ln(abs(-sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 4*x - 2))
- 2))
```

3.122 $\int \frac{1}{\sqrt{-2+4x-3x^2}}\,dx$

Optimal. Leaf size=33

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{\sqrt{3}}$$

```
[Out] -(ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x - 3*x^2])]/Sqrt[3])
```

Rubi [A] time = 0.0203906, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{3}\sqrt{-3x^2+4x-2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 + 4*x - 3*x^2],x]

[Out] -(ArcTan[(2 - 3*x)/(Sqrt[3]*Sqrt[-2 + 4*x - 3*x^2])]/Sqrt[3])
```

Rubi in Sympy [A] time = 1.48579, size = 34, normalized size = 1.03

$$-\frac{\sqrt{3}\,\operatorname{atan}\left(\frac{\sqrt{3}(-6x+4)}{6\sqrt{-3x^2+4x-2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+4*x-2)**(1/2),x)

[Out] -sqrt(3)*atan(sqrt(3)*(-6*x + 4)/(6*sqrt(-3*x**2 + 4*x - 2)))/3
```

Mathematica [A] time = 0.0224558, size = 28, normalized size = 0.85

$$-\frac{\tan^{-1}\left(\frac{2-3x}{\sqrt{-9x^2+12x-6}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 + 4*x - 3*x^2],x]
```

```
[Out] -(ArcTan[(2 - 3*x)/Sqrt[-6 + 12*x - 9*x^2]]/Sqrt[3])
```

Maple [A] time = 0.003, size = 26, normalized size = 0.8

$$\frac{\sqrt{3}}{3}\arctan\left(\sqrt{3}\left(x-\frac{2}{3}\right)\frac{1}{\sqrt{-3\,x^{2}+4\,x-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+4*x-2)^(1/2),x)
```

```
[Out] 1/3*3^(1/2)*arctan(3^(1/2)*(x-2/3)/(-3*x^2+4*x-2)^(1/2))
```

Maxima [A] time = 0.796606, size = 22, normalized size = 0.67

$$-\frac{1}{3}i\,\sqrt{3}\,\mathrm{arsinh}\left(\frac{1}{2}\,\sqrt{2}(3\,x-2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^2 + 4*x - 2),x, algorithm="maxima")
```

```
[Out] -1/3*I*sqrt(3)*arcsinh(1/2*sqrt(2)*(3*x - 2))
```

Fricas [A] time = 0.223021, size = 86, normalized size = 2.61

$$\frac{1}{6}\,\sqrt{3}\left(-i\,\log\left(\frac{2i\,\sqrt{3}\sqrt{-3\,x^{2}+4\,x-2}-6\,x+4}{x}\right)+i\,\log\left(\frac{-2i\,\sqrt{3}\sqrt{-3\,x^{2}+4\,x-2}-6\,x+4}{x}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^2 + 4*x - 2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*(-I*log((2*I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) - 6*x + 4
)/x) + I*log((-2*I*sqrt(3)*sqrt(-3*x^2 + 4*x - 2) - 6*x + 4)/x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^2 + 4x - 2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+4*x-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**2 + 4*x - 2), x)
```

GIAC/XCAS [A] time = 0.215377, size = 24, normalized size = 0.73

$$-\frac{1}{3}\sqrt{3}i \arcsin\left(\frac{1}{2}\sqrt{2}i(3x - 2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^2 + 4*x - 2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*i*arcsin(1/2*sqrt(2)*i*(3*x - 2))
```

3.123 $\int \frac{1}{\sqrt{-2+5x+3x^2}}\,dx$

Optimal. Leaf size=35

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{\sqrt{3}}$$

```
[Out] ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[-2 + 5*x + 3*x^2])]/Sqrt[3]
```

Rubi [A] time = 0.0204264, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{\tanh^{-1}\left(\frac{6x+5}{2\sqrt{3}\sqrt{3x^2+5x-2}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 + 5*x + 3*x^2],x]

[Out] ArcTanh[(5 + 6*x)/(2*Sqrt[3]*Sqrt[-2 + 5*x + 3*x^2])]/Sqrt[3]
```

Rubi in Sympy [A] time = 1.49536, size = 32, normalized size = 0.91

$$\frac{\sqrt{3}\,\text{atanh}\left(\frac{\sqrt{3}(6x+5)}{6\sqrt{3x^2+5x-2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(3*x**2+5*x-2)**(1/2),x)

[Out] sqrt(3)*atanh(sqrt(3)*(6*x + 5)/(6*sqrt(3*x**2 + 5*x - 2)))/3
```

Mathematica [A] time = 0.0155857, size = 28, normalized size = 0.8

$$\frac{\log\left(2\sqrt{9x^2+15x-6}+6x+5\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 + 5*x + 3*x^2],x]
```

```
[Out] Log[5 + 6*x + 2*Sqrt[-6 + 15*x + 9*x^2]]/Sqrt[3]
```

Maple [A] time = 0.003, size = 30, normalized size = 0.9

$$\frac{\sqrt{3}}{3}\ln\left(\frac{\sqrt{3}}{3}\left(\frac{5}{2}+3\,x\right)+\sqrt{3\,x^{2}+5\,x-2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(3*x^2+5*x-2)^(1/2),x)
```

```
[Out] 1/3*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x-2)^(1/2))*3^(1/2)
```

Maxima [A] time = 0.813478, size = 38, normalized size = 1.09

$$\frac{1}{3}\,\sqrt{3}\log\left(2\,\sqrt{3}\sqrt{3\,x^{2}+5\,x-2}+6\,x+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^2 + 5*x - 2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 5*x - 2) + 6*x + 5)
```

Fricas [A] time = 0.222686, size = 54, normalized size = 1.54

$$\frac{1}{6}\,\sqrt{3}\log\left(\sqrt{3}\left(72\,x^{2}+120\,x+1\right)+12\,\sqrt{3\,x^{2}+5\,x-2}(6\,x+5)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^2 + 5*x - 2),x, algorithm="fricas")
```

```
[Out] 1/6*sqrt(3)*log(sqrt(3)*(72*x^2 + 120*x + 1) + 12*sqrt(3*x^2 + 5*
x - 2)*(6*x + 5))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{3x^2+5x-2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(3*x**2 + 5*x - 2), x)
```

GIAC/XCAS [A] time = 0.215905, size = 46, normalized size = 1.31

$$-\frac{1}{3}\,\sqrt{3}\ln\left(\left|-2\,\sqrt{3}\left(\sqrt{3}x-\sqrt{3\,x^2+5\,x-2}\right)-5\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(3*x^2 + 5*x - 2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(3)*ln(abs(-2*sqrt(3)*(sqrt(3)*x - sqrt(3*x^2 + 5*x - 2)
) - 5))
```

3.124 $\int \frac{1}{\sqrt{-2+5x-3x^2}}\,dx$

Optimal. Leaf size=13

$$-\frac{\sin^{-1}(5-6x)}{\sqrt{3}}$$

```
[Out] -(ArcSin[5 - 6*x]/Sqrt[3])
```

Rubi [A] time = 0.0107924, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$-\frac{\sin^{-1}(5-6x)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[-2 + 5*x - 3*x^2],x]

[Out] -(ArcSin[5 - 6*x]/Sqrt[3])
```

Rubi in Sympy [A] time = 1.52518, size = 34, normalized size = 2.62

$$-\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}(-6x+5)}{6\sqrt{-3x^2+5x-2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-3*x**2+5*x-2)**(1/2),x)

[Out] -sqrt(3)*atan(sqrt(3)*(-6*x + 5)/(6*sqrt(-3*x**2 + 5*x - 2)))/3
```

Mathematica [A] time = 0.0103975, size = 13, normalized size = 1.

$$-\frac{\sin^{-1}(5-6x)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[-2 + 5*x - 3*x^2],x]
```

```
[Out] -(ArcSin[5 - 6*x]/Sqrt[3])
```

Maple [A] time = 0.004, size = 12, normalized size = 0.9

$$\frac{\arcsin\left(-5+6\,x\right)\sqrt{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-3*x^2+5*x-2)^(1/2),x)
```

```
[Out] 1/3*arcsin(-5+6*x)*3^(1/2)
```

Maxima [A] time = 0.789, size = 15, normalized size = 1.15

$$\frac{1}{3}\sqrt{3}\arcsin\left(6\,x-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^2 + 5*x - 2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(3)*arcsin(6*x - 5)
```

Fricas [A] time = 0.222689, size = 38, normalized size = 2.92

$$\frac{1}{3}\sqrt{3}\arctan\left(\frac{\sqrt{3}(6\,x-5)}{6\sqrt{-3\,x^2+5\,x-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^2 + 5*x - 2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/6*sqrt(3)*(6*x - 5)/sqrt(-3*x^2 + 5*x - 2))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-3x^2 + 5x - 2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-3*x**2+5*x-2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-3*x**2 + 5*x - 2), x)
```

GIAC/XCAS [A] time = 0.214524, size = 15, normalized size = 1.15

$$\frac{1}{3}\,\sqrt{3}\arcsin\left(6\,x - 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/sqrt(-3*x^2 + 5*x - 2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*arcsin(6*x - 5)
```

3.125 $\int \frac{1}{\sqrt{\frac{b^2+4c}{4c}+bx+cx^2}}\,dx$

Optimal. Leaf size=22

$$\frac{\sinh^{-1}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] ArcSinh[(b + 2*c*x)/(2*Sqrt[c])]/Sqrt[c]
```

Rubi [A] time = 0.0292301, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074

$$\frac{\sinh^{-1}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(b^2 + 4*c)/(4*c) + b*x + c*x^2],x]

[Out] ArcSinh[(b + 2*c*x)/(2*Sqrt[c])]/Sqrt[c]
```

Rubi in Sympy [A] time = 6.63212, size = 42, normalized size = 1.91

$$\frac{\operatorname{atanh}\left(\frac{4b+8cx}{4\sqrt{c}\sqrt{\frac{b^2}{c}+4bx+4cx^2+4}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(2/((b**2+4*c)/c+4*b*x+4*c*x**2)**(1/2),x)

[Out] atanh((4*b + 8*c*x)/(4*sqrt(c)*sqrt(b**2/c + 4*b*x + 4*c*x**2 + 4
)))/sqrt(c)
```

Mathematica [A] time = 0.0449615, size = 22, normalized size = 1.

$$\frac{\sinh^{-1}\left(\frac{b+2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(b^2 + 4*c)/(4*c) + b*x + c*x^2],x]
```

```
[Out] ArcSinh[(b + 2*c*x)/(2*Sqrt[c])]/Sqrt[c]
```

Maple [B] time = 0.015, size = 51, normalized size = 2.3

$$\frac{\sqrt{4}}{2}\ln\left(\frac{(4\,cx+2\,b)\sqrt{4}}{4}\frac{1}{\sqrt{c}}+\sqrt{\frac{b^2+4\,c}{c}+4\,bx+4\,cx^2}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(2/((b^2+4*c)/c+4*b*x+4*c*x^2)^(1/2),x)
```

```
[Out] 1/2*ln(1/4*(4*c*x+2*b)*4^(1/2)/c^(1/2)+((b^2+4*c)/c+4*b*x+4*c*x^2
)^(1/2))*4^(1/2)/c^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2/sqrt(4*c*x^2 + 4*b*x + (b^2 + 4*c)/c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.237165, size = 1, normalized size = 0.05

$$\left[\frac{\log\left(-\left(4\,c^2x^2+4\,bcx+b^2+2\,c\right)\sqrt{c}-\left(2\,c^2x+bc\right)\sqrt{\frac{4\,c^2x^2+4\,bcx+b^2+4\,c}{c}}\right)}{2\,\sqrt{c}},\ \frac{\arctan\left(\frac{(2\,cx+b)\sqrt{-c}}{c\sqrt{\frac{4\,c^2x^2+4\,bcx+b^2+4\,c}{c}}}\right)}{\sqrt{-c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2/sqrt(4*c*x^2 + 4*b*x + (b^2 + 4*c)/c),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(4*c^2*x^2 + 4*b*c*x + b^2 + 2*c)*sqrt(c) - (2*c^2*x +
b*c)*sqrt((4*c^2*x^2 + 4*b*c*x + b^2 + 4*c)/c))/sqrt(c), arctan((
2*c*x + b)*sqrt(-c)/(c*sqrt((4*c^2*x^2 + 4*b*c*x + b^2 + 4*c)/c))
)/sqrt(-c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int \frac{1}{\sqrt{\frac{b^2}{c} + 4bx + 4cx^2 + 4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2/((b**2+4*c)/c+4*b*x+4*c*x**2)**(1/2),x)
```

```
[Out] 2*Integral(1/sqrt(b**2/c + 4*b*x + 4*c*x**2 + 4), x)
```

GIAC/XCAS [A] time = 0.233485, size = 66, normalized size = 3.

$$-\frac{\ln\left(\left|-\left(2\sqrt{c}x - \sqrt{4\,cx^2 + 4\,bx + \frac{b^2+4\,c}{c}}\right)\sqrt{c} - b\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(2/sqrt(4*c*x^2 + 4*b*x + (b^2 + 4*c)/c),x, algorithm="giac")
```

```
[Out] -ln(abs(-(2*sqrt(c)*x - sqrt(4*c*x^2 + 4*b*x + (b^2 + 4*c)/c))*sq
rt(c) - b))/sqrt(c)
```

3.126 $\int \frac{1}{\sqrt{\frac{-b^2+4c}{4c}+bx-cx^2}}\,dx$

Optimal. Leaf size=23

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] -(ArcSin[(b - 2*c*x)/(2*Sqrt[c])]/Sqrt[c])
```

Rubi [A] time = 0.0283524, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(-b^2 + 4*c)/(4*c) + b*x - c*x^2],x]

[Out] -(ArcSin[(b - 2*c*x)/(2*Sqrt[c])]/Sqrt[c])
```

Rubi in Sympy [A] time = 6.40854, size = 44, normalized size = 1.91

$$-\frac{\operatorname{atan}\left(\frac{4b-8cx}{4\sqrt{c}\sqrt{-\frac{b^2}{c}+4bx-4cx^2+4}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(2/((-b**2+4*c)/c+4*b*x-4*c*x**2)**(1/2),x)

[Out] -atan((4*b - 8*c*x)/(4*sqrt(c)*sqrt(-b**2/c + 4*b*x - 4*c*x**2 +
4)))/sqrt(c)
```

Mathematica [A] time = 0.0447288, size = 23, normalized size = 1.

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{2\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(-b^2 + 4*c)/(4*c) + b*x - c*x^2],x]
```

```
[Out] -(ArcSin[(b - 2*c*x)/(2*Sqrt[c])]/Sqrt[c])
```

Maple [B] time = 0.018, size = 44, normalized size = 1.9

$$1\arctan\left(2\sqrt{c}\left(x-1/2\,\frac{b}{c}\right)\frac{1}{\sqrt{-4\,cx^2+4\,bx-\frac{b^2-4\,c}{c}}}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(2/((-b^2+4*c)/c+4*b*x-4*c*x^2)^(1/2),x)
```

```
[Out] 1/c^(1/2)*arctan(2*c^(1/2)*(x-1/2*b/c)/(-4*c*x^2+4*b*x-(b^2-4*c)/
c)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/sqrt(-4*c*x^2 + 4*b*x - (b^2 - 4*c)/c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.236473, size = 1, normalized size = 0.04

$$\left[\frac{\log\left(\left(4\,c^2x^2-4\,bcx+b^2-2\,c\right)\sqrt{-c}+\left(2\,c^2x-bc\right)\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-4\,c}{c}}\right)}{2\sqrt{-c}},\ \frac{\arctan\left(\frac{2\,cx-b}{\sqrt{c}\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-4\,c}{c}}}\right)}{\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/sqrt(-4*c*x^2 + 4*b*x - (b^2 - 4*c)/c),x, algorithm="fricas")
```

```
[Out] [1/2*log((4*c^2*x^2 - 4*b*c*x + b^2 - 2*c)*sqrt(-c) + (2*c^2*x -
b*c)*sqrt(-(4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)/c))/sqrt(-c), arctan
((2*c*x - b)/(sqrt(c)*sqrt(-(4*c^2*x^2 - 4*b*c*x + b^2 - 4*c)/c))
)/sqrt(c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int \frac{1}{\sqrt{-\frac{b^2}{c} + 4bx - 4cx^2 + 4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b**2+4*c)/c+4*b*x-4*c*x**2)**(1/2),x)
```

```
[Out] 2*Integral(1/sqrt(-b**2/c + 4*b*x - 4*c*x**2 + 4), x)
```

GIAC/XCAS [A] time = 0.229464, size = 72, normalized size = 3.13

$$-\frac{\ln\left(\left|\left(2\sqrt{-c}x - \sqrt{-4\,cx^2 + 4\,bx - \frac{b^2-4\,c}{c}}\right)\sqrt{-c} + b\right|\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/sqrt(-4*c*x^2 + 4*b*x - (b^2 - 4*c)/c),x, algorithm="giac")
```

```
[Out] -ln(abs((2*sqrt(-c)*x - sqrt(-4*c*x^2 + 4*b*x - (b^2 - 4*c)/c))*s
qrt(-c) + b))/sqrt(-c)
```

3.127 $\int \frac{1}{\sqrt{\frac{-b^2+c}{4c}+bx-cx^2}}\,dx$

Optimal. Leaf size=20

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

```
[Out] -(ArcSin[(b - 2*c*x)/Sqrt[c]]/Sqrt[c])
```

Rubi [A] time = 0.0246918, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Int[1/Sqrt[(-b^2 + c)/(4*c) + b*x - c*x^2],x]

[Out] -(ArcSin[(b - 2*c*x)/Sqrt[c]]/Sqrt[c])
```

Rubi in Sympy [A] time = 6.57462, size = 44, normalized size = 2.2

$$-\frac{\operatorname{atan}\left(\frac{4b-8cx}{4\sqrt{c}\sqrt{4bx-4cx^2+\frac{-b^2+c}{c}}}\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(2/((-b**2+c)/c+4*b*x-4*c*x**2)**(1/2),x)

[Out] -atan((4*b - 8*c*x)/(4*sqrt(c)*sqrt(4*b*x - 4*c*x**2 + (-b**2 + c
)/c)))/sqrt(c)
```

Mathematica [A] time = 0.0424198, size = 20, normalized size = 1.

$$-\frac{\sin^{-1}\left(\frac{b-2cx}{\sqrt{c}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[1/Sqrt[(-b^2 + c)/(4*c) + b*x - c*x^2],x]
```

```
[Out] -(ArcSin[(b - 2*c*x)/Sqrt[c]]/Sqrt[c])
```

Maple [B] time = 0.016, size = 44, normalized size = 2.2

$$1\arctan\left(2\sqrt{c}\left(x-1/2\,\frac{b}{c}\right)\frac{1}{\sqrt{-4\,cx^2+4\,bx-\frac{b^2-c}{c}}}\right)\frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(2/((-b^2+c)/c+4*b*x-4*c*x^2)^(1/2),x)
```

```
[Out] 1/c^(1/2)*arctan(2*c^(1/2)*(x-1/2*b/c)/(-4*c*x^2+4*b*x-(b^2-c)/c)
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/sqrt(-4*c*x^2 + 4*b*x - (b^2 - c)/c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.24696, size = 1, normalized size = 0.05

$$\left[\frac{\log\left(\left(8\,c^2x^2-8\,bcx+2\,b^2-c\right)\sqrt{-c}+2\,\left(2\,c^2x-bc\right)\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-c}{c}}\right)}{2\,\sqrt{-c}},\ \frac{\arctan\left(\frac{2\,cx-b}{\sqrt{c}\sqrt{-\frac{4\,c^2x^2-4\,bcx+b^2-c}{c}}}\right)}{\sqrt{c}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/sqrt(-4*c*x^2 + 4*b*x - (b^2 - c)/c),x, algorithm="fricas")
```

```
[Out] [1/2*log((8*c^2*x^2 - 8*b*c*x + 2*b^2 - c)*sqrt(-c) + 2*(2*c^2*x
- b*c)*sqrt(-(4*c^2*x^2 - 4*b*c*x + b^2 - c)/c))/sqrt(-c), arctan
((2*c*x - b)/(sqrt(c)*sqrt(-(4*c^2*x^2 - 4*b*c*x + b^2 - c)/c)))/
sqrt(c)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$2\int \frac{1}{\sqrt{-\frac{b^2}{c} + 4bx - 4cx^2 + 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/((-b**2+c)/c+4*b*x-4*c*x**2)**(1/2),x)
```

```
[Out] 2*Integral(1/sqrt(-b**2/c + 4*b*x - 4*c*x**2 + 1), x)
```

GIAC/XCAS [A] time = 0.230213, size = 72, normalized size = 3.6

$$-\frac{\ln\left(\left|\left(2\sqrt{-c}x - \sqrt{-4\,cx^2 + 4\,bx - \frac{b^2-c}{c}}\right)\sqrt{-c} + b\right|\right)}{\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(2/sqrt(-4*c*x^2 + 4*b*x - (b^2 - c)/c),x, algorithm="giac")
```

```
[Out] -ln(abs((2*sqrt(-c)*x - sqrt(-4*c*x^2 + 4*b*x - (b^2 - c)/c))*sqr
t(-c) + b))/sqrt(-c)
```

3.128 $\int \frac{1}{(2+3x+x^2)^{3/2}}\,dx$

Optimal. Leaf size=19

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

```
[Out] (-2*(3 + 2*x))/Sqrt[2 + 3*x + x^2]
```

Rubi [A] time = 0.0107709, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(2 + 3*x + x^2)^(-3/2),x]

[Out] (-2*(3 + 2*x))/Sqrt[2 + 3*x + x^2]
```

Rubi in Sympy [A] time = 1.19142, size = 17, normalized size = 0.89

$$-\frac{4x+6}{\sqrt{x^2+3x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(x**2+3*x+2)**(3/2),x)

[Out] -(4*x + 6)/sqrt(x**2 + 3*x + 2)
```

Mathematica [A] time = 0.0122003, size = 19, normalized size = 1.

$$-\frac{2(2x+3)}{\sqrt{x^2+3x+2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(2 + 3*x + x^2)^(-3/2),x]
```

```
[Out] (-2*(3 + 2*x))/Sqrt[2 + 3*x + x^2]
```

Maple **[A]** time = 0.005, size = 24, normalized size = 1.3

$$-2\,\frac{(2+x)\,(1+x)\,(2\,x+3)}{\left(x^{2}+3\,x+2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(x^2+3*x+2)^(3/2),x)
```

```
[Out] -2*(2+x)*(1+x)*(2*x+3)/(x^2+3*x+2)^(3/2)
```

Maxima **[A]** time = 0.745951, size = 35, normalized size = 1.84

$$-\frac{4\,x}{\sqrt{x^{2}+3\,x+2}}-\frac{6}{\sqrt{x^{2}+3\,x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3*x + 2)^(-3/2),x, algorithm="maxima")
```

```
[Out] -4*x/sqrt(x^2 + 3*x + 2) - 6/sqrt(x^2 + 3*x + 2)
```

Fricas **[A]** time = 0.221172, size = 42, normalized size = 2.21

$$\frac{2}{2\,x^{2}-\sqrt{x^{2}+3\,x+2}(2\,x+3)+6\,x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3*x + 2)^(-3/2),x, algorithm="fricas")
```

```
[Out] 2/(2*x^2 - sqrt(x^2 + 3*x + 2)*(2*x + 3) + 6*x + 4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 + 3x + 2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(x**2+3*x+2)**(3/2),x)
```

```
[Out] Integral((x**2 + 3*x + 2)**(-3/2), x)
```

GIAC/XCAS [A] time = 0.213333, size = 23, normalized size = 1.21

$$-\frac{2\,(2\,x + 3)}{\sqrt{x^2 + 3\,x + 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 3*x + 2)^(-3/2),x, algorithm="giac")
```

```
[Out] -2*(2*x + 3)/sqrt(x^2 + 3*x + 2)
```

3.129 $\int \frac{1}{(27-24x+4x^2)^{3/2}}\, dx$

Optimal. Leaf size=23

$$\frac{3-x}{9\sqrt{4x^2-24x+27}}$$

[Out] (3 - x)/(9*Sqrt[27 - 24*x + 4*x^2])

Rubi [A] time = 0.0102923, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{3-x}{9\sqrt{4x^2-24x+27}}$$

Antiderivative was successfully verified.

[In] Int[(27 - 24*x + 4*x^2)^(-3/2),x]

[Out] (3 - x)/(9*Sqrt[27 - 24*x + 4*x^2])

Rubi in Sympy [A] time = 1.28625, size = 19, normalized size = 0.83

$$\frac{-16x+48}{144\sqrt{4x^2-24x+27}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi_integrate(1/(4*x**2-24*x+27)**(3/2),x)

[Out] (-16*x + 48)/(144*sqrt(4*x**2 - 24*x + 27))

Mathematica [A] time = 0.015467, size = 23, normalized size = 1.

$$\frac{3-x}{9\sqrt{4x^2-24x+27}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(27 - 24*x + 4*x^2)^(-3/2),x]
```

```
[Out] (3 - x)/(9*Sqrt[27 - 24*x + 4*x^2])
```

Maple [A] time = 0.004, size = 28, normalized size = 1.2

$$-\frac{(-3+2x)(2x-9)(x-3)}{9}\left(4x^2-24x+27\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(4*x^2-24*x+27)^(3/2),x)
```

```
[Out] -1/9*(-3+2*x)*(2*x-9)*(x-3)/(4*x^2-24*x+27)^(3/2)
```

Maxima [A] time = 0.763091, size = 41, normalized size = 1.78

$$-\frac{x}{9\sqrt{4x^2-24x+27}}+\frac{1}{3\sqrt{4x^2-24x+27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 24*x + 27)^(-3/2),x, algorithm="maxima")
```

```
[Out] -1/9*x/sqrt(4*x^2 - 24*x + 27) + 1/3/sqrt(4*x^2 - 24*x + 27)
```

Fricas [A] time = 0.209984, size = 42, normalized size = 1.83

$$\frac{1}{2\left(4x^2-2\sqrt{4x^2-24x+27}(x-3)-24x+27\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 24*x + 27)^(-3/2),x, algorithm="fricas")
```

```
[Out] 1/2/(4*x^2 - 2*sqrt(4*x^2 - 24*x + 27)*(x - 3) - 24*x + 27)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(4x^2 - 24x + 27)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(4*x**2-24*x+27)**(3/2),x)
```

```
[Out] Integral((4*x**2 - 24*x + 27)**(-3/2), x)
```

GIAC/XCAS [A] time = 0.212368, size = 23, normalized size = 1.

$$-\frac{x - 3}{9\sqrt{4\,x^2 - 24\,x + 27}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 - 24*x + 27)^(-3/2),x, algorithm="giac")
```

```
[Out] -1/9*(x - 3)/sqrt(4*x^2 - 24*x + 27)
```

3.130 $\int \frac{x}{(5-4x-x^2)^{3/2}} \, dx$

Optimal. Leaf size=23

$$\frac{5-2x}{9\sqrt{-x^2-4x+5}}$$

```
[Out] (5 - 2*x)/(9*Sqrt[5 - 4*x - x^2])
```

Rubi [A] time = 0.0187094, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062

$$\frac{5-2x}{9\sqrt{-x^2-4x+5}}$$

Antiderivative was successfully verified.

```
[In]  Int[x/(5 - 4*x - x^2)^(3/2),x]

[Out] (5 - 2*x)/(9*Sqrt[5 - 4*x - x^2])
```

Rubi in Sympy [A] time = 3.75704, size = 17, normalized size = 0.74

$$\frac{-8x+20}{36\sqrt{-x^2-4x+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(x/(-x**2-4*x+5)**(3/2),x)

[Out] (-8*x + 20)/(36*sqrt(-x**2 - 4*x + 5))
```

Mathematica [A] time = 0.0220283, size = 23, normalized size = 1.

$$\frac{5-2x}{9\sqrt{-x^2-4x+5}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[x/(5 - 4*x - x^2)^(3/2),x]

[Out] (5 - 2*x)/(9*Sqrt[5 - 4*x - x^2])
```

Maple [A] time = 0.005, size = 26, normalized size = 1.1

$$\frac{(x+5)(-1+x)(-5+2x)}{9}\left(-x^2-4x+5\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(x/(-x^2-4*x+5)^(3/2),x)

[Out] 1/9*(x+5)*(-1+x)*(-5+2*x)/(-x^2-4*x+5)^(3/2)
```

Maxima [A] time = 0.780488, size = 41, normalized size = 1.78

$$-\frac{2x}{9\sqrt{-x^2-4x+5}}+\frac{5}{9\sqrt{-x^2-4x+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x^2 - 4*x + 5)^(3/2),x, algorithm="maxima")

[Out] -2/9*x/sqrt(-x^2 - 4*x + 5) + 5/9/sqrt(-x^2 - 4*x + 5)
```

Fricas [A] time = 0.220065, size = 39, normalized size = 1.7

$$\frac{\sqrt{-x^2-4x+5}(2x-5)}{9\left(x^2+4x-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x^2 - 4*x + 5)^(3/2),x, algorithm="fricas")

[Out] 1/9*sqrt(-x^2 - 4*x + 5)*(2*x - 5)/(x^2 + 4*x - 5)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\left(-\left(x-1\right)\left(x+5\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x**2-4*x+5)**(3/2),x)
```

```
[Out] Integral(x/(-(x - 1)*(x + 5))**(3/2), x)
```

GIAC/XCAS [A] time = 0.214025, size = 39, normalized size = 1.7

$$\frac{\sqrt{-x^2-4\,x+5}(2\,x-5)}{9\left(x^2+4\,x-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(x/(-x^2 - 4*x + 5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/9*sqrt(-x^2 - 4*x + 5)*(2*x - 5)/(x^2 + 4*x - 5)
```

3.131 $\int \frac{1}{(5-4x-x^2)^{5/2}}\,dx$

Optimal. Leaf size=43

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}}+\frac{x+2}{27\left(-x^2-4x+5\right)^{3/2}}$$

```
[Out] (2 + x)/(27*(5 - 4*x - x^2)^(3/2)) + (2*(2 + x))/(243*Sqrt[5 - 4*
x - x^2])
```

Rubi [A] time = 0.0199797, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143

$$\frac{2(x+2)}{243\sqrt{-x^2-4x+5}}+\frac{x+2}{27\left(-x^2-4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Int[(5 - 4*x - x^2)^(-5/2),x]
```

```
[Out] (2 + x)/(27*(5 - 4*x - x^2)^(3/2)) + (2*(2 + x))/(243*Sqrt[5 - 4*
x - x^2])
```

Rubi in Sympy [A] time = 1.68306, size = 36, normalized size = 0.84

$$\frac{2x+4}{54\left(-x^2-4x+5\right)^{\frac{3}{2}}}+\frac{4x+8}{486\sqrt{-x^2-4x+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate(1/(-x**2-4*x+5)**(5/2),x)
```

```
[Out] (2*x + 4)/(54*(-x**2 - 4*x + 5)**(3/2)) + (4*x + 8)/(486*sqrt(-x*
*2 - 4*x + 5))
```

Mathematica [A] time = 0.0269016, size = 31, normalized size = 0.72

$$-\frac{(x+2)\left(2x^2+8x-19\right)}{243\left(-x^2-4x+5\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(5 - 4*x - x^2)^(-5/2),x]
```

```
[Out] -((2 + x)*(-19 + 8*x + 2*x^2))/(243*(5 - 4*x - x^2)^(3/2))
```

Maple [A] time = 0.004, size = 36, normalized size = 0.8

$$\frac{(x+5)(-1+x)\left(2x^3+12x^2-3x-38\right)}{243}\left(-x^2-4x+5\right)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int(1/(-x^2-4*x+5)^(5/2),x)
```

```
[Out] 1/243*(x+5)*(-1+x)*(2*x^3+12*x^2-3*x-38)/(-x^2-4*x+5)^(5/2)
```

Maxima [A] time = 0.749633, size = 80, normalized size = 1.86

$$\frac{2x}{243\sqrt{-x^2-4x+5}}+\frac{4}{243\sqrt{-x^2-4x+5}}+\frac{x}{27\left(-x^2-4x+5\right)^{\frac{3}{2}}}+\frac{2}{27\left(-x^2-4x+5\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 - 4*x + 5)^(-5/2),x, algorithm="maxima")
```

```
[Out] 2/243*x/sqrt(-x^2 - 4*x + 5) + 4/243/sqrt(-x^2 - 4*x + 5) + 1/27*
x/(-x^2 - 4*x + 5)^(3/2) + 2/27/(-x^2 - 4*x + 5)^(3/2)
```

Fricas [A] time = 0.218417, size = 66, normalized size = 1.53

$$-\frac{\left(2x^3+12x^2-3x-38\right)\sqrt{-x^2-4x+5}}{243\left(x^4+8x^3+6x^2-40x+25\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 - 4*x + 5)^(-5/2),x, algorithm="fricas")
```

```
[Out] -1/243*(2*x^3 + 12*x^2 - 3*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^4 + 8*
x^3 + 6*x^2 - 40*x + 25)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-x^2 - 4x + 5)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate(1/(-x**2-4*x+5)**(5/2),x)
```

```
[Out] Integral((-x**2 - 4*x + 5)**(-5/2), x)
```

GIAC/XCAS [A] time = 0.215895, size = 49, normalized size = 1.14

$$-\frac{((2\,(x+6)x-3)x-38)\sqrt{-x^2-4\,x+5}}{243\,(x^2+4\,x-5)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 - 4*x + 5)^(-5/2),x, algorithm="giac")
```

```
[Out] -1/243*((2*(x + 6)*x - 3)*x - 38)*sqrt(-x^2 - 4*x + 5)/(x^2 + 4*x
 - 5)^2
```

3.132 $\int (a+bx+cx^2)^p \, dx$

Optimal. Leaf size=122

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx+cx^2\right)^{p+1}\,{}_2F_1\left(-p,p+1;p+2;\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

```
[Out] -((2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
))^(-1 - p)*(a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p
, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/
(Sqrt[b^2 - 4*a*c]*(1 + p)))
```

Rubi [A] time = 0.0865864, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{2^{p+1}\left(-\frac{-\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p-1}\left(a+bx+cx^2\right)^{p+1}\,{}_2F_1\left(-p,p+1;p+2;\frac{b+2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{(p+1)\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In]  Int[(a + b*x + c*x^2)^p,x]
```

```
[Out] -((2^(1 + p)*(-((b - Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]
))^(-1 - p)*(a + b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p
, 2 + p, (b + Sqrt[b^2 - 4*a*c] + 2*c*x)/(2*Sqrt[b^2 - 4*a*c])])/
(Sqrt[b^2 - 4*a*c]*(1 + p)))
```

Rubi in Sympy [A] time = 4.81927, size = 105, normalized size = 0.86

$$-\frac{\left(\frac{-\frac{b}{2}-cx+\frac{\sqrt{-4ac+b^2}}{2}}{\sqrt{-4ac+b^2}}\right)^{-p-1}\left(a+bx+cx^2\right)^{p+1}\,{}_2F_1\left(\left.\begin{matrix}-p,p+1\\p+2\end{matrix}\right|\frac{\frac{b}{2}+cx+\frac{\sqrt{-4ac+b^2}}{2}}{\sqrt{-4ac+b^2}}\right)}{(p+1)\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((c*x**2+b*x+a)**p,x)
```

```
[Out] -((-b/2 - c*x + sqrt(-4*a*c + b**2)/2)/sqrt(-4*a*c + b**2))**(-p
- 1)*(a + b*x + c*x**2)**(p + 1)*hyper((-p, p + 1), (p + 2,), (b/
```

```
2 + c*x + sqrt(-4*a*c + b**2)/2)/sqrt(-4*a*c + b**2))/((p + 1)*sq
rt(-4*a*c + b**2))
```

Mathematica [A] time = 0.154597, size = 126, normalized size = 1.03

$$\frac{2^{p-1}\left(-\sqrt{b^2-4ac}+b+2cx\right)\left(\frac{\sqrt{b^2-4ac}+b+2cx}{\sqrt{b^2-4ac}}\right)^{-p}(a+x(b+cx))^p\,{}_2F_1\left(-p,p+1;p+2;\frac{-b-2cx+\sqrt{b^2-4ac}}{2\sqrt{b^2-4ac}}\right)}{c(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(a + b*x + c*x^2)^p,x]
```

```
[Out] (2^(-1 + p)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x)*(a + x*(b + c*x))^p*H
ypergeometric2F1[-p, 1 + p, 2 + p, (-b + Sqrt[b^2 - 4*a*c] - 2*c*
x)/(2*Sqrt[b^2 - 4*a*c])])/(c*(1 + p)*((b + Sqrt[b^2 - 4*a*c] + 2
*c*x)/Sqrt[b^2 - 4*a*c])^p)
```

Maple [F] time = 0.157, size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((c*x^2+b*x+a)^p,x)
```

```
[Out] int((c*x^2+b*x+a)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(cx^2+bx+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^p,x, algorithm="fricas")

[Out] integral((c*x^2 + b*x + a)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+bx+cx^2\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x**2+b*x+a)**p,x)

[Out] Integral((a + b*x + c*x**2)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(cx^2+bx+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((c*x^2 + b*x + a)^p,x, algorithm="giac")

[Out] integrate((c*x^2 + b*x + a)^p, x)
```

3.133 $\int \left(3+4x+5x^2\right)^p \, dx$

Optimal. Leaf size=37

$$5^{-p-1}11^p(5x+2)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{11}(5x+2)^2\right)$$

```
[Out] 5^(-1 - p)*11^p*(2 + 5*x)*Hypergeometric2F1[1/2, -p, 3/2, -(2 + 5
*x)^2/11]
```

Rubi [A] time = 0.0372294, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$5^{-p-1}11^p(5x+2)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{11}(5x+2)^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x + 5*x^2)^p,x]
```

```
[Out] 5^(-1 - p)*11^p*(2 + 5*x)*Hypergeometric2F1[1/2, -p, 3/2, -(2 + 5
*x)^2/11]
```

Rubi in Sympy [A] time = 2.11355, size = 27, normalized size = 0.73

$$\frac{\left(\frac{11}{5}\right)^p (10x+4)\,{}_2F_1\left(\begin{matrix} -p, \frac{1}{2} \\ \frac{3}{2} \end{matrix}\middle| -\frac{(10x+4)^2}{44}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((5*x**2+4*x+3)**p,x)
```

```
[Out] (11/5)**p*(10*x + 4)*hyper((-p, 1/2), (3/2,), -(10*x + 4)**2/44)/
10
```

Mathematica [C] time = 0.110446, size = 93, normalized size = 2.51

$$\frac{11^{p/2}\left(5x-i\sqrt{11}+2\right)\left(-5ix+\sqrt{11}-2i\right)^{-p}\left(10x^2+8x+6\right)^p\ {}_2F_1\left(-p,p+1;p+2;\frac{5ix+\sqrt{11}+2i}{2\sqrt{11}}\right)}{5(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(3 + 4*x + 5*x^2)^p,x]
```

```
[Out] (11^(p/2)*(2 - I*Sqrt[11] + 5*x)*(6 + 8*x + 10*x^2)^p*Hypergeomet
ric2F1[-p, 1 + p, 2 + p, (2*I + Sqrt[11] + (5*I)*x)/(2*Sqrt[11])]
)/(5*(1 + p)*(-2*I + Sqrt[11] - (5*I)*x)^p)
```

Maple [F] time = 0.26, size = 0, normalized size = 0.

$$\int \left(5 x^{2}+4 x +3\right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((5*x^2+4*x+3)^p,x)
```

```
[Out] int((5*x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int { {\left(5 \, x^{2} + 4 \, x + 3\right)}^{p}\,{d x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 4*x + 3)^p,x, algorithm="maxima")
```

```
[Out] integrate((5*x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left({\left(5 \, x^{2} + 4 \, x + 3\right)}^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 4*x + 3)^p,x, algorithm="fricas")
```

```
[Out] integral((5*x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(5x^2 + 4x + 3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x**2+4*x+3)**p,x)
```

```
[Out] Integral((5*x**2 + 4*x + 3)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(5\,x^2 + 4\,x + 3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((5*x^2 + 4*x + 3)^p,x, algorithm="giac")
```

```
[Out] integrate((5*x^2 + 4*x + 3)^p, x)
```

3.134 $\int \left(3+4x+4x^2\right)^p \, dx$

Optimal. Leaf size=32

$$2^{p-1}(2x+1)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{2}(2x+1)^2\right)$$

```
[Out] 2^(-1 + p)*(1 + 2*x)*Hypergeometric2F1[1/2, -p, 3/2, -(1 + 2*x)^2
/2]
```

Rubi [A] time = 0.033132, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$2^{p-1}(2x+1)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{2}(2x+1)^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x + 4*x^2)^p,x]
```

```
[Out] 2^(-1 + p)*(1 + 2*x)*Hypergeometric2F1[1/2, -p, 3/2, -(1 + 2*x)^2
/2]
```

Rubi in Sympy [A] time = 1.91681, size = 26, normalized size = 0.81

$$\frac{2^p\,(8x+4)\,{}_2F_1\left(\begin{matrix}-p,\ \frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\,-\frac{(8x+4)^2}{32}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((4*x**2+4*x+3)**p,x)
```

```
[Out] 2**p*(8*x + 4)*hyper((-p, 1/2), (3/2,), -(8*x + 4)**2/32)/8
```

Mathematica [C] time = 0.145667, size = 94, normalized size = 2.94

$$\frac{2^{\frac{3p}{2}-1}\left(2x-i\sqrt{2}+1\right)\left(-2ix+\sqrt{2}-i\right)^{-p}\left(4x^2+4x+3\right)^p\,{}_2F_1\left(-p,p+1;p+2;\frac{1}{4}\left(2i\sqrt{2}x+i\sqrt{2}+2\right)\right)}{p+1}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(3 + 4*x + 4*x^2)^p,x]
```

```
[Out] (2^(-1 + (3*p)/2)*(1 - I*Sqrt[2] + 2*x)*(3 + 4*x + 4*x^2)^p*Hyper
geometric2F1[-p, 1 + p, 2 + p, (2 + I*Sqrt[2] + (2*I)*Sqrt[2]*x)/
4])/((1 + p)*(-I + Sqrt[2] - (2*I)*x)^p)
```

Maple [F] time = 0.231, size = 0, normalized size = 0.

$$\int \left(4 x^{2}+4 x +3\right)^{p}d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((4*x^2+4*x+3)^p,x)
```

```
[Out] int((4*x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int {\left(4 \, x^{2} + 4 \, x + 3\right)}^{p}\,{d x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 4*x + 3)^p,x, algorithm="maxima")
```

```
[Out] integrate((4*x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$${\rm integral}\left({\left(4 \, x^{2} + 4 \, x + 3\right)}^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 4*x + 3)^p,x, algorithm="fricas")
```

```
[Out] integral((4*x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4x^2 + 4x + 3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x**2+4*x+3)**p,x)
```

```
[Out] Integral((4*x**2 + 4*x + 3)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(4\,x^2 + 4\,x + 3\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x^2 + 4*x + 3)^p,x, algorithm="giac")
```

```
[Out] integrate((4*x^2 + 4*x + 3)^p, x)
```

3.135 $\int \left(3+4x+3x^2\right)^p \, dx$

Optimal. Leaf size=37

$$3^{-p-1}5^p(3x+2)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{5}(3x+2)^2\right)$$

```
[Out] 3^(-1 - p)*5^p*(2 + 3*x)*Hypergeometric2F1[1/2, -p, 3/2, -(2 + 3*
x)^2/5]
```

Rubi [A] time = 0.0359645, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$3^{-p-1}5^p(3x+2)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{1}{5}(3x+2)^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x + 3*x^2)^p,x]
```

```
[Out] 3^(-1 - p)*5^p*(2 + 3*x)*Hypergeometric2F1[1/2, -p, 3/2, -(2 + 3*
x)^2/5]
```

Rubi in Sympy [A] time = 1.98427, size = 27, normalized size = 0.73

$$\frac{\left(\frac{5}{3}\right)^p(6x+4)\,{}_2F_1\left(\begin{matrix}-p,\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|-\frac{(6x+4)^2}{20}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3*x**2+4*x+3)**p,x)
```

```
[Out] (5/3)**p*(6*x + 4)*hyper((-p, 1/2), (3/2,), -(6*x + 4)**2/20)/6
```

Mathematica [C] time = 0.109086, size = 93, normalized size = 2.51

$$\frac{5^{p/2}\left(3x-i\sqrt{5}+2\right)\left(-3ix+\sqrt{5}-2i\right)^{-p}\left(6x^2+8x+6\right)^p\,{}_2F_1\left(-p,p+1;p+2;\frac{3ix+\sqrt{5}+2i}{2\sqrt{5}}\right)}{3(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(3 + 4*x + 3*x^2)^p,x]
```

```
[Out] (5^(p/2)*(2 - I*Sqrt[5] + 3*x)*(6 + 8*x + 6*x^2)^p*Hypergeometric
2F1[-p, 1 + p, 2 + p, (2*I + Sqrt[5] + (3*I)*x)/(2*Sqrt[5])])/(3*
(1 + p)*(-2*I + Sqrt[5] - (3*I)*x)^p)
```

Maple [F] time = 0.184, size = 0, normalized size = 0.

$$\int (3\,x^2+4\,x+3)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3*x^2+4*x+3)^p,x)
```

```
[Out] int((3*x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (3\,x^2+4\,x+3)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 4*x + 3)^p,x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((3\,x^2+4\,x+3)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 4*x + 3)^p,x, algorithm="fricas")
```

```
[Out] integral((3*x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(3x^2+4x+3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x**2+4*x+3)**p,x)
```

```
[Out] Integral((3*x**2 + 4*x + 3)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(3\,x^2+4\,x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3*x^2 + 4*x + 3)^p,x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 4*x + 3)^p, x)
```

3.136 $\int \left(3+4x+2x^2\right)^p\,dx$

Optimal. Leaf size=21

$$(x+1)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-2(x+1)^2\right)$$

```
[Out] (1 + x)*Hypergeometric2F1[1/2, -p, 3/2, -2*(1 + x)^2]
```

Rubi [A] time = 0.0227508, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$(x+1)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-2(x+1)^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x + 2*x^2)^p,x]
```

```
[Out] (1 + x)*Hypergeometric2F1[1/2, -p, 3/2, -2*(1 + x)^2]
```

Rubi in Sympy [A] time = 1.72968, size = 17, normalized size = 0.81

$$(x+1)\,{}_2F_1\left(\begin{matrix}-p,\ \frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\,-2\,(x+1)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((2*x**2+4*x+3)**p,x)
```

```
[Out] (x + 1)*hyper((-p, 1/2), (3/2,), -2*(x + 1)**2)
```

Mathematica [C] time = 0.100943, size = 92, normalized size = 4.38

$$\frac{2^{\frac{3p}{2}-1}\left(2x-i\sqrt{2}+2\right)\left(-2ix+\sqrt{2}-2i\right)^{-p}\left(2x^2+4x+3\right)^p\,{}_2F_1\left(-p,p+1;p+2;\frac{2ix+\sqrt{2}+2i}{2\sqrt{2}}\right)}{p+1}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(3 + 4*x + 2*x^2)^p,x]
```

```
[Out] (2^(-1 + (3*p)/2)*(2 - I*Sqrt[2] + 2*x)*(3 + 4*x + 2*x^2)^p*Hyper
geometric2F1[-p, 1 + p, 2 + p, (2*I + Sqrt[2] + (2*I)*x)/(2*Sqrt[
2])])/((1 + p)*(-2*I + Sqrt[2] - (2*I)*x)^p)
```

Maple [F]   time = 0.228, size = 0, normalized size = 0.

$$\int \left(2 x^2 + 4 x + 3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((2*x^2+4*x+3)^p,x)
```

```
[Out] int((2*x^2+4*x+3)^p,x)
```

Maxima [F]   time = 0., size = 0, normalized size = 0.

$$\int \left(2 x^2 + 4 x + 3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 4*x + 3)^p,x, algorithm="maxima")
```

```
[Out] integrate((2*x^2 + 4*x + 3)^p, x)
```

Fricas [F]   time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(2 x^2 + 4 x + 3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 4*x + 3)^p,x, algorithm="fricas")
```

```
[Out] integral((2*x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(2x^2+4x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x**2+4*x+3)**p,x)

[Out] Integral((2*x**2 + 4*x + 3)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(2\,x^2+4\,x+3\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((2*x^2 + 4*x + 3)^p,x, algorithm="giac")

[Out] integrate((2*x^2 + 4*x + 3)^p, x)
```

3.137 $\int \left(3+4x+x^2\right)^p \, dx$

Optimal. Leaf size=54

$$-\frac{2^{2p+1}(-2x-2)^{-p-1}\left(x^2+4x+3\right)^{p+1}{}_2F_1\left(-p,p+1;p+2;\frac{x+3}{2}\right)}{p+1}$$

```
[Out] -((2^(1 + 2*p)*(-2 - 2*x)^(-1 - p)*(3 + 4*x + x^2)^(1 + p)*Hyperg
eometric2F1[-p, 1 + p, 2 + p, (3 + x)/2])/(1 + p))
```

Rubi [A] time = 0.0280078, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{2^{2p+1}(-2x-2)^{-p-1}\left(x^2+4x+3\right)^{p+1}{}_2F_1\left(-p,p+1;p+2;\frac{x+3}{2}\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x + x^2)^p,x]
```

```
[Out] -((2^(1 + 2*p)*(-2 - 2*x)^(-1 - p)*(3 + 4*x + x^2)^(1 + p)*Hyperg
eometric2F1[-p, 1 + p, 2 + p, (3 + x)/2])/(1 + p))
```

Rubi in Sympy [A] time = 1.93508, size = 44, normalized size = 0.81

$$-\frac{\left(-\frac{x}{2}-\frac{1}{2}\right)^{-p-1}\left(x^2+4x+3\right)^{p+1}{}_2F_1\left(\begin{matrix}-p,p+1\\ p+2\end{matrix}\middle|\frac{x}{2}+\frac{3}{2}\right)}{2\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((x**2+4*x+3)**p,x)
```

```
[Out] -(-x/2 - 1/2)**(-p - 1)*(x**2 + 4*x + 3)**(p + 1)*hyper((-p, p +
1), (p + 2,), x/2 + 3/2)/(2*(p + 1))
```

Mathematica [A] time = 0.0292909, size = 48, normalized size = 0.89

$$\frac{2^p(x+1)(x+3)^{-p}\left(x^2+4x+3\right)^p{}_2F_1\left(-p,p+1;p+2;\frac{1}{2}(-x-1)\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 4*x + x^2)^p,x]
```

```
[Out] (2^p*(1 + x)*(3 + 4*x + x^2)^p*Hypergeometric2F1[-p, 1 + p, 2 + p
, (-1 - x)/2])/((1 + p)*(3 + x)^p)
```

Maple [F] time = 0.068, size = 0, normalized size = 0.

$$\int \left(x^2 + 4x + 3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((x^2+4*x+3)^p,x)
```

```
[Out] int((x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2 + 4x + 3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 4*x + 3)^p,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(x^2 + 4x + 3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 4*x + 3)^p,x, algorithm="fricas")
```

```
[Out] integral((x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2 + 4x + 3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x**2+4*x+3)**p,x)
```

```
[Out] Integral((x**2 + 4*x + 3)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(x^2 + 4\,x + 3\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((x^2 + 4*x + 3)^p,x, algorithm="giac")
```

```
[Out] integrate((x^2 + 4*x + 3)^p, x)
```

3.138 $\int (3+4x)^p\,dx$

Optimal. Leaf size=18

$$\frac{(4x+3)^{p+1}}{4(p+1)}$$

```
[Out] (3 + 4*x)^(1 + p)/(4*(1 + p))
```

Rubi [A] time = 0.00803477, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{(4x+3)^{p+1}}{4(p+1)}$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x)^p,x]

[Out] (3 + 4*x)^(1 + p)/(4*(1 + p))
```

Rubi in Sympy [A] time = 1.48413, size = 12, normalized size = 0.67

$$\frac{(4x+3)^{p+1}}{4\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((3+4*x)**p,x)

[Out] (4*x + 3)**(p + 1)/(4*(p + 1))
```

Mathematica [A] time = 0.00842931, size = 17, normalized size = 0.94

$$\frac{(4x+3)^{p+1}}{4p+4}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 4*x)^p,x]

[Out] (3 + 4*x)^(1 + p)/(4 + 4*p)
```

Maple [A] time = 0.003, size = 17, normalized size = 0.9

$$\frac{(3+4x)^{1+p}}{4p+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((3+4*x)^p,x)

[Out] 1/4*(3+4*x)^(1+p)/(1+p)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x + 3)^p,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 0.235928, size = 26, normalized size = 1.44

$$\frac{(4x+3)^p(4x+3)}{4(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x + 3)^p,x, algorithm="fricas")

[Out] 1/4*(4*x + 3)^p*(4*x + 3)/(p + 1)
```

Sympy [A] time = 0.073729, size = 20, normalized size = 1.11

$$\frac{\begin{cases}\frac{(4x+3)^{p+1}}{p+1} & \text{for } p \neq -1\\ \log(4x+3) & \text{otherwise}\end{cases}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((3+4*x)**p,x)
```

```
[Out] Piecewise(((4*x + 3)**(p + 1)/(p + 1), Ne(p, -1)), (log(4*x + 3),
 True))/4
```

GIAC/XCAS [A] time = 0.206089, size = 22, normalized size = 1.22

$$\frac{(4\,x+3)^{p+1}}{4\,(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((4*x + 3)^p,x, algorithm="giac")
```

```
[Out] 1/4*(4*x + 3)^(p + 1)/(p + 1)
```

3.139 $\int \left(3+4x-x^2\right)^p\,dx$

Optimal. Leaf size=31

$$-7^p(2-x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{7}(2-x)^2\right)$$

```
[Out] -(7^p*(2 - x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - x)^2/7])
```

Rubi [A] time = 0.0279102, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-7^p(2-x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{7}(2-x)^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x - x^2)^p,x]
```

```
[Out] -(7^p*(2 - x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - x)^2/7])
```

Rubi in Sympy [A] time = 1.95643, size = 26, normalized size = 0.84

$$-\frac{7^p\,(-2x+4)\,{}_2F_1\left(\begin{matrix}-p,\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{(-2x+4)^2}{28}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-x**2+4*x+3)**p,x)
```

```
[Out] -7**p*(-2*x + 4)*hyper((-p, 1/2), (3/2,), (-2*x + 4)**2/28)/2
```

Mathematica [B] time = 0.0801263, size = 83, normalized size = 2.68

$$-\frac{\left(-x+\sqrt{7}+2\right)\left(-x^2+4x+3\right)^p\left(\frac{x-\sqrt{7}-2}{2\sqrt{7}}+1\right)^{-p}\,{}_2F_1\left(-p,p+1;p+2;-\frac{x-\sqrt{7}-2}{2\sqrt{7}}\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 4*x - x^2)^p,x]
```

```
[Out] -(((2 + Sqrt[7] - x)*(3 + 4*x - x^2)^p*Hypergeometric2F1[-p, 1 +
p, 2 + p, -(-2 - Sqrt[7] + x)/(2*Sqrt[7])])/((1 + p)*(1 + (-2 - S
qrt[7] + x)/(2*Sqrt[7]))^p))
```

Maple [F] time = 0.151, size = 0, normalized size = 0.

$$\int \left(-x^2+4x+3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-x^2+4*x+3)^p,x)
```

```
[Out] int((-x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^2+4x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 4*x + 3)^p,x, algorithm="maxima")
```

```
[Out] integrate((-x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-x^2+4x+3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 4*x + 3)^p,x, algorithm="fricas")
```

```
[Out] integral((-x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^2 + 4x + 3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x**2+4*x+3)**p,x)
```

```
[Out] Integral((-x**2 + 4*x + 3)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-x^2 + 4\,x + 3\right)^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-x^2 + 4*x + 3)^p,x, algorithm="giac")
```

```
[Out] integrate((-x^2 + 4*x + 3)^p, x)
```

3.140 $\int \left(3+4x-2x^2\right)^p\,dx$

Optimal. Leaf size=31

$$-5^p(1-x)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{2}{5}(1-x)^2\right)$$

```
[Out] -(5^p*(1 - x)*Hypergeometric2F1[1/2, -p, 3/2, (2*(1 - x)^2)/5])
```

Rubi [A] time = 0.0264776, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-5^p(1-x)\,_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{2}{5}(1-x)^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x - 2*x^2)^p,x]

[Out] -(5^p*(1 - x)*Hypergeometric2F1[1/2, -p, 3/2, (2*(1 - x)^2)/5])
```

Rubi in Sympy [A] time = 2.03983, size = 26, normalized size = 0.84

$$-\frac{5^p\left(-4x+4\right)\,_2F_1\left(\begin{matrix}-p,\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{(-4x+4)^2}{40}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-2*x**2+4*x+3)**p,x)

[Out] -5**p*(-4*x + 4)*hyper((-p, 1/2), (3/2,), (-4*x + 4)**2/40)/4
```

Mathematica [B] time = 0.172804, size = 86, normalized size = 2.77

$$-\frac{2^{\frac{3p}{2}-1}5^{p/2}\left(-2x+\sqrt{10}+2\right)\left(2x+\sqrt{10}-2\right)^{-p}\left(-2x^2+4x+3\right)^p\,_2F_1\left(-p,p+1;p+2;-\frac{x}{\sqrt{10}}+\frac{1}{\sqrt{10}}+\frac{1}{2}\right)}{p+1}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(3 + 4*x - 2*x^2)^p,x]
```

```
[Out] -((2^(-1 + (3*p)/2)*5^(p/2)*(2 + Sqrt[10] - 2*x)*(3 + 4*x - 2*x^2
)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, 1/2 + 1/Sqrt[10] - x/Sqrt
[10]])/((1 + p)*(-2 + Sqrt[10] + 2*x)^p))
```

Maple [F] time = 0.154, size = 0, normalized size = 0.

$$\int \left(-2\,x^{2}+4\,x+3\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-2*x^2+4*x+3)^p,x)
```

```
[Out] int((-2*x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-2\,x^{2}+4\,x+3\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x^2 + 4*x + 3)^p,x, algorithm="maxima")
```

```
[Out] integrate((-2*x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-2\,x^{2}+4\,x+3\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x^2 + 4*x + 3)^p,x, algorithm="fricas")
```

```
[Out] integral((-2*x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-2x^2+4x+3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x**2+4*x+3)**p,x)
```

```
[Out] Integral((-2*x**2 + 4*x + 3)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-2\,x^2+4\,x+3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-2*x^2 + 4*x + 3)^p,x, algorithm="giac")
```

```
[Out] integrate((-2*x^2 + 4*x + 3)^p, x)
```

3.141 $\int \left(3+4x-3x^2\right)^p \, dx$

Optimal. Leaf size=38

$$-3^{-p-1}13^p(2-3x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{13}(2-3x)^2\right)$$

```
[Out] -(3^(-1 - p)*13^p*(2 - 3*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 -
3*x)^2/13])
```

Rubi [A] time = 0.0317743, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-3^{-p-1}13^p(2-3x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{13}(2-3x)^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x - 3*x^2)^p,x]
```

```
[Out] -(3^(-1 - p)*13^p*(2 - 3*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 -
3*x)^2/13])
```

Rubi in Sympy [A] time = 2.06834, size = 27, normalized size = 0.71

$$-\frac{\left(\frac{13}{3}\right)^p(-6x+4)\,{}_2F_1\left(\begin{matrix}-p,\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{(-6x+4)^2}{52}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-3*x**2+4*x+3)**p,x)
```

```
[Out] -(13/3)**p*(-6*x + 4)*hyper((-p, 1/2), (3/2,), (-6*x + 4)**2/52)/
6
```

Mathematica [B] time = 0.112104, size = 81, normalized size = 2.13

$$-\frac{13^{p/2}\left(-3x+\sqrt{13}+2\right)\left(3x+\sqrt{13}-2\right)^{-p}\left(-6x^2+8x+6\right)^p\,{}_2F_1\left(-p,p+1;p+2;\frac{-3x+\sqrt{13}+2}{2\sqrt{13}}\right)}{3(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(3 + 4*x - 3*x^2)^p,x]
```

```
[Out] -(13^(p/2)*(2 + Sqrt[13] - 3*x)*(6 + 8*x - 6*x^2)^p*Hypergeometri
c2F1[-p, 1 + p, 2 + p, (2 + Sqrt[13] - 3*x)/(2*Sqrt[13])])/(3*(1
+ p)*(-2 + Sqrt[13] + 3*x)^p)
```

Maple [F] time = 0.157, size = 0, normalized size = 0.

$$\int \left(-3\,x^{2}+4\,x+3\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-3*x^2+4*x+3)^p,x)
```

```
[Out] int((-3*x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-3\,x^{2}+4\,x+3\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x^2 + 4*x + 3)^p,x, algorithm="maxima")
```

```
[Out] integrate((-3*x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-3\,x^{2}+4\,x+3\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x^2 + 4*x + 3)^p,x, algorithm="fricas")
```

```
[Out] integral((-3*x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-3x^2 + 4x + 3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x**2+4*x+3)**p,x)
```

```
[Out] Integral((-3*x**2 + 4*x + 3)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-3\,x^2 + 4\,x + 3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-3*x^2 + 4*x + 3)^p,x, algorithm="giac")
```

```
[Out] integrate((-3*x^2 + 4*x + 3)^p, x)
```

3.142 $\int \left(3+4x-4x^2\right)^p\,dx$

Optimal. Leaf size=35

$$-2^{2p-1}(1-2x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{4}(1-2x)^2\right)$$

```
[Out] -(2^(-1 + 2*p)*(1 - 2*x)*Hypergeometric2F1[1/2, -p, 3/2, (1 - 2*x
)^2/4])
```

Rubi [A] time = 0.0327874, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-2^{2p-1}(1-2x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{4}(1-2x)^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x - 4*x^2)^p,x]
```

```
[Out] -(2^(-1 + 2*p)*(1 - 2*x)*Hypergeometric2F1[1/2, -p, 3/2, (1 - 2*x
)^2/4])
```

Rubi in Sympy [A] time = 1.95474, size = 26, normalized size = 0.74

$$-\frac{4^p\,(-8x+4)\,{}_2F_1\left(\begin{matrix}-p,\frac{1}{2}\\\frac{3}{2}\end{matrix}\middle|\frac{(-8x+4)^2}{64}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-4*x**2+4*x+3)**p,x)
```

```
[Out] -4**p*(-8*x + 4)*hyper((-p, 1/2), (3/2,), (-8*x + 4)**2/64)/8
```

Mathematica [A] time = 0.0360669, size = 58, normalized size = 1.66

$$\frac{2^{2p-1}(2x-3)(2x+1)^{-p}\left(-4x^2+4x+3\right)^p\ {}_2F_1\left(-p,p+1;p+2;\frac{3}{4}-\frac{x}{2}\right)}{p+1}$$

Antiderivative was successfully verified.

```
[In]  Integrate[(3 + 4*x - 4*x^2)^p,x]
```

```
[Out] (2^(-1 + 2*p)*(-3 + 2*x)*(3 + 4*x - 4*x^2)^p*Hypergeometric2F1[-p
, 1 + p, 2 + p, 3/4 - x/2])/((1 + p)*(1 + 2*x)^p)
```

Maple [F] time = 0.096, size = 0, normalized size = 0.

$$\int \left(-4 x^2 + 4 x + 3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-4*x^2+4*x+3)^p,x)
```

```
[Out] int((-4*x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-4 x^2 + 4 x + 3\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 4*x + 3)^p,x, algorithm="maxima")
```

```
[Out] integrate((-4*x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-4 x^2 + 4 x + 3\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 4*x + 3)^p,x, algorithm="fricas")
```

```
[Out] integral((-4*x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-4x^2+4x+3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x**2+4*x+3)**p,x)

[Out] Integral((-4*x**2 + 4*x + 3)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-4\,x^2+4\,x+3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-4*x^2 + 4*x + 3)^p,x, algorithm="giac")

[Out] integrate((-4*x^2 + 4*x + 3)^p, x)
```

3.143 $\int \left(3+4x-5x^2\right)^p\,dx$

Optimal. Leaf size=38

$$-5^{-p-1}19^p(2-5x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{19}(2-5x)^2\right)$$

```
[Out] -(5^(-1 - p)*19^p*(2 - 5*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 -
5*x)^2/19])
```

Rubi [A] time = 0.0304637, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167

$$-5^{-p-1}19^p(2-5x)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{1}{19}(2-5x)^2\right)$$

Antiderivative was successfully verified.

```
[In]  Int[(3 + 4*x - 5*x^2)^p,x]
```

```
[Out] -(5^(-1 - p)*19^p*(2 - 5*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 -
5*x)^2/19])
```

Rubi in Sympy [A] time = 2.10219, size = 27, normalized size = 0.71

$$-\frac{\left(\frac{19}{5}\right)^p(-10x+4)\,{}_2F_1\left(\begin{matrix}-p,\frac{1}{2}\\ \frac{3}{2}\end{matrix}\middle|\frac{(-10x+4)^2}{76}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  rubi_integrate((-5*x**2+4*x+3)**p,x)
```

```
[Out] -(19/5)**p*(-10*x + 4)*hyper((-p, 1/2), (3/2,), (-10*x + 4)**2/76
)/10
```

Mathematica [B] time = 0.113515, size = 81, normalized size = 2.13

$$-\frac{19^{p/2}\left(-5x+\sqrt{19}+2\right)\left(5x+\sqrt{19}-2\right)^{-p}\left(-10x^2+8x+6\right)^p\,{}_2F_1\left(-p,p+1;p+2;\frac{-5x+\sqrt{19}+2}{2\sqrt{19}}\right)}{5(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In]  Integrate[(3 + 4*x - 5*x^2)^p,x]
```

```
[Out] -(19^(p/2)*(2 + Sqrt[19] - 5*x)*(6 + 8*x - 10*x^2)^p*Hypergeometr
ic2F1[-p, 1 + p, 2 + p, (2 + Sqrt[19] - 5*x)/(2*Sqrt[19])])/(5*(1
 + p)*(-2 + Sqrt[19] + 5*x)^p)
```

Maple [F] time = 0.158, size = 0, normalized size = 0.

$$\int \left(-5 x^{2}+4 x+3\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  int((-5*x^2+4*x+3)^p,x)
```

```
[Out] int((-5*x^2+4*x+3)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-5 x^{2}+4 x+3\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-5*x^2 + 4*x + 3)^p,x, algorithm="maxima")
```

```
[Out] integrate((-5*x^2 + 4*x + 3)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-5 x^{2}+4 x+3\right)^{p}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-5*x^2 + 4*x + 3)^p,x, algorithm="fricas")
```

```
[Out] integral((-5*x^2 + 4*x + 3)^p, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-5x^2 + 4x + 3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-5*x**2+4*x+3)**p,x)
```

```
[Out] Integral((-5*x**2 + 4*x + 3)**p, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-5\,x^2 + 4\,x + 3\right)^p \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In]  integrate((-5*x^2 + 4*x + 3)^p,x, algorithm="giac")
```

```
[Out] integrate((-5*x^2 + 4*x + 3)^p, x)
```

4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```

```
ExpnType[expn_] :=
  If[AtomQ[expn], 1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]
HypergeometricFunctionQ[func_] := MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
AppellFunctionQ[func_] := MemberQ[{AppellF1},func]
```

```
# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);
  #This check below actually is not needed, since I only call this grading only for
  #passed integrals. i.e. I check for "F" before calling this.

  if not type(result,freeof('int')) then
     return "F";
  end if;

  if ExpnType_result<=ExpnType_optimal then
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
         if leaf_count_result<=2*leaf_count_optimal then
            return "A";
         else
            return "B";
         end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      return "C";
  end if
end proc:
```

```
# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
    max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
      max(2,ExpnType(op(1,expn))) end if else
    max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:
```

```
ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
  apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```